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Darcy velocity and Péclet number analysis from underground thermal data

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ABSTRACT We propose a geophysical technique for the analysis of underground thermal data, which can help in the study of shallow aquifers with low velocity flows due to hydraulic gradient. The Darcy velocity and quantitative information on the dominant process of heat transport (Péclet number) can be extracted from thermal logs in boreholes. Examples of application are given for a set of field data, recorded within porosity permeable horizons affected by groundwater flow. Hydrothermal parameters are determined by matching temperature and thermal gradient data with analytical models incorporating both heat and water transfer. As isotherms and equipotential lines are neither purely horizontal nor purely vertical, the technique, based on constant heat and water flow, also assumes that the temperature varies linearly along the horizontal.

1. Introduction

Several strategies have been developed to explore the circulation of geofluids, which can yield heat transport over large spatial scales (see e.g., Jessop, 1987; Haenel *et al.*, 1988; Anderson, 2005). Permeability of igneous and metamorphic rocks is generally negligible and only secondary permeability from tectonic fracturing may again allow re-infiltration. Except for halite and other evaporites, no sedimentary rock is totally impermeable, and there may be leakage between aquifers over wide areas if the hydraulic head is sufficient. Mixing and dispersion effects occur on the granular scale, but they can only be perceived on the scale of formation thickness.

Any aquifer formation is thus an extremely complex system of interconnected channels, whose analysis, in terms of thermal effects, should be carried out on a wide variety of scales. Along the vertical, transport of heat takes place on a scale of bed thickness. The scale of lateral transport is many times larger, and it is of the order of the formation extent (Jessop, 1990; Pfister and Rybach, 1996; Swanson and Bahr, 2004). Data interpretation or prediction of reservoir behaviour can be done only on the basis of average properties and simplified models. For this reason, it is difficult to achieve accurate, quantitative descriptions of the analysed aquifer system, and a qualitative understanding of the processes involved is often the best that can be obtained.

This paper presents some practical thermal methods for inferring water transfer in shallow, low porosity, permeable horizons. Examples of application are then given for some thermal logs. We show that underground temperature and thermal gradient data can give quantitative information about the water and heat flow.

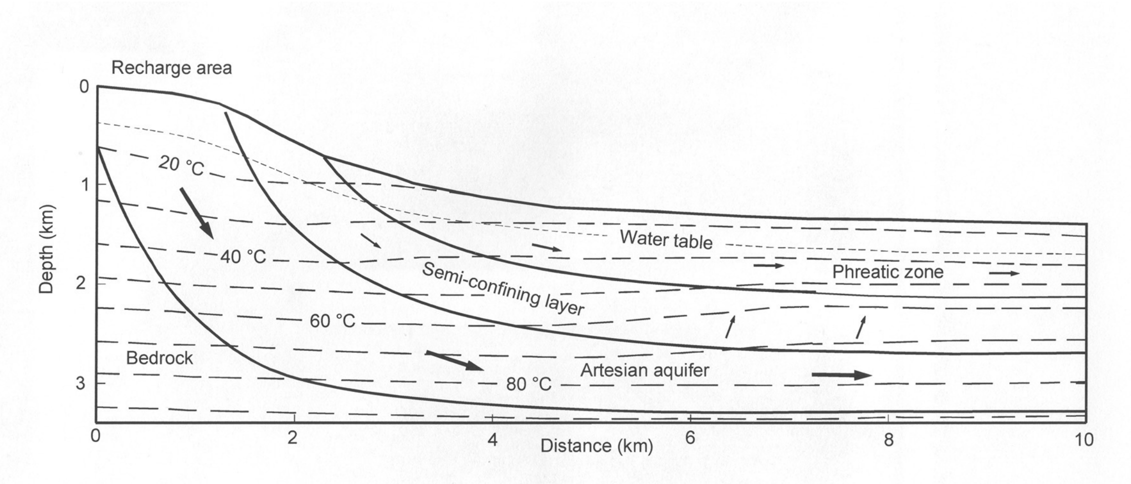

Fig. 1 - Conceptual model of an advectively-disturbed thermal regime of a typical aquifer system. Notice that the isotherms are not always horizontal. Equipotential lines are perpendicular to water velocity which is indicated by arrows.

2. Analytical models

2.1. Vertical flow

Groundwater flow from recharge areas, where precipitation seeps downwards beneath the ground surface and reaches the saturated zone, to discharge areas, where subsurface water is discharged to streams, lakes, ponds or swamps, forms an additional mechanism of heat transfer to pure conduction, which is generally assumed as the underground thermal regime (Fig. 1). If thermal gradient in the horizontal direction is negligible, the differential equation for combined conductive and groundwater advective heat transfer is (Stallman, 1963, 1965; Bredehoeft and Papadopulos, 1965)

$$\frac{d^2T}{dz^2} - \frac{c_w \rho_w u_z}{\kappa}\frac{dT}{dz} = 0 \tag{1}$$

where c_w and ρ_w are the water specific heat and density, respectively, u_z is the Darcy velocity in the vertical direction z (positive downwards), κ is the bulk thermal conductivity and T is temperature.

The solution of Eq. (1) is

$$T = T_1 + \left(T_2 - T_1\right)\frac{\exp\left(\beta_z\, z/h\right) - 1}{\exp\beta_z - 1} \tag{2}$$

where β_z $(= c_w\rho_w u_z h/\kappa)$ is a dimensionless parameter, T_1 and T_2 are the temperatures at the top and the bottom of the investigated depth range h of the aquifer. The quantity β_z is positive or negative depending on whether u_z is downwards or upwards.

Eq. (2) is valid for steady-state thermal conditions and for uniform, isotropic, homogeneous, and saturated porous media. The flow rate is assumed to be constant and sufficiently small to maintain thermal equilibrium between the water and the rock matrix. The thermal field is influenced only by the flow of water parallel to the geothermal gradient, whereas horizontal water flow has no effect.

2.2. Horizontal and vertical flow

Since most layers are sloping and since surface topographic relief usually exists across the aquifer, heat and water flow, particularly in semi-confining layers, is neither purely horizontal nor purely vertical (Fig. 1). In this case, Lu and Ge (1996) demonstrated that the solution is an extension of Eq. (1). Assuming a linear variation of temperature, in the left-hand member of the equation we must add the term $-\Gamma_o\,(\beta_o/h)$ that accounts for the constant horizontal flow of heat and fluid. Therefore we have

$$\frac{d^2T}{dz^2}-\frac{\beta_z}{h}\frac{dT}{dz}-\frac{\beta_o}{h}\overline{\Gamma}_o=0 \tag{3}$$

where $\beta_o = c_w\,\rho_w\,u_o\,h/\kappa$, Γ_o and u_o are the horizontal components of the thermal gradient and the Darcy velocity, respectively. In this case, the temperature as a function of depth is given by

$$T=T_1+\left(T_2-T_1\right)\left[\frac{\exp\left(\beta_z\,z/h\right)-1}{\exp\beta_z-1}+\frac{\Gamma_o\,u_o}{\Gamma_z\,u_z}\left(\frac{\exp\left(\beta_z\,z/h\right)-1}{\exp\beta_z-1}-\frac{z}{h}\right)\right] \tag{4}$$

where Γ_z is the vertical thermal gradient. In the absence of horizontal heat or water flow ($\Gamma_o = 0$ or $u_o = 0$), Eq. (4) reduces to Eq. (2).

Reiter (2001) suggested that the analysis of the thermal effect of groundwater can be practicable by comparing the thermal gradient Γ_z with temperature and depth. Integrating Eq. (3) once yields

$$\overline{\Gamma}_z=\frac{\beta_z T}{h}+\frac{\beta_o\overline{\Gamma}_o\,z}{h}+C \tag{5}$$

where C is the integration constant. Eq. (5) can be seen as a plane whose slopes to the axes of temperature and depth contain information on the vertical and horizontal components of the Darcy velocity, respectively.

2.3. Péclet number

The quantities β_z and β_o may be taken as a measure of the relative efficiency of a porous

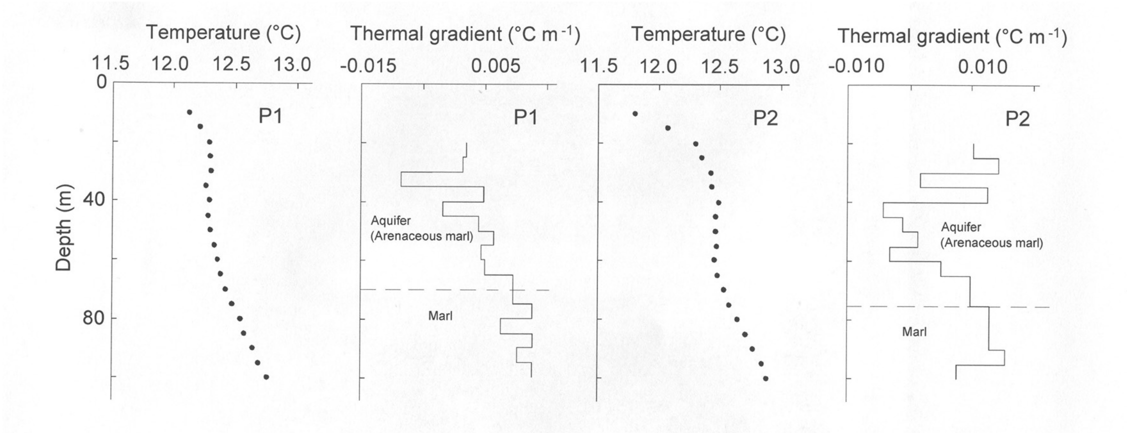

Fig. 2 - Thermal log, thermal gradient and stratigraphy of boreholes P1 and P2.

horizon for the simultaneous transport of heat by groundwater flow and pure conduction. They appear to be analogues of the thermal Péclet number, which quantifies the potential for advection to perturb the temperature-depth distribution. The vertical Péclet number Pe_z can be expressed as the ratio of the advected heat flux q_{ad} and the conducted heat flux q_c over a characteristic length L

$$Pe_z = \frac{q_{ad}}{q_c} = \frac{\rho_w c_w |u_z| (T_2 - T_1)}{\kappa (T_2 - T_1)/L} = \frac{\rho_w c_w |u_z| L}{\kappa}. \tag{6}$$

When $Pe_z >> 1$ vertical advection dominates, while the conductive component prevails for

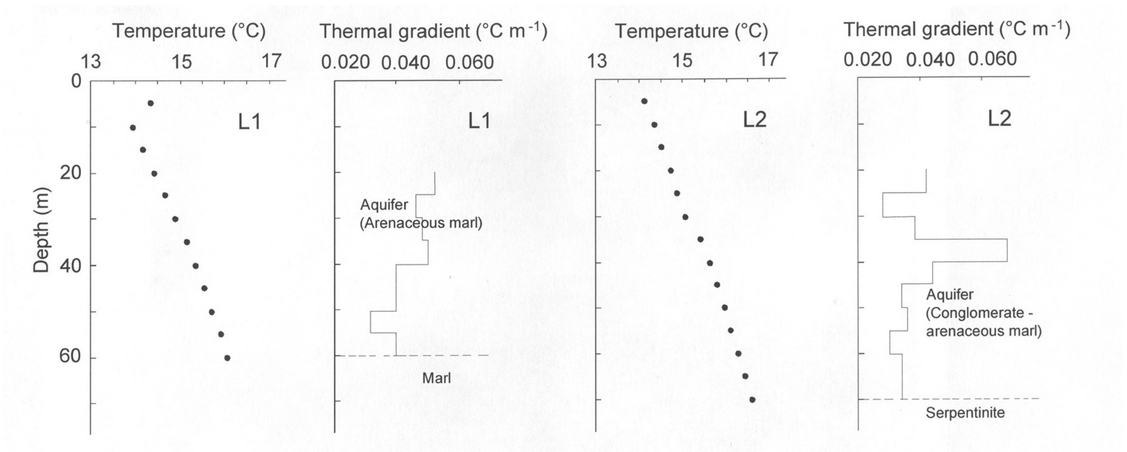

Fig. 3 - Thermal log, thermal gradient and stratigraphy of boreholes L1 and L2.

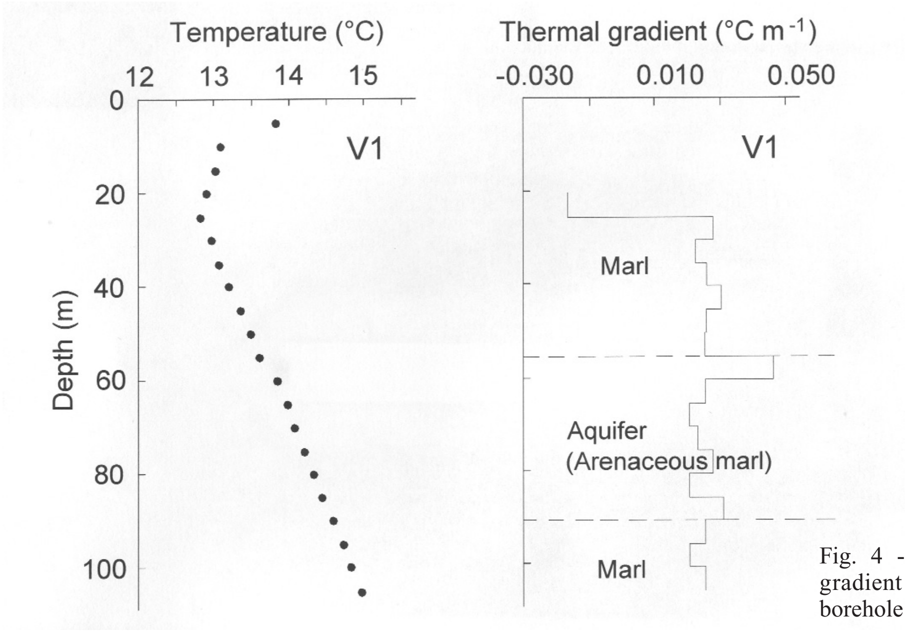

Fig. 4 - Thermal log, thermal gradient and stratigraphy of borehole V1.

Pe_z << 1. The length L can be chosen in many ways, often selected on the basis of the scale of the flow system (see e.g., Domenico and Palciauskas, 1973). For our problem, it was assumed to correspond to the thickness of the aquifer. Substituting u_o to u_z into Eq. (6), one derives also the horizontal Péclet number Pe_o.

3. Analysis of borehole data

The foregoing analytical solutions for heat and groundwater flow have been applied to five thermal logs recorded in boreholes located in northwestern Italy, three in Piedmont (P1, P2 and V1) and two in Liguria (L1 and L2). The boreholes were drilled for geothermal exploration. The thermal logs, carried out by means of a thermo-resistance equipment having uncertainty less that 0.03 °C, are shown in Figs. 2 to 4. Information is completed by a set of thermal conductivity measurements carried out on core samples recovered during drillings.

Previous geothermal studies by Pasquale *et al*. (2000) and Verdoya *et al*. (2007) on the areas where the boreholes are located showed that the underground thermal regime contains a discernible climatic signal, explainable with an increase of ground surface temperature over the past few decades. This has caused a positive shift in the temperature–depth data that is maximum (0.3 °C) at 25 m and decreases with depth, becoming nearly negligible at 50 m depth. Thus, temperature data were preliminarily treated for such a climatic noise, according to suggestions from earlier studies.

Boreholes P1 and P2 (100 m depth) and V1 (105 m depth) show some stratigraphic homogeneity. They crossed marly sedimentary successions with embedded thin arenaceous layers belonging to the so-called Tertiary Piedmont Basin. P1 and P2 are very close to each other (about

Table 1 – Darcy velocity, Péclet number and statistics of the models used to fit thermal data. The summed square of residual (SSE) and the root mean squared error (RMSE) are given in °C for model A and B, and in °C m^{-1} for model C.

Borehole code (Location)	Depth range (m)	Model	Darcy velocity (m s^{-1})		Péclet number		*SSE*	*RMSE*
			vert.	horiz.	vert.	horiz.		
P1 (Ponti, AL)	25 – 70	A	$4.12\ 10^{-8}$	–	3.9	–	0.0048	0.0262
		B	$0.21\ 10^{-8}$	$-1.89\ 10^{-8}$	0.2	1.8	0.0011	0.0135
		C	$1.98\ 10^{-8}$	$1.47\ 10^{-8}$	1.9	1.4	0.0001	0.0038
P2 (Ponti, AL)	25 – 75	A	$0.05\ 10^{-8}$	–	0.1	–	0.0068	0.0292
		B	$1.42\ 10^{-7}$	$-1.18\ 10^{-7}$	14.9	12.3	0.0055	0.0280
		C	$0.20\ 10^{-7}$	$0.14\ 10^{-7}$	2.1	1.5	0.0001	0.0035
V1 (Verduno, CN)	50 – 90	A	$0.02\ 10^{-9}$	–	0.0	–	0.0052	0.0294
		B	$0.80\ 10^{-9}$	$-0.40\ 10^{-7}$	0.1	3.4	0.0040	0.0283
		C	$0.69\ 10^{-7}$	$-0.57\ 10^{-6}$	5.8	47.7	0.0001	0.0041
L1 (Lerca, GE)	25 – 60	A	$-0.87\ 10^{-8}$	–	0.6	–	0.0008	0.0126
		B					0.0137	0.0585
		C	$-0.08\ 10^{-12}$	$-0.15\ 10^{-5}$	0.0	104.6	0.0001	0.0045
L2 (Lerca, GE)	25 – 70	A	$-0.87\ 10^{-8}$	–	0.8	–	0.0057	0.0285
		B					0.0435	0.0851
		C	$-0.32\ 10^{-8}$	$-1.16\ 10^{-8}$	0.3	1.0	0.0011	0.0125

50 m), and their thermal logs are characterized by an evident upward concave profile, which is maintained even after climatic correction and may denote both horizontal flow of cold water and downward leakage. Such a groundwater regime is likely as the two boreholes are located in the mid-upper part of a gently dipping hill. Groundwater was encountered only to about 70 m during drilling in P1 and to 75 m depth in P2. No particular distortion is instead visible in the thermal log of V1, for which a weak groundwater flow was observed at a depth of 55-90 m. However, this hole lies in a morphological situation similar to P1 and P2. Thus, it is likely that also the groundwater regime is similar.

The two wells L1 and L2 reached depths of 60 and 70 m, respectively, and present some lithological variability. Borehole L1 crossed Oligo-Pliocene thin layers of arenaceous marls, terminating with impermeable marls at the hole bottom. L2 penetrated mostly conglomerates with embedded thin layers of arenaceous marls, characterized by moderate presence of groundwater, and finally encountered the impermeable crystalline bedrock (serpentinites) at 70 m depth. L2 is located on a hillside at about 1 km from L1, which lies close to the valley floor.

Both thermal logs denote slightly downward concave profiles, which may denote both upward and horizontal flow of relatively warmer water along the entire borehole section.

The analysis procedure consisted in matching temperature and thermal gradient data with theoretical curves (models A, B and C) obtained from Eqs. (2), (4) and (5). Model A derives from Eq. (2), which can be re-arranged in the simplified form

$$T(z) = a_1 + b_1 \exp (c_1 z) \tag{7}$$

where $c_1 = u_z \rho_w c_w / \kappa$. Models B and C are obtained by rewriting Eqs. (4) and (5), respectively, as

$$T(z) = a_2 + b_2 \exp (c_2 z) - d_2 z \tag{8}$$

$$\bar{\Gamma}_Z (z, T) = a_3 + b_3 z + c_3 T \tag{9}$$

with $c_2 = u_z \rho_w c_w / \kappa$, $d_2 = u_o \Gamma_o / u_z$, $b_3 = c_w \rho_w u_o \Gamma_o / \kappa$ and $c_3 = c_w \rho_w u_z / \kappa$.

The model coefficients c_1, c_2 and c_3 thus contain information about u_z, whereas b_3 and d_2 give u_o, provided that the water volumetric heat capacity ($\rho_w c_w$), the average bulk thermal conductivity κ and the horizontal thermal gradient are known for the depth interval of length h over which measures were carried out. A least-square fitting procedure can be used to determine model coefficients [see Verdoya *et al.* (2008) for details on the analysis technique]. After velocities are determined and the characteristic length L is fixed, the Péclet number can be estimated.

Table 1 shows the values of Darcy velocity, vertical and horizontal components, and Péclet number, as inferred from the model coefficients. The density and specific heat of water were respectively assumed to be 1000 kg m^{-3} and 4186 J kg^{-1} K^{-1}. The determination of u_o requires the horizontal thermal gradient to be known. From the thermal logs of the boreholes P1 and P2, the horizontal thermal gradient is estimated to vary between $1.3 \cdot 10^{-3}$ and $5.3 \cdot 10^{-3}$ °C m^{-1}. Thus for these two wells as well as for the well V1, all belonging to the same geological unit, a representative average value of $3.3 \cdot 10^{-3}$ °C m^{-1} was taken for calculations of the horizontal component of the Darcy velocity. The data of the pair of wells L1 and L2 show, instead, an average horizontal thermal gradient of $0.2 \pm 0.1 \cdot 10^{-3}$ °C m^{-1}.

The thermal conductivity, measured on core specimens of arenaceous marl of the wells P1, P2 and V1 is on average 2.0 W m^{-1} K^{-1}, whereas for the arenaceous marl of the wells L1 and L2 it is 2.1 W m^{-1} K^{-1}. The conglomerate thermal conductivity was assumed to be 2.2 W $m^{-1}K^{-1}$ as a result of the average between the value of 2.1 W $m^{-1}K^{-1}$ of the matrix (arenaceous marl, 50% of whole rock) and of 2.3 W $m^{-1}K^{-1}$ of the clasts (serpentinite).

The goodness of fit of the three models can be evaluated from the summed square of the residual (labelled as *SSE*) and the root mean squared error (*RMSE*), all shown in Table 1. The latter is a measure of the variation of observed values around the calculated values. For both *SSE* and *RMSE* values closer to zero indicate a better fit. Figs. 5 and 6 show, as an example, the fitting curves for the models A, B and C superimposed on thermal data of the hole P1, together with a plot of the differences between the values obtained with the models and those measured in the hole. Calculations were carried out only for the sections of the holes affected by water circulation.

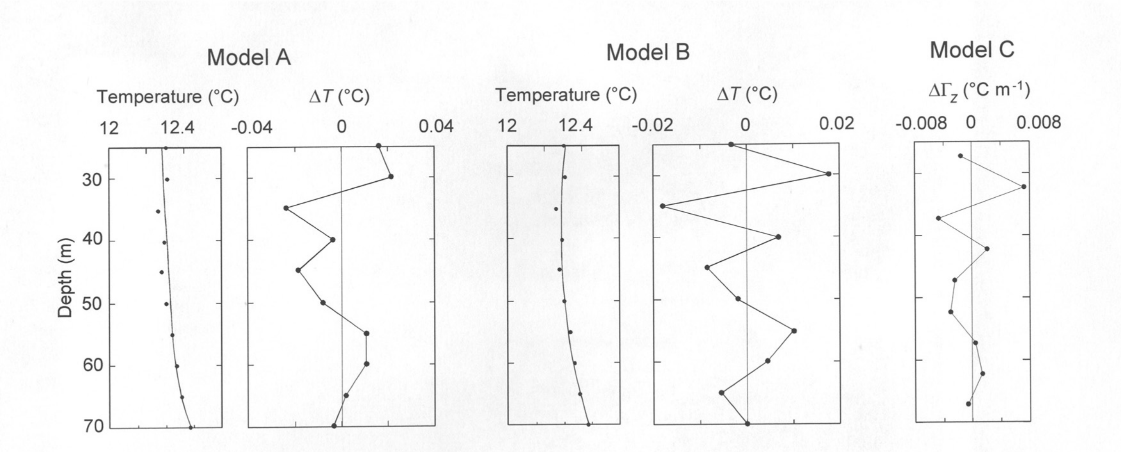

Fig. 5 - Curve fit to observed temperatures and difference between observed and modelled temperature ΔT (models A and B), and difference between observed and modelled thermal gradient $\Delta\Gamma_z$ (model C) for the borehole P1 (see Table 1).

4. Discussion

Our approach assumes that thermal parameters are constant along the section of the borehole where groundwater movement occurs. Under natural conditions, this is not always the case, and curvatures in thermal logs can be also explained by variation of such parameters. However, for the investigated boreholes, thermal conductivity measurements carried out on several core-samples show a composed variation to the average not larger than 10%, thus excluding that distortion in logs is due to lithological variation. This implies an uncertainty on the Darcy velocity of the same order of magnitude. Moreover, the temperature and pressure dependence of thermal parameters can be neglected, as the investigated depth range is relatively shallow.

Thermal conductivity variations may, in principle, affect also the determination of the Péclet number. These may cause an uncertainty of the same order of magnitude of the uncertainty in thermal conductivity. The Péclet number also depends on the length L of the section where advection occurs. The latter parameter may be inferred from stratigraphic information as the depth to some low permeability material, such as clay layers or crystalline basement. However, this depth may be not always reached by drillholes. Thus, it is common practice to assume that L is the length of that part of borehole where the permeable formation was encountered and over which the regression is performed. In this case, L may represent a minimum value, and the Péclet number can be thus underestimated. This problem does not hold in our calculations, because aquifers were entirely crossed by thermal logging, and, consequently, the investigated aquifer depth range h coincides with L. This involves that Péclet number has the same uncertainty of the Darcy velocity.

It must be stressed that the type of distortion of the thermal log depends on the direction of the vertical flow (upwards or downwards) and/or the thermal effect (cooling or heating) of the horizontal groundwater movement, which is reflected by the sign of the Darcy velocity components [see e.g., Reiter (2001), Verdoya *et al.* (2008), for a detailed discussion]. From the statistical point of view, different models may give similar good fits to data, but results may be

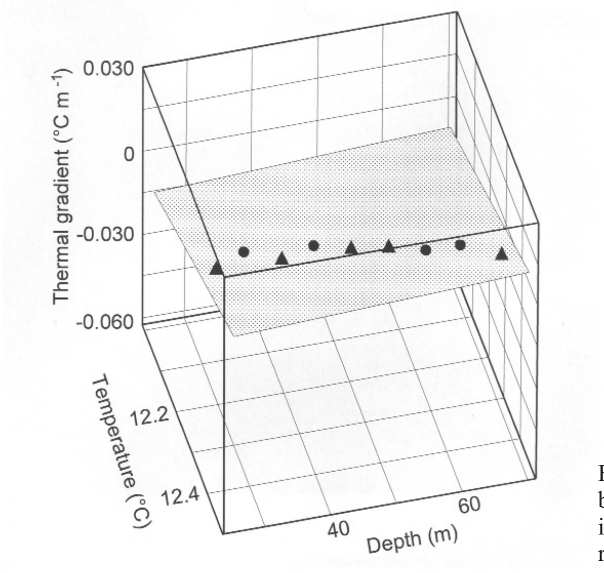

Fig. 6 - Planar fit to observed thermal gradients of borehole P1 (model C). Full circles and triangles indicate data lying above and below the plane, respectively.

contradictory in terms of flow direction.

This bias can be tackled using the shape of the thermal logs as a qualitative descriptor of the type of flow. A downward concave thermal log may be accounted for by downward flow or horizontal leakage of cold water, whereas an opposite shape can be the result of upward movements or a warm horizontal flow. The accepted model and Darcy velocity estimates are the ones that are both statistically and qualitatively consistent, and in agreement with the available hydrogeological and geothermal information. Thus, in the models, the sign of the coefficients has to be constrained to give regression results consistent with the curvature of the thermal logs. Besides, the hydraulic parameters were not estimated for the models that gave a poorer fitting, i.e., *RMSE* larger than temperature uncertainty.

The results show that the thermal logs of holes P1, P2 and Vl are consistent with a slow downward flow of groundwater with vertical Darcy velocity varying from $0.02 \cdot 10^{-9}$ to $1.42 \cdot 10^{-7}$ m s^{-1}, i.e., from about only 0.6 mm to 4.5 m per year. Less variability was obtained for the horizontal component, resulting in the range 0.5–18.0 m per year. Of course, model A cannot reveal any horizontal movement of groundwater, as it assumes that thermal gradient is zero. Both Darcy velocity components almost coincide in the hole P2 when the model B is applied. It should be stressed that the goodness of fit (smaller *RMSE*) is, in any event, larger for model B. In both holes P1 and P2, the data analysis in terms of vertical thermal gradient (model C) confirms that the horizontal and vertical components of the Darcy velocity are comparable. In borehole V1, the horizontal component of the Darcy velocity appears one order of magnitude larger than the vertical one. These results seem, as a whole, compatible with the hydrogeological situations of this set of holes, all drilled in a low permeability formation.

Different results were obtained for the thermal logs of boreholes L1 and L2. L1 shows the largest horizontal velocity (about 47 m per year), whereas the vertical upward velocity is less than 0.3 m per year. These results are compatible with the different hydrogeological conditions of these boreholes, especially for L1, where larger permeability and then Darcy velocity has to be expected.

A similar trend is reflected by the vertical Péclet number, which is in absolute value almost always < 1. Only holes P1 and P2 denote a significant heat transfer by advection ($2 < Pe_z < 15$). Concerning the horizontal Péclet number, it is always ≥ 1 and it is at its maximum (about 105) in L1 for model C, where the largest horizontal velocity was inferred.

In summary, the thermal data analysis reinforces the interpretation that the two borehole sets have different hydrogeological characteristics. In holes P1, P2 and V1, the expressions incorporating both u_o and u_z (models B and C) show a better fit than the model that considers only the vertical component (model A). The former models predict a horizontal flow of the same order of magnitude as the vertical component (downward flow of cold water). For borehole V1, model A points to an almost negligible vertical velocity. Both components of the Darcy velocity are significantly small, in agreement with the low permeability expected in the formation. In boreholes L1 and L2, the fit appears good for model A, but hydrogeological conditions lead to prefer the results of model C that indicate a significant horizontal component, which is consistent with an expected larger permeability in L1.

5. Conclusions

The applied analytical solutions for temperature-depth data in fully saturated porous layers, allow a quantitative evaluation of water flow and a relative measurement of heat transport by the bulk motion of water with respect that due to pure conduction. They appear well suited for application in shallow aquifers with low-velocity flows. Variations of water volumetric heat capacity and bulk thermal conductivity may introduce scatter into the comparison of the theoretical curves and the observational data. Thus, calculations of the hydrothermal parameters based on average properties depend on the assumption of a reasonable uniformity.

Despite the possible limitations, the proposed approach shows that underground temperatures are a natural and sensitive tracer of subsurface flows. It requires simple temperature measurements that can be combined with other measurements commonly carried out in wells during geotechnical surveys. The basic information obtained on the hydrothermal state can be valuable in case of subsequent numerical modelling for the analysis of aquifer-wide processes in two and three dimensions.

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