Statistical evaluation of vulnerability and expected seismic damage of residential buildings in the Veneto-Friuli area (NE Italy)

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ABSTRACT The European macroseismic scale EMS98, through the classification of buildings in vulnerability classes with a progressively decreasing level of seismic risk (from A to F) gives a coherent conceptual frame for a large scale evaluation of the seismic vulnerability of buildings. The theory of the random sets is used to derive the upper and lower bounds of relative frequencies of the damage grades for every class and macroseismic intensity from the qualitative measures of the scale. Preliminary classifications of the number of buildings and their volumes in every census section based on "poor" data obtained from national statistics, are verified through a first-level survey form in a sample of census sections in the provinces of Belluno, Treviso and Pordenone (NE Italy). The comparison allows us to make some proposals of corrective coefficients, based on simple linear or better robust regression.

1. Introduction

The traditional macroseismic scales contain, even though in a fuzzy and not explicit way, matrices of probability of damage for single building types: more precisely, more diffuse building types in the interval of time and territories that the authors of the proposal have directly or indirectly observed through the documentation of their behavior during earthquakes of various intensity (Bernardini, 2004). In the successive applications of the scale the same matrices of damage probability have been applied, to measure the local macroseismic intensity, in uniform way in the time and on the territory, without an explicit control of the variations of vulnerability of the existing types or of the appearance of new building types (as an example those designed with more modern methods of seismic protection).

The new European macroseismic scale EMS98 (Grünthal, 1998) seems overcome this ambiguity in the definition of the scale, through the explicit distinction between building type and class of vulnerability (which is directly connected to a matrix of damage probabilities) as well as with the introduction, besides the traditional classes A, B and C of the ordinary buildings designed without explicit controls of seismic resistance, of 3 classes (D, E and F) of buildings with levels of progressively increasing protection.

The potentialities of the EMS98 scale for an analysis of the seismic vulnerability of the ordinary buildings have been recently emphasized in Giovinazzi and Lagomarsino (2001): in

Table 1 - Correlation between EMS98 classes and *Iv* index [modified from Giovinazzi *et al.*(2001)].

EMS 98 Class	A	B	C	D	E	F
Iv (mean)	60	40	20	0	-20	(-40)

particular, a binomial formulation for the matrices of implicit probabilities of damage in the EMS98 was proposed in such paper. A good correlation has been shown of such model with the forecasting of damage obtained with the 2nd level method (Benedetti and Petrini, 1984) of the GNDT (using an index of vulnerability *Iv* variable between -20 and 100), when the value of *Iv* equal to 60 is assumed for the mainly vulnerable class (A) and equal constant decrements equal to 20 for the classes of smaller vulnerability (from B to E). The correspondence would be that indicated in Table 1, between class of vulnerability EMS98 and index of vulnerability *Iv*, extended also to the F class (although exceptional in the field of the ordinary buildings).

A good correlation has been shown, moreover, between the same model and the binomial matrices of probability of damage for the classes A, B and C, proposed in Braga *et al.* (1982), by optimizing the reports of damage observed after the earthquake of Irpinia, 1980 and making reference to the macroseismic scale MSK.

The present relation introduces the methodology for forecasting the vulnerability of the ordinary residential buildings in the classified seismic zones of the provinces of Belluno, Treviso and Pordenone in the Veneto-Friuli area (NE Italy), by classifying them into “classes of vulnerability” according to the definitions of the macroseismic scale EMS98. Starting from acquainting ourselves with the inventory of the buildings supplied from data ISTAT91 (ISTAT, 1991), the classification criterion has been calibrated with the observation of a sample of buildings opportunely selected and surveyed with a first level vulnerability form. These have returned some coefficients that correct the preliminary classifications done on original ISTAT91 data.

The criteria for the preliminary classification from ISTAT91 data and from the directly observed buildings [with the “First level Form of survey and safety evaluation for ordinary buildings in the post-seismic emergency: AeDES (Bernardini, 2000)] are reassumed. Finally, the statistical derivations of the corrective coefficients of the preliminary ISTAT91 classifications, to be used in the successive steps of the research to evaluate maps of risk and deterministic scenarios of damage for selected earthquakes, are presented.

2. Fuzzy Damage Probability Matrices implicitly given by the EMS98 scale

The qualitative judgements given by the EMS98 macroseismic scale (Grünthal, 1998) on frequencies of buildings with a different grade of damage, for every vulnerability class (from A to F) and macroseismic intensity (from V to XII), are shown in Table 2.

It is remarkable that:

- at lower intensities (V and VI) the same damage is forecast for classes A and B;
- from intensities VII to XI a simple rule of diagonal shifting of the frequencies of the same

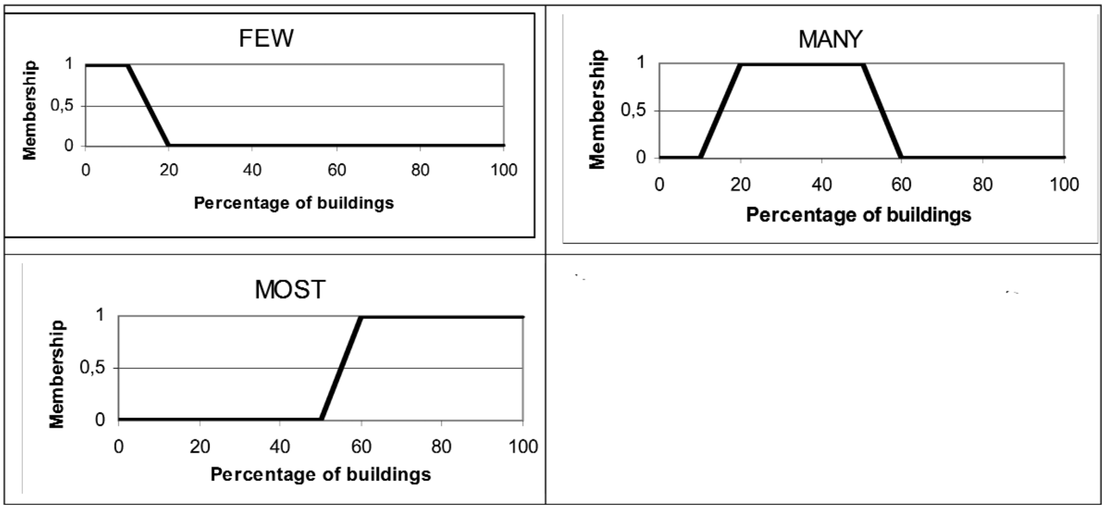

Fig. 1 - FEW, MANY and MOST fuzzy measures.

class is generally respected (increasing of one level of intensities increases 1 grade of damage to the same percentage of buildings).

The quantitative meaning, in terms of frequencies of damaged buildings, of the employed adjectives (FEW, MANY, MOST) is qualitatively suggested by the scale in a graphical fuzzy manner; therefore it seems reasonable to assume that their mathematical description are trapezoidal fuzzy subsets, all together giving a fuzzy partition of the percentage frequency interval [0, 100] (Fig. 1).

Moreover, the fuzzy sets can be considered as consonants random sets (Dubois and Prade, 1991; Bernardini, 1999) with focal elements given by the nested family of their α-cuts and uniform value $d\alpha$ of the basic probabilistic assignment. Therefore, for every fuzzy set upper, lower and "white" cumulative distribution functions (second order probabilities of the probability of damage grade conditional to macroseismic intensity) can be easily computed, and finally upper, lower and "white" values of the expected frequencies can be derived (Bernardini, 2008).

Taking into account that for every macroseismic intensity the sum of the percentages in the different grades of damage must give 100, it is clear that the linguistic damage matrix shown in Table 2 is incomplete. A reasonable linguistic extension of Table 2 can be obtained according to the following criteria:

- for every intensity and for every class the scale gives, explicitly, the "linguistic" frequencies of the grades with greater damage; therefore, the linguistic frequency NONE (i. e. numerically 0) for all higher grades of damage is assumed here;
- for lower grades, the extension of every row is performed in such a way that the sum of the expected "white" probabilities should be in any case equal to 100 (linguistically: ALL); moreover, the rule of diagonal shifting is at the best respected.

When the second criterion is assumed, the following remarkable and useful property is obtained: the expected "white" probabilities of the grades of each class, at every intensity level,

Table 2 - Linguistic damage probabilities according to EMS98 (Grünthal, 1998).

Damage Grade/ Intensity	0	1 Negligible	2 Moderate	3 Substantial to Heavy	4 Very Heavy	5 Destruction
V		Few A or B				
VI		Many A or B, Few C	Few A or B			
VII			Many B, Few C	Many A, Few B	Few A	
VIII			Many C, Few D	Many B, Few C	Many A, Few B	Few A
IX			Many D, Few E	Many C, Few D	Many B, Few C	Many A, Few B
X			Many E, Few F	Many D, Few E	Many C, Few D	Most A, Many B, Few C
XI			Many F	Many E, Few F	Most C Many D, Few E	Most B, Many C Few D
XII						All A or B Nearly All C, Most D or E or F

Table 3 - Completed linguistic DPM of Classes A, and C. Light grey cells: EMS98 definitions. Strong grey cells: modified definitions. White cells: extension according to rules in Table 2.

Damage Grade / Intensity	0	1 Negligible	2 Moderate	3 Substantial to Heavy	4 Very Heavy	5 Destruction
CLASS A						
V	All - Few	Few	None	None	None	None
VI	Most – 8/3*Few	Many	Few	None	None	None
VII	Few	2*Few	Many	Many	Few	None
VIII	None	Few	2* Few	Many	Many	Few
IX	None	Few	Few	2*Few	Many	Many
X	None	None	None	Few	2* Few	Most
XI	None	None	None	None	None	All
XII	None	None	None	None	None	All
CLASS C						
V	None	None	None	None	None	None
VI	All - Few	Few	None	None	None	None
VII	Most – 8/3*Few	Many	Few	None	None	None
VIII	3*Few	Many	Many	Few	None	None
IX	Few	2*Few	Many	Many	Few	None
X	None	Few	2*Few	Many	Many	Few
XI	None	None	None	1/3 *Few	Most- Few	Many- Few
XII	None	None	None	None	1/3 * Few	Nearly All

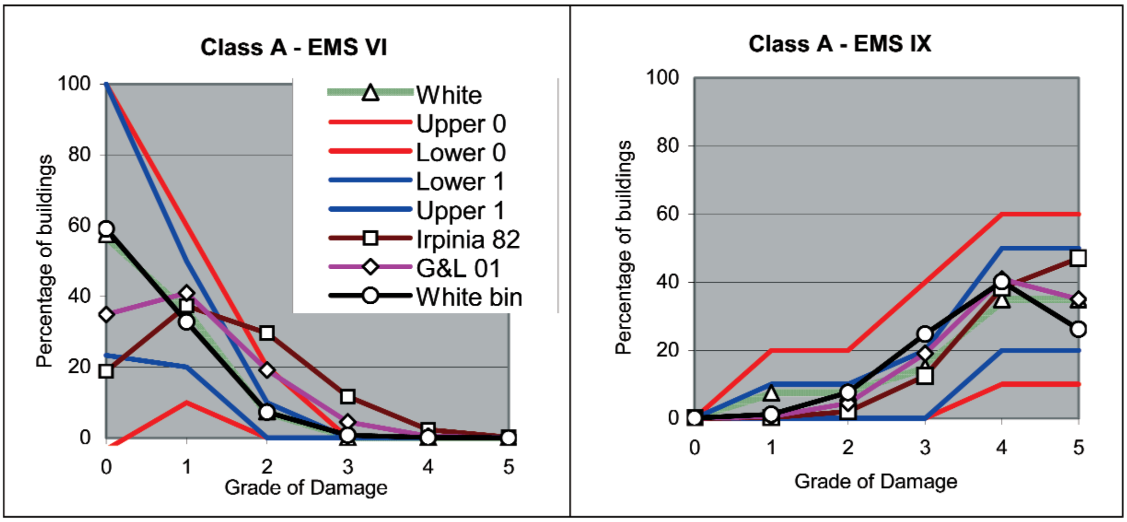

Fig. 2 - Class A: extremes of the α-cuts (for $\alpha = 0$ and $\alpha = 1$) of fuzzy sets measuring percentage of buildings in the grades of damage for macroseismic intensity VI and IX and comparison with expected "white" probabilities and binomial distributions proposed by Giovinazzi and Lagomarsino (2001: G&L 01) and Braga *et al.* (1982: Irpinia 82).

give an effective discrete probability distribution (the sum of percentages equals 100), while obviously, this is not true for their upper and lower bounds.

For example, in Table 3, the extended linguistic Damage Probability Matrices (DPMs) are displayed for classes A and C. Light grey cells contain linguistic measures suggested by EMS98 definitions (Table 2), while extensions in white cells have been obtained with the criteria

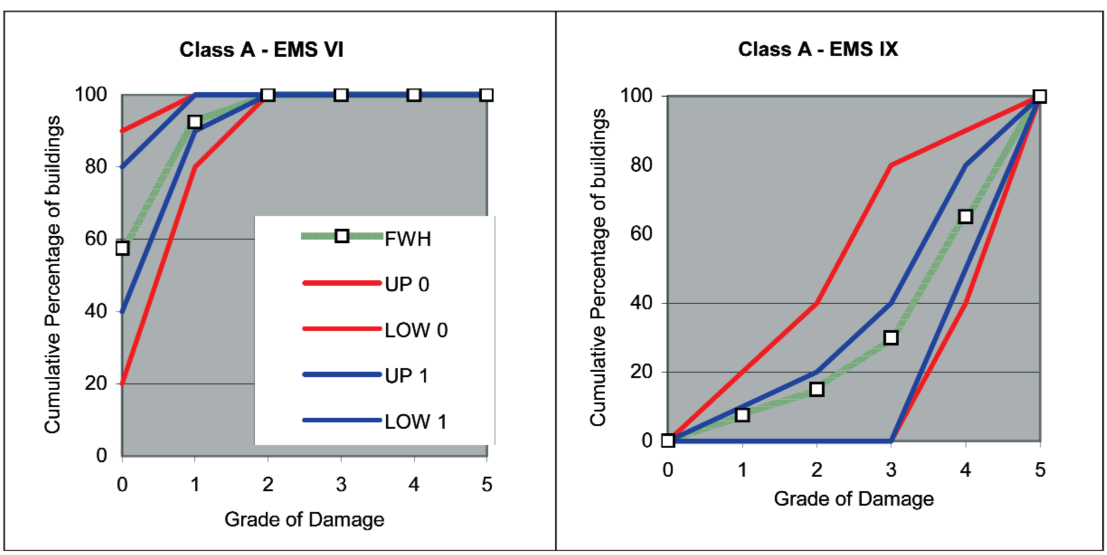

Fig. 3 - Class A, intensity EMS98 VI and IX: cumulative upper and lower damage distributions for $\alpha = 0$ and $\alpha = 1$ and mean "white" probabilities (FWH).

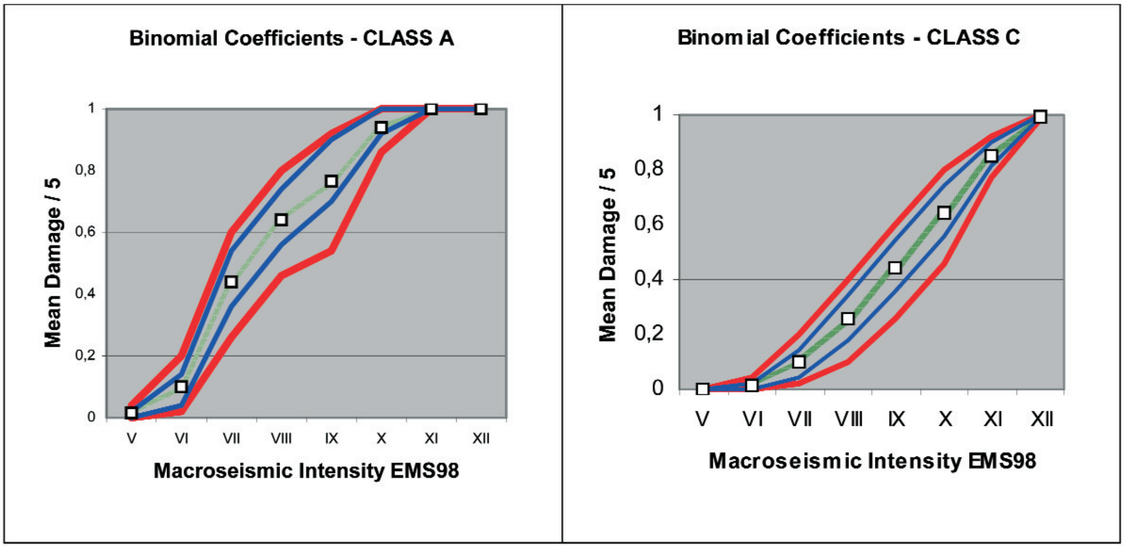

Fig. 4 - Binomial coefficients of expected “white” probabilities, for $\alpha = 0$ ed $\alpha = 1$; classes A and C and intensities from V to XII.

indicated above. Strong grey cells contain linguistic values slightly modified with respect to Table 2 [for example: Class C, intensity XI: the expected “white” frequencies MOST (grade 4) + MANY (grade 5) should overcome 100].

Combining Table 3 and the α-cuts associated to every linguistic definition through the above indicated criteria (Bernardini, 2005), the corresponding α-cuts and expected “white” probabilities of the percentages of buildings in the grades of damage can be easily derived. For example, with reference to class A and intensities VI and IX, in Fig. 2 the bounds computed for $\alpha = 0$ and $\alpha = 1$ and the expected “white” probabilities are compared with two binomial distributions, respectively proposed by Giovinazzi and Lagomarsino (2001) and Braga *et al.* (1982), from statistics of damage recorded in the 1980 Irpinia earthquake.

Finally, for every prescribed value of α, the effective upper and lower cumulative distribution functions can be computed by solving the following linear programming problems:

$$\begin{aligned} &\textit{objective function}: \quad \sum_{j=0}^{5} f_j \cdot j = \begin{matrix}\min\\ \max\end{matrix} \\ &\textit{subjected to}: \quad \sum_{0}^{5} f_j \cdot = 100; \quad f_{j,\mathrm{LOWER}} \le f_j \le f_{j,\mathrm{UPPER}}, \; j = 0 \text{ to } 5. \end{aligned} \tag{1}$$

For example, with reference to class A and intensities VI and IX, in Fig. 3 the upper/lower cumulative distribution functions for $\alpha = 0$ and $\alpha = 1$ are displayed and compared with the expected “white” probabilities.

For practical applications, it is, moreover, convenient to derive binomial distributions that approximate the numerical upper/lower and “white” cumulative distribution functions. Taking

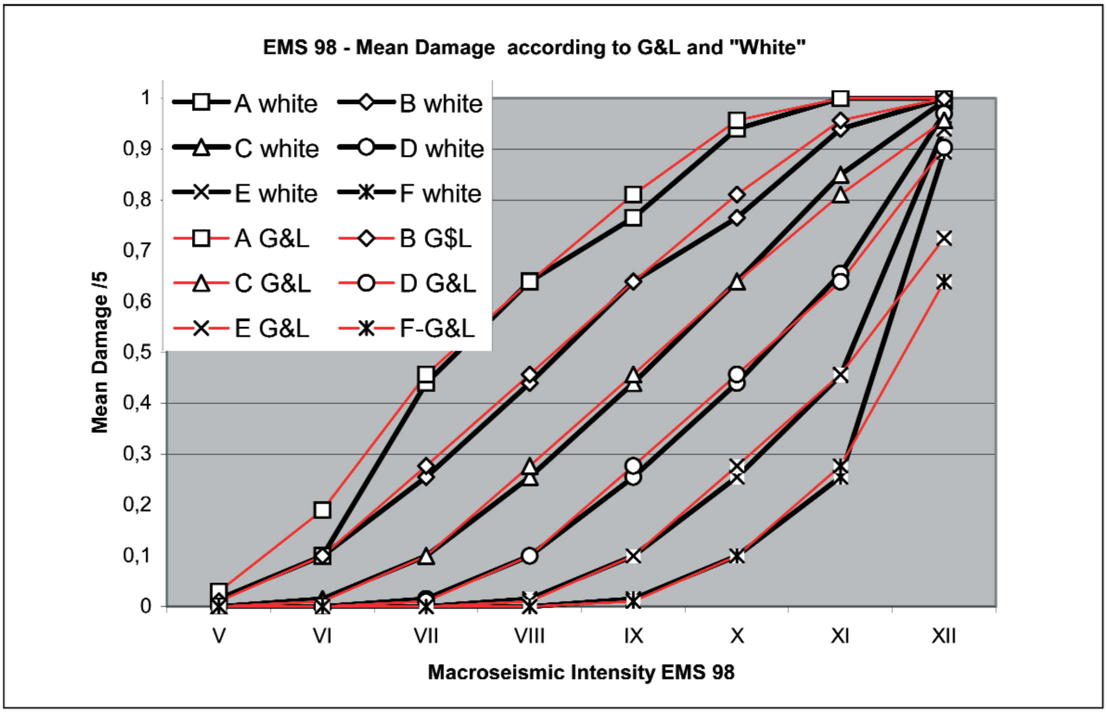

Fig. 5 - Comparison between binomial coefficients (mean damage/5) proposed in Giovinazzi and Lagomarsino (2001: G&L) and derived from mean "white" probabilities.

into account that a binomial distribution is defined completely by its mean value, the approximating binomial distributions are simply defined by the mean value of the numerical cumulative distributions. For example, the upper/lower resulting binomial coefficients for $\alpha = 0$ and $\alpha = 1$ and their expected "white" values are displayed in Fig. 4 for classes A and C and intensities from V to XII.

Finally, in Fig. 5, the expected "white" binomial coefficients are compared with the binomial coefficients proposed in Giovinazzi and Lagomarsino (2001). A substantial agreement between the two proposals can be observed. However in the formulation here proposed at intensity VI the vulnerability of class A is lower and equal to the vulnerability of class B, following the judgements of EMS98: see Table 2. Moreover, at intensity XII the vulnerability of classes D, E, F is clearly higher and yet more coherent with EMS98 linguistic measures.

3. Models for EMS98 classifications

3.1. Classification from ISTAT 1991 data

The ISTAT91 data allow us to determine the frequencies (in every census section) of groups of "homogenous" buildings (in first approximation: equally vulnerable) with respect to a number of typological parameters (typology of the vertical structures, age of construction, number of storeys, state of maintenance, state of aggregation with adjacent buildings). Italian medium

Table 4 - Parameters for the classifications from ISTAT91 data.

***k* (type)**	**1 (pilotis)**	**2 (r.c.)**	**3 (masonry)**	**4 (other)**	**5 (unknown)**
$I_v{}^1{}_1$ (*k*)	50	45	60	55	52
Delta_ j(*k*)	-20	-20	-25	-20	-22
Delta_ j(*k*)	-10	-15	-15	-15	-15
Manut(*k*)	-10	-10	-10	-10	-10
Clas(*k*)	-10	-20	-10	-10	-15

values of *Iv* for such groups have been estimated in a reliably enough way in Meroni *et al.* (2000), at least for the masonry buildings, and indicate values in the interval [13, 44], with elevated coefficients of variation (between 0.20 and 0.30). It seems, therefore, reasonable to suppose that the greater part of the buildings in existing masonry in Italy, belongs to the classes of vulnerability EMS98 B and C, when the class of vulnerability A remains confined to narrow groups of buildings of very poor quality and in bad state of maintenance. This conclusion seems also coherent with the typological definitions of the EMS98, with reference obviously to a range of typologies at an international level, comprising those in territories substantially not involved by the modernization.

More difficult is the classification of the reinforced concrete buildings: the interval [32, 36] (with coefficient of variation above 0.4) indicated from Meroni *et al.* (2000) seems excessively pessimistic, even if the same EMS98 does not exclude the possibility of classification in B class of r. c. buildings designed without criteria of seismic protection. On the other hand, the reports of the damage due to the recent earthquakes of Greece and Turkey seem to confirm the particular vulnerability of such types, in particular, for earthquakes of high seismic intensity.

In the present proposal (Bernardini, 2004), the age of buildings compared to the date of seismic classification of the area is further considered in the classification. This parameter is consistent with the criteria suggested by EMS98 scale, when introducing the different classes D, E, F to classify buildings designed according to progressively more severe rules of seismic protection; it is particularly important for r. c. construction types, but surely also relevant for masonry buildings. In fact, although only after 1996 does the Italian Code prescribe an effective analysis of the seismic response of masonry buildings, the design rules in the previous codes were enough, when respected, to greatly reduce their vulnerability.

The proposal depends on the 5 parameters, specified in Table 4 for the 5 types of vertical structure sconsidered in the ISTAT91 data. Type 4, generally combines r.c. columns or panels and masonry walls, or in some cases totally or partially wood or steel vertical elements; its vulnerability therefore seems clearly intermediate between vulnerabilities of r.c. (1 + 2) and masonry (3) types. Finally, when any typological information on the vertical structures is lacking (5), the assumed vulnerability parameters correspond to the mean values of the classified types, taking into account that in evaluating vulnerability and risk (differently with respect to safety evaluation) both positive and negative errors should be avoided.

The *i* and *j* indexes in the second and third row of Table 4, respectively, refer to intervals *i* of the age of construction (or total retrofitting) of the buildings and to typological factors *j* specified

Table 5 - Influence of age of construction and typological factors.

i	Range of age of constr.	*j*	Typological Factors	
			Aggregation	Number of storeys
1	< 1919	1	2 (yes)	3 (>4)
2	1919-1945	2	2 (yes)	2 (3-4)
3	1946-1961	3	1 (no)	3 (>4)
4	1962-1971	4	2 (yes)	1 (1-2)
5	1972-1981	5	1 (no)	2 (3-4)
5,4	1982-1984	6	1 (no)	1 (1-2)
6	1984-1991			

in Table 5. The Delta parameters, in Table 4, give the total negative variations, to be applied linearly with the corresponding indexes to the *Iv* of the worst case ($i = j = 1$) specified, for every *k*, in the first row. Therefore, the values in the first row are the mean value of *Iv* for aggregate buildings of type *k,* built before 1919, and, in any case, before the year of seismic classification of the area, code of the number of floors 3 (> 4), in bad state of maintenance.

The "Manut" and "Clas" parameters specify the reductions of Iv to be considered respectively when:

- the group of buildings is in a good state of maintenance (in 1991);
- the group of buildings has been built up after the date of the seismic classification of the area.

Therefore, the mean value of *Iv* of each group of buildings is defined by the relation:

$$Iv(i,j,k) = I^{1}_{v\,1}(k) + Delta\_i(k) \quad (i-1)/5 + Delta\_j(k) \quad (j-1)/5 + Manut(k) + Clas(k). \quad (2)$$

The last decrease in the formula is considered for the interval of age of construction $i > i_c$, where i_c is the age interval of the year of seismic classification of the area. For example, if the municipality has been classified in 1979, it is reasonable to assume $i_c = 5$; if the classification was in 1972 it is better to assume $i_c = 4$.

Finally, the classification in EMS98 classes is performed according to the rules specified in Table 6.

The *Iv* index above defined assumes values in the range [0, 50] for pilotis buildings (classes from B to D), [0, 60] for masonry (classes from A to D), [-20, 45] for r.c. buildings (classes from B to E).

The result seems substantially coherent with the classes suggested by the EMS98 scale for each type.

3.2. Classification of buildings from AeDES data

The AeDES survey form (Bernardini, 2000) is actually officially recommended by the Italian Civil Protection Department to inspect ordinary multi-storey buildings in seismic emergencies,

Table 6 - Criterion of classification in EMS98 classes.

EMS 98 Class	A	B	C	D	E	F
Iv (mean)	50 < *Iv*	30 < *Iv* <=50	10 < *Iv* <=30	-10 < *Iv* < =10	-30 < *Iv* <=-10	*Iv* <=- 30

to evaluate the damage and judge the usability of the buildings, taking into account also a list of observed and registered seismic vulnerability factors; therefore, outside emergencies, the form allows us to derive more precise vulnerability classifications for each building, and has been used here to classify all the buildings in a sample of census sections. In the research here described the form has been used mainly to evaluate the reliability of the classifications obtained by the ISTAT91 data, according to the model above described in chapter 3.1.

For every building, the AeDES form gives more extensive, qualitative and quantitative, information by comparison to ISTAT data for groups of buildings in the same census sections; this information, therefore, can be used to evaluate more precisely the vulnerability index *Iv* and the corresponding vulnerability EMS98 class. This more precise classification, however, should be given in a coherent way, with respect to the previous classification rules based on ISTAT data. In fact, the mean values of the *Iv* index obtained from AeDES data for the different groups of Italian buildings assumed equally vulnerable on the basis of ISTAT91 parameters, should coincide with the *Iv* values used in the classification based on ISTAT91 data.

When this coherence is satisfied, the derived corrective coefficients of ISTAT91 classifications, are going to be evaluated by comparising them with AeDES classifications, and we will take into account both characteristics of the local types modifying their vulnerability with respect to the mean national value, and errors in the ISTAT91 survey (in the quantitative computations of number of buildings and volumes and in the registration of their vulnerability factors).

However, the national statistical distributions of the *Iv* index for these groups of buildings are actually not known: therefore, the coherence of the classification criteria cannot be demonstrated but only supposed on the basis of the reasonable hypotheses described in the following. Particularly:

- evaluating typological factors considered also by the ISTAT91 survey (e.g. number of storeys or age interval of the year of costruction/retrofitting), the criterion of coherence requires that the same *Iv* values should be used;
- evaluating typological factors not considered by the ISTAT91 survey (e.g. roof or the specific combination of wall and floor types in a masonry building) a symmetrical national distribution should be assumed, and, therefore, symmetrical variation of *Iv* when positive or negative vulnerability factors are observed in single buildings;
- evaluating exceptional typological factors (e.g. columns in masonry buildings or structural damage), not relevant in the national statistical distributions, the variation should be applied in a not-symmetrical way with respect to the *Iv* value of the most usual condition.

3.2.1 Basic vulnerability

As specified in the first row of Table 4 for *Iv* index from ISTAT91 data, a specific value I_b of the index *Iv* is given for all the types described in the AeDES survey form, referring to a building

Table 7 - Basic vulnerability I_b of masonry buildings (k = 3).

AeDES References to floors / walls	**A (Unknown)**	**B Irregular without ties**	**C Irregular with ties**	**D Regular without ties**	**E Regular with ties**	**F**
1 - unknown	57	65	60	55	50	With isolated Columns add +5
2 - vaults without ties	67	75	70	65	60	
3 - vaults with ties	62	70	65	60	55	
4 - deformable slab	57	65	60	55	50	
5 - semi-rigid slab	52	60	55	50	45	
6 - rigid slab	47	55	50	45	40	

of the oldest age of construction (i = 1) (and an associated hypothesis of bad maintenance, not explicitly given in the AeDES form but here supposed, assuming that the quality of the maintenance decreases with time) and with the worst combination of typological factors considered also in ISTAT91 data (j = 1). The values of I_b are specified in Tables 7 and 8, respectively for masonry buildings (k=3 in ISTAT91 data) and "other types" (generally r.c. buildings: k = 1 e 2). In any case, when information is lacking (row 1 and column A of Table 7, and moreover in Table 10 and 11) the mean value is assumed.

The basic vulnerability of buildings with masonry, wall mixed to other typologies or reinforced masonry walls (k = 4) is obtained addig the variations displayed in Table 9 to the basic value of the plain masonry type .

3.2.2 Typological factors considered also in the ISTAT91 data

We consider here the age interval of the year of construction/retrofitting and the associated level of maintenance, the number of storeys and, in the case of aggregated buildings, the position in the group. The evaluation is yet to be obtained through the values i and j of a linear scale and the overall quantity of the variations, for each type k, *Delta_i(k)* and *Delta_j(k)*, as specified in

Table 8 - Basic vulnerability I_b of other typologies (k = 1 and 2) depending on factors of regularity: 1 - plan or elevation, 2 - infilled or curtain wall disposition.

AeDES References	**Factors of regularity**			
	1 irreg-2 irreg	1 irreg - 2 reg	1 reg- 2 irreg	1 reg – 2 reg
7 – R.C. frames	55	50	50	45
8 – R.C. walls	50	45	45	40
9 - Steel	45	40	40	35

Table 9 - Variation of basic vulnerability I_b of buildings with masonry mixed to others typologies (k = 4) or reinforced.

AeDES References	**Variation of I_b**
G1 – R.C. storeys over masonry storeys	0
G2 - masonry storeys over R.C. storeys	+10
G3 - R.C. and masonry in the same storey	0
H1 - masonry with injections or plain plasters	-10
H2 – reinforced masonry or masonry with reinforced plasters	-20
H3 - masonry with others or not identified reinforces	-10

Tables 10 and 11. Moreover, the vulnerability index is reduced when the building has been constructed or retrofitted in an area subjected to legal rules of seismic protection.

Two alternative hypotheses (CR or RR) have been used to evaluate, within this classification, the influence of the retrofitting intervention in reducing the vulnerability of the original building (CR: negligible, except when made in conformity to a seismic protection law in force for the municipality; RR: corresponding to the safety requirement of a new building built in the same year). More precise inspections of some retrofitted buildings suggest that the first hypothesis unfortunately seems much more realistic. However, this uncertainty should also be considered in damage forecasting.

3.2.3 Typological factors not considered in the ISTAT91 data

This information is recorded in Section 3 (Roof), 4 (pre-existing Damage) and 7 (Morphology of the site and settlements) of the AeDES form. The corresponding variations of vulnerability F_r are displayed in Table 12.

3.2.4 Evaluation of *Iv* and classification

Finally, the *Iv* index of the building will be computed with the following formula:

$$Iv = I_{b\,1}^{1}(k) + Delta\_i(k)\ \ (i-1)/5 + Delta\_j(k)/(j-1)\ \ 5 + Clas(k) + \sum_r F_r \qquad (3)$$

and the EMS98 class of the building will be identified with the same criteria previously used for classifications based on ISTAT91 data (Table 6).

For buildings without evidence of pre-existing damage or settlement and constructed before

Table 10 - Parameters of classification from AeDES.

***k* (tipology)**	**3 (masonry)**	**1 and 2 (r.c./steel)**	**4 (mixed or reinforced)**	**5 (unknown)**
Delta_i(*k*)	-30	-25	-30	-27
Delta_j(*k*)	-15	-15	-15	-15
Clas(*k*)	-10	-20	-10	-15

Table 11 - Influence of age of construction and typological factors.

i	**Range of age of constr.**	*j*	**Typological Factors**	
			Aggregation	Number of storeys
1	< 1919	1	extreme or corner	>4
2	1919_1945	2	extreme or corner	>2 and <5 or unknown
3	1946_1961 or unknown	3	internal or isolated or unknown	>4
4	1962_1971	4	extreme or corner	1 or 2
5	1972_1981	5	internal or isolated or unknown	>2 and <5 or unknown
6	1982_1991	6	internal or isolated or unknown	1 or 2
7	1991_2001			
8	> 2001			

1991, the *Iv* index belongs in the following intervals:

[-15, 90] (classes from A to D) for masonry;

[-25, 65] (classes from A to E) for r.c. buildings.

These intervals seem appropriate, taking into account that the variability of *Iv* for single buildings should be clearly higher than the variability of mean values of groups of buildings; in fact particular buildings with a very different vulnerability with respect to the mean national value for the general type cannot be excluded.

4. Comparison of EMS98 vulnerability classifications

4.1. Introduction

The main reason for this comparison is to evaluate the reliability of the data taken from ISTAT91 and to elaborate corrective coefficients that allow a more accurate EMS98 classification from the rough classification available for the entire population (of the census sections).

Table 12 - Typological factors not considered in the ISTAT91 data.

AeDES References	**1 Thrusting and heavy**	**2 Not thrusting and heavy**	**3 Thrusting and light**	**4 Not thrusting and light**
ROOF	+5	0	0	-5
AeDES References	**Very serious**	**Serious**	**Light > 1/3**	**None or light < 1/3**
PRE-EXISTING DAMAGE	+40	+20	+10	0
AeDES References	**1 Top**	**2 Strong slope**	**3 Slight slope or unknown**	**4 Plain**
MORPHOLOGY OF THE SITE	+5	+5	0	0
AeDES References	**A Absent or unknown**	**B Created by earth.**	**C Increased by earth.**	**D Pre-existing**
SETTLEMENTS	0	+5	+5	+5

Moreover, it will take into account the retrofitting building work and the construction of new residential buildings since 1991.

Detailed information about the criteria employed in the selection of 18 municipalities in the total area of interest (212 municipalities in the provinces of Belluno, Treviso and Pordenone) and of the representative census sections in each municipality is presented in Bernardini (2008). In the selected 46 census sections 4808 residential buildings have been surveyed and stored in the database.

The following notation will be used throughout the text unless otherwise stated:

- ${}^{g}Y_i$, $g=\{A, B, C, D, E, F\}$, $i=1,..., 46$, is the number of buildings in the g-vulnerability class of the i-th census section derived from the AeDES survey;
- ${}^{g}X_i$, $g=\{A, B, C, D, E, F\}$, $i=1,..., 46$, is the number of buildings in the g-vulnerability class of the i-th census section derived from ISTAT91;
- ${}^{g}W_i$, $g=\{A, B, C, D, E, F\}$, $i=1,..., 46$, is the volume of buildings in the g-vulnerability class of the i-th census section derived from the AeDES survey 2000;
- ${}^{g}V_i$, $g=\{A, B, C, D, E, F\}$, $i=1,..., 46$, is the volume of buildings in the g-vulnerability class of the i-th census section derived from ISTAT91;
- M_i, $i=1,..., 46$, the percentage of brickwork out of the total volume in the i-th census section derived from ISTAT91.
- $i=1,..., 46$ represents the 46 census sections of 18 municipalities considered in the AeDES sample. If g does not appear in the superscript, we simply refer to the complete variable, e.g. $Y_i = \Sigma_g {}^{g}Y_i$.

Thus, pairs of observations (${}^{g}X_i$, ${}^{g}Y_i$) or (${}^{g}V_i$, ${}^{g}W_i$), respectively relating to the number of buildings and the volumes, are available. Since the AeDES survey provides more accurate information than the national statistics, we can consider the observations on Y and W as the "true" values (respectively number of buildings and volumes) observed in the considered sample. Note that accessories, and not residential buildings, have been excluded from the computation of frequencies and volumes. Moreover, the AeDES data set may or may not be restricted to buildings built before 1991, but in any case observed in the year 2002. In the first case, a more uniform comparison with ISTAT data is possible; in the second, the comparison allows us to evaluate the subsequent modification of building stock as well, and therefore, evaluate the actual vulnerability of present constructions. Finally, class F is not significant in the data and is therefore excluded from the analysis.

The two different hypotheses (CR and RR, see chapter 3.2.2) have been considered in the AeDES 2000-based classification regarding the influence of the building's age on vulnerability. Therefore, two data sets, corresponding respectively to "hypothesis CR" and "hypothesis RR", are taken into consideration for the analyses. In the following chapters we refer only to variables X and Y; however, for volumes, we proceed in the same manner as with number of buildings, and the same consideration holds for V and W.

4.2. Initial comparison of AeDES and ISTAT data

For the time being let us consider variables X and Y that represent two different measurements of the same quantity (number of buildings). For this reason, they should be linked by an exact straight line relationship (with a slope equal to one), i.e. the plot of Y against X more or less gives a straight line through the origin. We say "more or less" because, although the relationship is currently exact, our measurements (that is to say ISTAT91 data and the method used for their classification) may be subject to errors and the plotted points would thus be unlikely to fall exactly on the line. Instead, they would vary randomly either side. In this case, we would use the identity relationship with random errors of the form $Y_i = X_i + \varepsilon_i$, i=1, ..., 46, where ε_i represents the assumed random measurement error of the i-th observation. We can write $\hat{Y}_i = X_i$, where $\hat{Y}$ denotes the predicted value of Y given X, and $e_i = Y_i - \hat{Y}$ is the residual, that are our "estimates of the errors ε_i". The residuals contain all available information on how the fitted model fails to properly explain the variation observed in dependent variable Y. However, if the model is not correct, i.e. if bias error $B_i = E[Y_i]-E[\hat{Y}_i] \neq 0$, then the residuals contain both random and systematic components. Note that the extent of the bias depends not only on the postulated model and the true model but also on the values of the X-variable (thus the bias may depend on inaccurate information obtained from the ISTAT91 questionnaire and through interviewer errors, except the proposed method of classification). In the simple case of fitting a straight line, bias error can usually be detected merely by examining a plot of the data: if we plot the pair (X_i, Y_i), we find there is an (average) underestimation of the number of buildings derived from national statistics (X) compared with those derived from AeDES (Y). In order to provide statistical evidence, we test the null hypothesis that and have the same mean value. The "paired-sample t test" is significant for both data set "CR" and "RR", thus we can conclude that there are systematic error components in X which cause an average underestimate of Y. Now, we shall analyse whether this difference is homogenous over the various vulnerability classifications. Let us consider the following quantities:

- $r = \sum_{i=1}^{46} Y_i / \sum_{i=1}^{46} X_i$ that is the ratio between the total number of buildings derived from AeDES and the number derived from national statistics;
- ${}^g\widetilde{X} = {}^gX_i \cdot r$ that is the number of buildings derived from national statistics corrected through r;
- ${}^gZ_i = {}^g\widetilde{X}_i - {}^gY_i$.

Now, we are interested in testing whether the different vulnerability classes A,…, E have different effects on the differences ${}^gZ_i$; clearly $\overline{Z} = \sum_i Z_i / 46 = 0$ but we wish to test the null hypothesis of equality of group means (the ANOVA null hypothesis). The F statistic is significant for both data set "CR" and "RR", showing there is evidence that the vulnerability classes produce different effects. These results suggest that we should calculate different corrective coefficients, one for each vulnerability class; hence, throughout the text, analyses stratified by groups will be used.

4.3. An initial proposal for corrective coefficients

Taking into account the conclusions suggested by the previous analyses, an initial proposal for corrective coefficients, based on the ratio of the summations on the sample of census sections separately for each class (or on the mean error for each class), could be given simply assuming:

$$ {}^g\widetilde{X}_i = {}^gX_i \cdot {}^gr = {}^gX_i \cdot \frac{{}^gY}{{}^gX} = {}^gX_i \cdot \left(1 + \frac{{}^g\varepsilon}{{}^gX}\right). \qquad (4) $$

The above criterion for calculating the corrective coefficients satisfies the requirement of obtaining the same total number of objects (i.e. buildings and volumes) for ISTAT91 "corrected" and AeDES data in each vulnerability class. Nevertheless, this criterion can be improved by substituting it with one that minimizes the sum of squares of deviation from a fitted straight line, in order to support a more consistent inferential extrapolation to the entire target population, i.e. all census sections located in the seismic zone of the provinces of Belluno, Treviso and Pordenone.

4.4. Corrective coefficients through linear regression

The main purpose of regression is to explore the dependence of one variable on another. From what we stated in the introduction, we can consider the observations on *Y* as the "true" number of buildings, therefore, we would estimate the actual values from those available for the entire population. In particular, in a simple linear regression we have the following relationship:

$$^{g}Y_i = {^{g}\beta_o} + {^{g}\beta_1}\,{^{g}x_1} + {^{g}\varepsilon_1} \qquad (5)$$

where the quantities $^{g}\beta_0$ and $^{g}\beta_1$, the intercept and the slope of the regression, are assumed to be fixed unknown parameters and $^{g}\varepsilon_1$ is a random variable. The main purpose of regression is to predict $^{g}Y_1$ (the "true" number of buildings) from knowledge of $^{g}x_1$ (the "incorrect" number of buildings available for the entire population of the census section; we use $^{g}x_1$ instead of $^{g}X_1$ to highlight the fact that $^{g}x_1$ is fixed and known or that it is a realization of the observable random variable $^{g}X_1$), using a relationship such as Eq. (5), (Casella and Berger, 1990; Draper and Smit, 1998).

4.4.1. Robust regression

It seems, from our analysis, that the errors have a non-normal distribution. We might, therefore, consider a robust regression method, particularly in cases where the error distribution is heavier-tailed than the normal distribution, i.e. when there is more probability in the tails than in the normal. Such heavier-tailed distributions are likely to generate more large errors than normal. A least-square analysis weighs each observation equally when getting parameter estimates. Robust methods enable the observations to be weighted unequally. Essentially, observations that produce large residuals are down-weighted by a robust estimation method. From the various weight functions given in the literature, we chose the bisquare weight function.

4.4.2. Regression using a dummy variable

Up to this point, we have considered the first-order linear regression model in one variable g_x. Usually more complex linear models are needed: knowledge of more than one predictor variable (the "incorrect" number of buildings in the census section) is needed to obtain better understanding and/or a better prediction. The considered variable is the percentage $^{g}m_i$ of brickwork out of the total volume of the *g*-vulnerability class in the *i*-th census section, available for all (528) census sections in the 18 municipalities: if $^{g}m_i$<0.53945 (this cut-off value is obtained through a K-means cluster analysis), then the *g*-vulnerability class belongs to the second group, otherwise to the first (note below that we call this variable 'Group'). The way we used dummy variable 'Group' allowed us to fit a no-intercept linear model with different slopes

Table 13 - Comparison of simple (S*) and robost (R*) convective coefficients (non interception solutions) to weighted mean solutions (WM).

Buildings	**Hypothesis CR**					**Hypothesis RR**				
class	WM	S*	*rto*	R*	*rto*	WM	S*	*rto*	R*	*rto*
A	1.415	1.008	.71	0.994	.70	1.189	0.839	.70	0.994	.83
B	1.145	1.076	.94	0.942	.82	0.788	0.737	.93	0.942	1.19
C	1.473	1.324	.90	1.255	.85	1.464	1.296	.88	1.255	.86
D	1.206	1.152	.95	1.157	.96	1.714	1.459	.85	1.157	.67
E	1.000	0.529	.53	0.346	.35	1.188	0.609	.51	0.346	.29
Volumes	**Hypothesis CR**					**Hypothesis RR**				
class	WM	S*	*rto*	R*	*rto*	WM	S*	*rto*	R*	*rto*
A	2.253	1.667	.74	1.691	.75	1.659	1.306	.79	1.342	.81
B	2.056	1.998	.97	1.595	.78	1.002	1.474	1.47	1.198	1.20
C	2.207	1.962	.89	1.920	.87	1.301	1.896	1.46	1.855	1.43
D	2.212	2.241	1.01	1.608	.73	2.721	2.532	.93	1.928	0.71
E	1.425	0.705	.49	1.466	1.03	1.937	0.784	.40	1.638	.85

depending on the group.

4.5. Results

Several statistical methodologies have been compared in this work to forecast, for every census section, actual frequencies of the number of residential buildings and their volumes in the EMS98 vulnerability classes, correcting preliminary classifications based on ISTAT91 statistical data.

Because we are essentially interested in extrapolating our inference outside the sample, the linear regression-based solution is more appropriate than the one based on the ratio of the summations on the sample of census sections separately for each class (WM: weighted mean solutions).

Moreover, still to be considered is the fact that solutions with high interception values, when applied to a very large set of census sections in the area of interest (nearly 5000 in 210 different municipalities), might produce an unjustified increase in buildings and volumes in the rural and mountainous areas, where ISTAT91 data give zero or nearly zero.

Hence, in the final choice, only the solutions based on a non-interception model are taken into account (see Table 13). In the tables, the ratio (*rto*) of each corrective coefficient derived from simple (S*) or robust (R*) linear correlation to the WM coefficient demonstrates the incongruity with the total number of buildings and volumes when the suggested coefficients are applied to the sample of 46 census sections surveyed using the AeDES form.

Furthermore, a second more interesting non-interception solution can be suggested for class A (the most critical class when it comes to forecasting damage scenarios), i.e. the solution based on the dummy variable described in chapter 4.2. Of course, choosing this solution depends on its

Table 14 - Proposed corrective coefficients for hypothesis CR (a) and RR (b). A_m is the percentage of brickwork out of the total volume for the class A.

a) Hypothesis CR

Class	**Number of buildings**		**Volumes**	
	$\hat{Y}$	Method	$\hat{W}$	Method
A	1.008 $\cdot^{A}x$	Simple	1.691 $\cdot^{A}v$	Robust
	1.490 $\cdot^{A}x$ 0.952 $\cdot^{A}x$	Dummy: $^{A}m>0.53945$ $^{A}m\leq0.53945$	2.986 $\cdot^{A}v$ 1.542 $\cdot^{A}v$	Dummy: $^{A}m>0.53945$ $^{A}m\leq0.53945$
B	1.076 $\cdot^{B}x$	Simple	1.998 $\cdot^{B}v$	Simple
C	1.324 $\cdot^{C}x$	Simple	1.962 $\cdot^{C}v$	Simple
D	1.157 $\cdot^{D}x$	Robust	2.241 $\cdot^{D}v$	Simple
E	0.529 $\cdot^{E}x$	Simple	1.466 $\cdot^{E}v$	Robust

b) Hypothesis RR

Class	**Number of buildings**		**Volumes**	
	$\hat{Y}$	Method	$\hat{W}$	Method
A	0.994 $\cdot^{A}x$	Simple	1.342 $\cdot^{A}v$	Robust
	1.320 $\cdot^{A}x$ 0.784 $\cdot^{A}x$	Dummy: $^{A}m>0.53945$ $^{A}m\leq0.53945$	2.675 $\cdot^{A}v$ 1.176 $\cdot^{A}v$	Dummy: $^{A}m>0.53945$ $^{A}m\leq0.53945$
B	0.737 $\cdot^{B}x$	Simple	1.198 $\cdot^{B}v$	Simple
C	1.255 $\cdot^{C}x$	Simple	1.855 $\cdot^{C}v$	Simple
D	1.459 $\cdot^{D}x$	Robust	2.532 $\cdot^{D}v$	Simple
E	0.609 $\cdot^{E}x$	Simple	1.638 $\cdot^{E}v$	Robust

computational feasibility: indeed, it requires the evaluation of parameter M (the ratio of the volume of masonry to the total volume) in every census section.

The final suggested choice of a non-interception solution is displayed in Table 14: in order to obtain estimates closer to the AeDES data in the sample, the corrective coefficients with an *rto* closer to 1 are selected for every class and separately for the number of buildings and volumes. This hypothesis seems reasonable taking into account that number of buildings and volumes (although logically strictly correlated) are evaluated very differently by ISTAT91 data: the number of buildings is derived roughly from the number of residential dwellings, while the volume is evaluated in a much more reliable manner from the recorded (but generally underestimated) surface area of the dwellings.

5. Conclusions

Implicit DPMs and appropriate classifications of the building stock can be used to evaluate seismic risk and damage scenarios at the level of the territorial units of the available data.

The uncertainty analysis, based on the ideas of the random set theory, seems particularly useful to capture and treat uncertainties of different nature, namely randomness, incompleteness of data, epistemic uncertainty about relevant aspects of the phenomena, with a unified procedure.

In the present paper, the fuzziness of the implicit DPMs suggested by the macroseismic scale EMS98 and the uncertainty about the influence of the past retrofitting interventions have been considered specifically.

Employment of systematic inventory of the residential buildings, although based on data of poor quality (in the present application the ISTAT91 data) and proper direct inspection of a representative sample, allow us do describe in a proper manner, the urban vulnerability and further, to evaluate seismic damage scenarios and seismic risk.

However, the proposed models to classify groups of buildings described by ISTAT91 data and single buildings observed by means of the AeDES survey form, although based on reasonable hypotheses and damage reports of past Italian earthquakes, should be more extensively tested in different areas, with different building typologies, before being confirmed and considered for general applicability.

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