Interpretation of global geomagnetic sounding data using stochastic inversion

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(Received: July 21, 2006; accepted: May 10, 2007)

ABSTRACT The stochastic inversion theory, formulated on stochastic processes and defined on real Hilbert spaces, has been utilized in the optimal solution to inverse the global geomagnetic sounding (GMS) problem to estimate vertical conductivity distribution of the Earth from long period geomagnetic continuum data in the bandwidth 0.2-0.01 cycles per day (CPD). The stochastic formalism, in essence, is based upon an iterative perturbation algorithm, which relates the changes in the model to first order changes in the data affected through the imposition of first order Taylor series expansion about some a priori/updated model parameters at frequencies and degrees of spherical harmonics. This approximation reduces the nonlinear GMS inversion problem to solving a system of linear perturbation equations, which are simultaneously underdetermined and overconstrained. The system of linear perturbation equations gets solved through the application of a stochastic inversion formula and the computed perturbations are added to the a priori/updated model parameters. This calculation sequence keeps iterating till an optimal model results (hopefully), satisfying a reduced chi-square statistic criterion, an indicator of goodness of fit test between observed GMS data and the theoretical data functionals corresponding to a finally accepted model. For the envisaged layered Earth structure, a priori resistivities and thicknesses alongwith noise and solution autocorrelation operators are required to initiate the stochastic inversion algorithm. Computer-assisted interactive forward modeling, corresponding to frequencies belonging to the mentioned frequency band and a spherical harmonic of degree one, has been undertaken to estimate a priori layer parameters. Fidelity of the estimated parameters in terms of parameter resolutions and total estimation error involved in respect of parameter estimations have also been studied in their entireties. Correlation coefficients of the estimated parameters have also been provided. Starting with a priori layer parameters, obtained by computer-assisted interactive modeling, stochastic inversion has been successful in retrieving the resistivity-depth distribution of the Earth from a depth of about 436.19±7.81 km from the Earth's surface with a resistivity low of 0.96±0.08 ohm·m, which gets further reduced to 0.86±0.16 ohm·m at a depth of 565.0±11.9 km which continues down to a depth of 845.62±17.74 km approximately with satisfactory parameter resolutions and minimum total estimation errors. Correlation coefficients of the estimated parameters are mostly characterized by zeroes/small values, indicating insignificant parameter correlations. The agreement between the observed data and the corresponding best-fit curve is finally obtained for a reduced chi-square statistic criterion less than unity. Non-retrieval of resistivity-depth distribution in respect of the upper surface and

conducting interior of the Earth is mostly caused by inadequate data bandwidth and noisy data, as actually available, utilized in the inversion process.

1. Introduction

Geomagnetic induction studies are primarily concerned with inductive response which the Earth's crust and mantle show as electrical conductors towards transient variations in the Earth's magnetic field. The cardinal parameter to be investigated is the electrical conductivity (reciprocal of resistivity).

The spectrum of natural geomagnetic time variations available for geomagnetic induction studies covers several orders of magnitude in frequency.

In this study, the Earth's model, to be investigated, is taken to be horizontally layered (Fig. 1). The general expression for surface impedance Z for a horizontally layered (each layer has a constant resistivity ρ_1 and thickness t_1) Earth model of N layers (Fig. 1), exposed to the electromagnetic field of external origin is given (Srivastava, 1966) as

$$Z = -i\omega\mu_0/M_1[M_1t_1 + \text{arccoth}\{M_1/M_2 \coth (M_2t_2 + \ldots + \ldots \text{arccoth } M_{N-1}/M_N)\}] \quad (1)$$

where M_1 is the propagation constant of the downward diffusing field in the 1^{th} layer given by

$$M_1^2 = i\omega\mu_1/\rho_1 + Q^2 \quad (2)$$

for 1 = 1,..., N; μ_1 and ρ_1 are the magnetic permeability and resistivity of the 1^{th} layer respectively, i is the imaginary quantity defined as $(-1)^{1/2}$, magnetic permeability inside the Earth is generally taken as the free space magnetic permeability μ_0 (μ_0 is taken as $4\pi \cdot 10^{-7}$ Volt·s·amp^{-1}·m^{-1} and π being a constant), the angular frequency ω is $2\pi f$ where f is the frequency of the inducing field, Q is the wave number of the inducing field given by $[n(n+1)]^{1/2}/R$ or n/R, according to n is small or large ($n \geq 5$) and R being the radius of the Earth. For the considered frequency range, as used in the global geomagnetic sounding (GMS) study, the displacement current may be neglected, therefore, the dielectric constant of the medium does not appear in Eq. (2) which may be separated into its real and imaginary components as $[(K+Q^2)/2]^{1/2}$ and $[(K-Q^2)/2]^{1/2}$ with K as

$$K^2 = Q^4 + (\omega\mu_0/\rho_1)^2 \quad (3)$$

where all the letters have the same significances as explained earlier.

The spatial non-uniformity of the inducing field defined by its wavelength W is

$$W = 2\pi/Q \quad (4)$$

where Q has the same significance enunciated earlier. In the GMS study, spatial non-uniformity of the inducing field is required to be considered. The propagation constant in the GMS study for the 1^{th} layer is, therefore, provided by Eq. (2). For a laterally uniform and horizontally layered

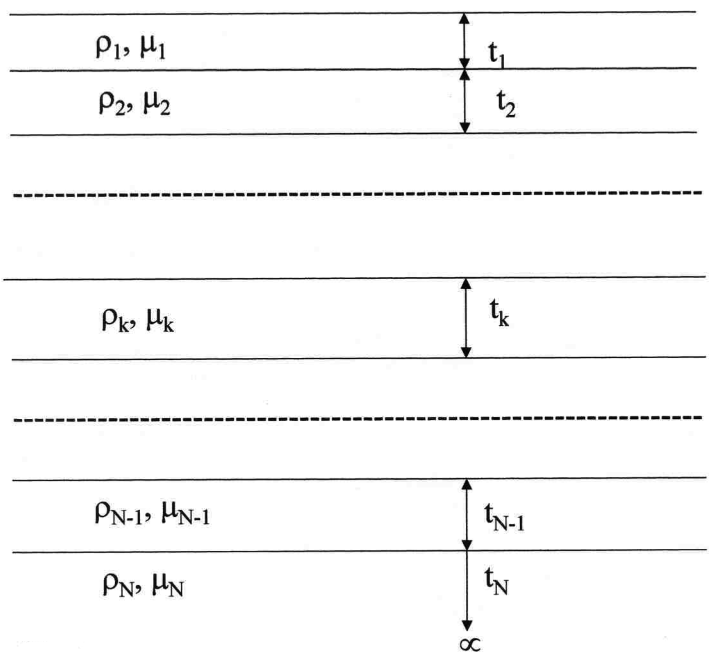

Fig. 1 - A horizontally stratified Earth model.

Earth model, the inductive response *Vn*, defined as the surface ratio of internal to external parts for a spherical harmonic of degree *n* is the same for vertical and horizontal magnetic field variations for given frequencies and degrees of considered harmonics. Inductive response is given by Schmucker and Jankowski (1971) as

$$V_n = (i\omega\mu_0 - QZ)/(i\omega\mu_0 + QZ) \tag{5}$$

where Z is the surface impedance corresponding to the propagation constant given by Eq. (2) and other letters appearing in Eq. (5) have the same significances as explained earlier. The corresponding apparent resistivity ρ_a is given by

$$\rho_a = |Z|^2 / \omega\mu_0 \tag{6}$$

where the letters have the same significances as mentioned earlier.

In the GMS study, Z is obtained through the application of Eq. (5) from the measured surface ratio V_n for various frequencies and degrees of harmonics defining the geometry of the generating source as elaborated by Schmucker (1970). It may be mentioned that the separation into its external and internal parts of the total magnetic field in the GMS study is based on the basic assumption of potential.

The geomagnetic response of Banks (1972) in the frequency bandwidth 0.2-0.01 cycles per day (CPD), which includes the geomagnetic continuum is utilized in the study. The choice of upper and lower frequencies is governed by the criteria enunciated by Parker (1970).

2. Stochastic inversion theory

An inversion theory is concerned with mapping a set of data (points in some finite vector space) into a model space and the mapping of finite data into the model space consists of linear functions only. Conceptually, it implies that the exact solution cannot be obtained since information supplied by the data is insufficient for various reasons. However, it is possible to estimate linear averages of the sought model. The stochastic technique used herein involves stochastic processes, defined on the real Hilbert space. That is, the model space is generalized to a Hilbert space having a fairly arbitrary norm and statistical interpretations of error and ergodicity which are invoked in this treatment. Consequently, the sought model may be classed according to some specified degree of confidence.

The stochastic inversion formalism, as developed by Franklin (1970), is only applicable to linear problems. Therefore, the nonlinear GMS forward problem is linearized so that stochastic inversion formula may be applicable to it. The first order Taylor series expansion of the GMS forward problem (a mathematical expression of the physics of the problem so that geomagnetic responses may be evaluated corresponding to a specified structure given the frequencies and degrees of the considered harmonics) about some a priori/corrected model parameters and frequencies may be expressed in terms of vector as

$$\delta\mu = A\delta p + \hat{n} \tag{7}$$

where A is a linear operator, the row elements of which are the Frechet Kernels of the data, δp is the difference between the actual parameter vector P of the envisaged Earth model and a priori/corrected parameter vector $\hat{P}$ and $\hat{n}$ is a vector being the difference between the observed data and theoretical data functionals corresponding to a priori/corrected parameter vector and is a vector containing the error components associated with the data.

The stochastic inversion formula due to Franklin (1970) may be written following the modification made by Jackson (1979) as

$$\delta p = (A^T F_{qq}^{-1} A + F_{pp}^{-1})^{-1} A^T F_{qq}^{-1}\ \delta\mu \tag{8}$$

If p be the number of unknowns (parameters) to be estimated and q be the total number of data used in the inversion process, then, in Eq. (8), A is the q x p system matrix and A^T refers to the transpose of A, F_{pp}^{-1} is the inverse of F_{qq}, the noise autocorrelation operator, F_{pp} is the p x p solution autocorrelation operator and F_{pp}^{-1} refers to the inverse of F_{pp}. The estimated vector P is, therefore, given by

$$P = \hat{P} + \delta p \tag{9}$$

where the letters have the same significances as explained earlier.

An inspection of Eq. (8) makes it evident that before a solution of this equation is attempted, the solution autocorrelation operator F_{pp} and noise autocorrelation operator F_{qq} ought to be obtained to initiate the inversion process. A justified choice of the solution autocorrelation operator is fraught with uncertainties. F_{pp} acts as a filtration operator which may be so parameterized as to ignore the unrealistic solutions based on physical considerations, geological constraints, etc.

The various issues involved in the formation of F_{pp} have been discussed at length by Wiggins (1972) and Jackson (1972). In the present study, F_{pp} is parameterized to become a p x p diagonal matrix in which the diagonal elements are the squares of the expected parameter variations. Such a parameterization of F_{pp} does imply that off-diagonal elements, which provide parameter cross-correlations are not taken into account. For non zero diagonal elements, F_{pp} has possessed the property of being non singular. A strong F_{pp} should work as a low pass filter.

For a N layered Earth model (Fig. 1), if $\Delta\rho_1, \Delta\rho_2, \ldots, \Delta\rho_l, \ldots, \Delta\rho_N$ be the expected variations in resistivities and $\Delta t_1, \Delta t_2, \ldots, \Delta t_l, \ldots, \Delta t_{N-1}$ be the expected variations in thicknesses, the solution autocorrelation operator has the representation as

$$Fpp = \mathrm{diag}\,[\,\Delta\rho_1^2, \Delta\rho_2^2, \ldots, \Delta\rho_l^2, \ldots, \Delta\rho_N^2, \Delta t_1^2, \Delta t_2^2, \ldots, \Delta t_l^2, \ldots, \Delta t_{N-1}^2\,] \qquad (10)$$

It is assumed that expected resistivity and the thickness variations are uncorrelated, thus F_{pp} may be generated.

The noise autocorrelation operator F_{qq} has the representation as

$$Fqq = \mathrm{diag}\,[\,\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2, \ldots, \sigma_q^2] \qquad (11)$$

where $\sigma_1, \sigma_2, \ldots, \sigma_q$ are standard deviations of the observed data. In this deviation, it is assumed that the noise components of the data are uncorrelated i.e., error in a data corresponding to a frequency is unrelated to the error in the data at another frequency.

The resolution of the estimated parameters in an inversion problem is given by Jackson (1972) as

$$U = (A^T F_{qq}^{-1} A + F_{pp}^{-1})\text{-}1\, A^T F_{qq}^{-1} A \qquad (12)$$

where the letters have the same significances as explained earlier.

The degree to which the matrix U resembles an identity matrix is a measure of the parameter resolution obtainable from the given data set. Closeness of rows to showing unity on diagonals and zeroes on the off-diagonal elements is called the deltaness criteria of the resolution matrix. The row elements of the resolution matrix are called the averaging kernels, that may also be regarded as windows through which the estimations of the inversion problem may be viewed (Jackson, 1972).

The total errors of the estimated parameters in an inversion problem may be obtained (Jackson, 1979) as

$$\mathrm{cov}\,(P) = (LA\text{-}I)\, F_{pp}\, (LA\text{-}I)^T + LF_{qq}\, L^T \qquad (13)$$

where P, being the parameter vector being estimated, I is the pxp identity matrix, $(LA\text{-}I)^T$ and L^T are the transposes of the matrices $(LA\text{-}I)$ and L, respectively, and L being the minimum variance estimator (Jackson, 1979) given as

$$L = (A^T F_{qq}^{-1} A + F_{pp}^{-1})^{-1} A^T F_{qq}^{-1} A \tag{14}$$

where the letters have the same significances as explained earlier. In Eq. (13), first term is the covariance of resolving errors, and the second term is the covariance of random errors propagated into the estimates by the data. With the help of Eq. (14), the Eq. (13) may be simplified as

$$\mathrm{cov}\,(P) = (A^T F_{qq} A + F_{pp}^{-1})^{-1} \tag{15}$$

where the letters have the same significances as illustrated earlier.

In Eq. (13), the diagonal elements are the variance terms of the estimation vector P and off-diagonal elements are the covariance terms of P.

The parameter correlation coefficients of an inversion problem may be obtained after Jackson (1972) as

$$[P]_{zz'} = \frac{[\mathrm{cov}(P)]_{zz'}}{[\mathrm{cov}(P)]_{zz}^{1/2}[\mathrm{cov}(P)]_{z'z'}^{1/2}} \tag{16}$$

where the letters have the same significances as explained earlier.

It provides the parameter correlation coefficient between the P_z and $P_{z'}$ parameters. If $[P]_{zz'}$ is non zero, then the parameters P_z and $P_{z'}$ are correlated. It $[P]_{zz'}$ shows that the value of the parameter P_z affects the size of the parameter $P_{z'}$. On the other hand, if $P_{zz'}$ is near unity, the parameters P_z and $P_{z'}$ are strongly correlated and (nearly) linearly dependent.

The reduced chi-square (χ^2) statistic criterion, used as a goodness of the fit-test (Mills and Fitch, 1977) between the observed data and the theoretical data corresponding to an estimated model obtained by inversion or otherwise is given by

$$\chi^2 = \frac{1}{(q-1)} \sum_{m=1}^{q} \left[\frac{g_m(P,S_m) - g_m(\hat{P},S_m)}{\sigma_m} \right]^2 \tag{17}$$

where $g_m\,(P, S_m)$, σ_m and $g_m\,(\hat{P}, S_m)$ are the observed data, the corresponding standard deviation and the theoretical data for an estimated model $\hat{P}$ respectively for the m-th frequency S_m in which S is a vector containing the frequency values and q being the number of data. In applying this criterion, it is assumed that the data are normally distributed with zero mean and known variances in the presence of random errors, which are also assumed to be normally distributed. If a model is to be acceptable, then

$$\chi^2 \leq 1 \tag{18}$$

should hold good.

3. State-of-the-art GMS inversion problem

The subject of one dimensional inversions of GMS data for finding the conductivity depth profile of the horizontally stratified/concentrically stratified (spherically symmetric) Earth model contains a wide class of diverse techniques.

Lahiri and Price (1939) obtained a five parameter model for GMS data for a spherically symmetric Earth model. In this technique, conductivity varies as an arbitrary inverse power of radius below some arbitrary depth. The GMS inversion technique of Eckhardt (1963) is based on the solution of a first order differential equation. Parker (1970) applied the linear average approach of Backus and Gilbert (1967, 1968, 1970) for the inversion of geomagnetic data. Parker (1970) assumed the conductivity to be a continuous function of the radius for a spherically symmetric Earth model. A simple technique, based on a two-layer approximation, for the inversion of GMS data is due to Schmucker (1970). The conductivity and thickness for each resolved frequency component can be uniquely obtained in this inversion technique.

Bailey (1970) and Weidelt (1972) presented mathematical proofs showing uniqueness of geomagnetic depth sounding inversion problem. Bailey's (1970) technique is based on the solution of a nonlinear integro-differential equation and Weidelt's (1972) technique uses the solution of an integral equation. The techniques of Bailey (1970) and Weidelt (1972) exploited the analytical properties of response functions in a complex frequency plane. These inversion techniques are reported to be unsatisfactory while dealing with noisy and bandlimited field data. Jady (1974), using a variational approach, developed an inversion technique for geosounding data in order to estimate the conductivity structure of a concentrically stratified Earth model. The general built-in shortcomings of iterative least-squares algorithms were discussed in detail by Anderssen (1975) in the context of inversion of global electromagnetic induction data. Rokityansky (1982) has presented an exhaustive treatment pertaining to the various aspects of the geoelectromagnetic investigation on the Earth's crust and mantle. He has depicted in detail the geomagnetic induction studies undertaken in Russia and east Europe.

4. Results of GMS inversion study

A nine layered, isotropic, homogeneous and horizontally stratified resistivity (reciprocal of conductivity) thickness model of the Earth is chosen following Banks (1972). The proposed model satisfies the global observation that conductivity of the Earth increases quite sharply with depth everywhere (Banks, 1972). For a horizontally stratified Earth model, the wave number of the inducing field in the GMS study is given by $n(n+1)^{1/2}/R$, where n is the degree of the spherical harmonic of the inducing field given by unity (Banks, 1972) and R being the Earth's average radius taken to be 6371 km. Thus, wave number of the inducing field is 2.22 x 10^{-7} m^{-1}. The diagonal elements of the noise autocorrelation operator F_{qq} are taken to be squares of the respective standard deviations of the geomagnetic data (Fig. 2). Fixation of diagonal elements of solution autocorrelation operators in regard to layer parameters is somewhat difficult. Variability limits of ρ_1, ρ_2, ρ_3, ρ_4, ρ_5, ρ_6, ρ_7 and ρ_8 are assigned values which are squares of 6% of the respective a priori values; t_1, t_2, t_3, t_4, t_5, t_6, t_7 and t_8 parameters are assigned variability limits that are squares of 13% of the respective a priori values.

A priori model parameters consisting of resistivites and thicknesses of the proposed nine layered Earth model are obtained through computer-assisted interactive modeling (based on the forward problem solution of GMS corresponding to the considered frequency range and degree of the harmonic) subject to satisfying a given reduced (χ^2) statistic criterion (Mills and Fitch, 1977) between the observed geomagnetic responses and the theoretical ones, corresponding to an assumed resistivity-thickness model of the Earth. In the interactive modeling, the resistivity-thickness model of the Earth is selected corresponding to a χ^2 criterion of 5.73, keeping in view that the Earth model, so obtained, would be updated (hopefully) through the application of a stochastic inversion. Thus, the standard requirement of χ^2 criterion ≤1 is not adhered to. It is also an arduous task indeed, to obtain a model having a χ^2 criterion ≤1, using an interactive modeling process. The conductivity of the core of the Earth corresponding to the resistivity ρ_9 = 3.3x10^{-5} ohm·m (Rokityansky, 1982) is kept constant in the forward modeling. This conductivity is so large that the skin depth is less than 10.0 km even for the frequency 0.01 CPD, the lower cutoff frequency limit.

The inversion of GMS data (apparent resistivities) is initiated with a priori parameters as ρ_1 = 150.0, ρ_2 = 2.45, ρ_3 = 1.5, ρ_4 = 2.0, ρ_5 = 0.2, ρ_6 = 0.35, ρ_7 = 0.02 , ρ_8 = 0.002 all in ohm·m; t_1 = 300.0, t_2 = 135.0, t_3 = 240.0, t_4 = 430.0, t_5 = 600.0, t_6 = 550.0, t_7 = 450.0 and t_8 = 77.0 all in km. The conductivity of the core of the Earth (corresponding resistivity ρ_9) is kept constant all through the inversion process as in the forward modeling. Table 1 shows the convergence of a priori parameters as a function of iterations. Near surface resistivities ρ_1 and ρ_2 do not show any convergences from their a priori values of 150.0 and 2.45 ohm·m, respectively. It is caused by the sharp high frequency cutoff imposed (Parker, 1970). A priori values for ρ_3 and ρ_4 show convergences and arrive at the values of 0.96 and 0.86 ohm·m respectively. A priori values of ρ_5, ρ_6, ρ_7 and ρ_8 show insignificant variations in the iterations (Table 1). The resolving kernel plots (row elements from first to sixteenth row of the 16x16 resolution matrix) are shown in Fig. 2a to 2p in the form of bar diagrams in respect of estimations of ρ_1, ρ_2, ρ_3, ρ_4, ρ_5, ρ_6, ρ_7, ρ_8, t_1, t_2, t_3, t_4, t_5, t_6, t_7 and t_8 parameters respectively. Solid and the hollow bars in respect of the suit of diagrams of Fig. 2a to 2p present diagonal elements (resolutions attained) and the relevant non-diagonal elements of the matrices in respect of the mentioned parameter estimations. With references to Fig 2c, 2d, 2k, and 2l, we observe ρ_3, ρ_4, t_3 and t_4 are estimated with good resolutions having the values of 0.86, 0.85, 0.83 and 0.81, respectively (solid bars); non-diagonal elements pertaining to these estimates are also small enough. Thus, high resolution values given by diagonal elements coupled with negligibly small non-diagonal elements ensure attainments of satisfactory deltaness criteria in respect of ρ_3, ρ_4, t_3 and t_4 estimates.

Fig. 3 provides the geomagnetic responses and the corresponding standard deviations in terms of apparent resistivities, as computed through Eqs. (5) and (6) from Banks (1972) continuum data, plotted against the resolved frequencies. The geomagnetic sounding curve and the, finally, estimated parameters with their respective total estimation errors are also shown in Fig. 3. A perusal of Fig. 3 indicates that the geomagnetic sounding curve, attained after six iterations with a reduced χ^2 statistic criterion of 0.86, has taken a mean path in respect of the input data. The error components of the estimated parameters, as shown in Table 2, indicate that the resolution

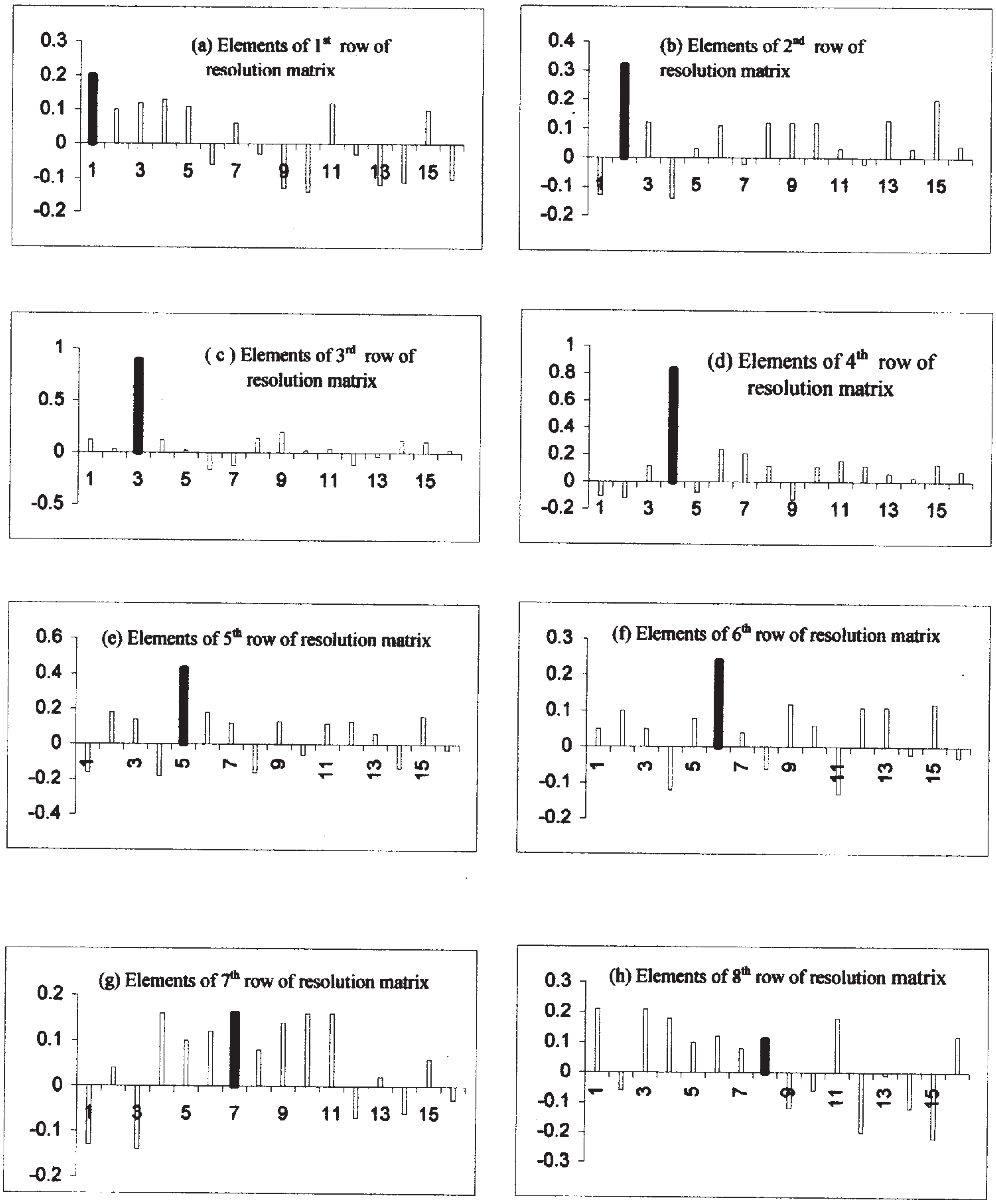

Fig. 2 - Bar diagram giving the elements of the estimated resolution matrix in respect of the parameter estimations.

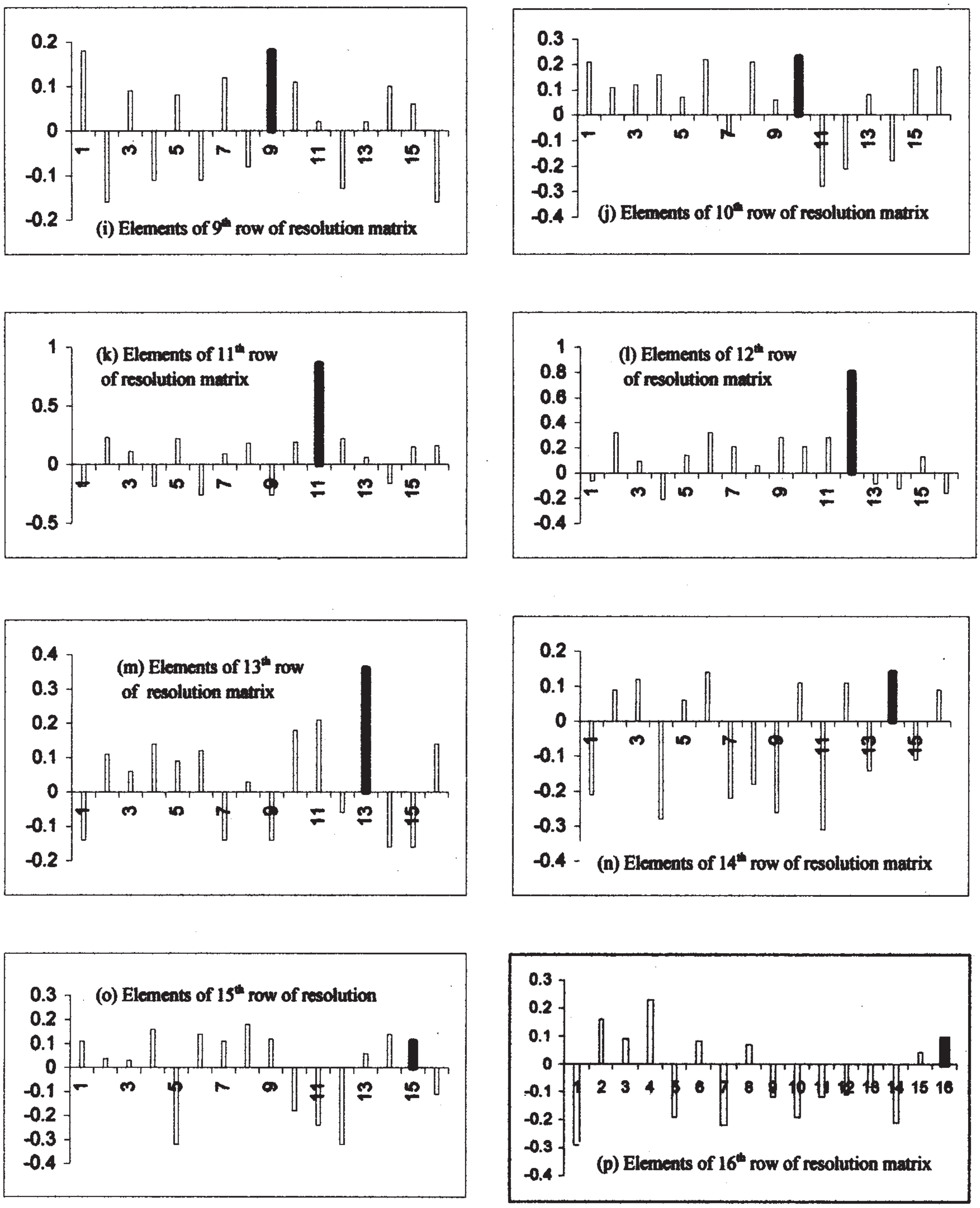

Fig. 2 - continued.

Table 1 - Convergence of a priori layer parameters as a function of iterations.

Parameters	A priori values	First iteration	Second iteration	Third iteration	Fourth iteration	Fifth iteration	Sixth iteration
ρ_1(ohm-m)	150.0	150.06	150.43	150.23	150.22	150.23	150.24
ρ_2 " ≤	2.45	2.41	2.43	2.43	2.43	2.43	2.44
ρ_3 " ≤	1.5	0.21	0.64	0.76	0.95	0.96	0.96
ρ_4 " ≤	2.0	0.73	0.73	0.73	0.87	0.86	0.86
ρ_5 " ≤	0.2	0.19	0.18	0.19	0.18	0.19	0.19
ρ_6 " ≤	0.35	0.35	0.38	0.34	0.34	0.34	0.34
ρ_7 " ≤	0.02	0.01	0.02	0.02	0.02	0.02	0.02
ρ_8 " ≤	0.002	0.002	0.002	0.001	0.002	0.002	0.002
t_1 (km)	300.0	301.27	301.73	301.76	301.78	301.78	301.72
t_2 " ≤	135.0	134.42	134.48	134.48	134.48	134.48	134.47
t_3 " ≤	240.0	190.23	208.37	195.36	140.36	137.36	128.81
t_4 " ≤	430.0	370.68	305.75	295.88	270.76	280.88	280.62
t_5 " ≤	600.0	585.75	590.78	592.7	592.82	592.76	592.88
t_6 " ≤	550.0	548.22	547.18	547.28	547.32	548.72	547.38
t_7 " ≤	450.0	450.12	450.28	450.29	450.18	450.21	450.28
t_8 " ≤	77.0	77.12	77.12	77.12	77.12	77.12	77.12

errors are higher than the corresponding random errors for the estimated ρ_1, ρ_2, ρ_3, ρ_4, ρ_5, ρ_6, ρ_7, ρ_8, t_1, t_2, t_3, t_4, t_5, t_6, t_7 and t_8 parameters of the Earth model. For ρ_3, resolution error equals random error and for ρ_4 , random error of 0.14 ohm·m exceeds the resolution error of 0.08 ohm·m. The parameter correlation coefficient matrix shown in Table 3 is characterized by the absence of strong correlations. Mention may be made of the correlations for ρ_2 - ρ_3 combination, ρ_3 - ρ_4 combination and ρ_4 - ρ_6 combination, showing correlation coefficients of –0.49, -0.38 and 0.4 respectively.

A study of the system matrix *A* (not shown here because of its huge size being a 35x16 matrix) appearing in each iteration shows that the geosounding apparent resistivity derivatives with respect to ρ_1, ρ_2, ρ_5, ρ_6, ρ_7, ρ_8, t_1, t_2, t_5, t_6, t_7 and t_8 parameters are negligibly small for most of the frequencies. Therefore, the lack of convergences in respect of the mentioned parameters may be attributable to the general insensitivity of GMS responses for these parameters, corresponding to the mentioned frequencies. According to Jupp and Vozoff (1975), such parameters may be called irrelevant ones. The insensitivity of the GMS responses to ρ_5, ρ_6, ρ_7, ρ_8, t_5, t_6, t_7 and t_8 is mostly put down to sharp low frequency cutoff clamped at 0.01 CPD, leading to a drastic reduction of skin depth and rapid fall in signal strength in the conducting interior of the Earth. The lack of convergences of t_1, t_2, ρ_1 and ρ_2 estimates are due to a sharp high frequency cutoff imposed at 0.2 CPD. In order to study if reduced noise level to the input data would lead to a possible improvement in parameter estimations, the geomagnetic inversion study was repeated with an approximately five percent noise level in the input data. The estimated parameters are not much different from the parameters already obtained. ρ_3, ρ_4, t_3, and t_4 estimates are marginally

Table 2 - Error components of the estimated parameters.

Estimation of resistivity values	Resolution error (ohm-m)	Random error (ohm-m)	Total estimation error (ohm-m)
ρ_1	5.05	0.01	5.05
ρ_2	0.41	0.04	0.41
ρ_3	0.06	0.06	0.08
ρ_4	0.08	0.14	0.16
ρ_5	0.15	0.06	0.16
ρ_6	0.20	0.08	0.22
ρ_7	0.06	0.01	0.06
ρ_8	0.02	0.0	0.02

Estimation of thickness values	Resolution error (km)	Random error (km)	Total estimation error (km)
t_1	4.5	1.18	4.65
t_2	2.93	1.19	3.16
t_3	3.83	1.44	4.09
t_4	5.55	1.82	5.84
t_5	6.45	1.38	6.60
t_6	6.52	0.0	6.52
t_7	5.87	0.0	5.87
t_8	2.83	0.0	2.83

improved over their previous values whilst ρ_1, ρ_2, ρ_5, ρ_6, ρ_7, ρ_8, t_1, t_2, t_5, t_6, t_7 and t_8 estimates continue to be the same as earlier.

In the present study, F_{pp} for GMS inversion problem is set up such that (i) the expected parameter variability limits should be able to provide a satisfactory trade-off between the resolution and estimation error of an estimate, (ii) the computed parameter perturbations should provide smooth values and (iii) a satisfactory deltaness criteria may be obtained for the resolution matrix. Through a perusal of Table 2, incorporating the total estimation errors in regard to the estimated layer parameters, it is observed that total estimation errors are less than the adopted standard deviations in regard to the a priori layer parameters obtained by computer-assisted interactive modeling. Therefore, stochastic inversion has improved upon the adopted standard deviations of the a priori layer parameters.

Through a perusal of Table 1, in conjunction with Table 2, tabulating the estimated layer parameters (resistivities and thicknesses) and the corresponding estimation errors respectively, it is observed that at a depth of 436.19±7.81 km (approx.) a resistivity of 0.96±0.08 ohm·m exists, which continues for a constant layer thickness of 128.81±4.09 km. Thus, the globally observed resistivity low starts at a depth of 436.19±7.81 km (approx.). Thereafter, the resistivity further diminishes to a value of 0.96±0.08 ohm·m, which goes for a thickness of 280.62±5.84 km starting from 565.0±11.9 km depth.

We thus confirm the low resistivity zone, starting at a depth of 436.19±7.81 km (approx.) continues up to a depth of 845.62±17.74 km (approx.). Estimated resistivities beyond the aforementioned depth of 845.62±17.74 km (approx.) are not reliable because of poor resolutions

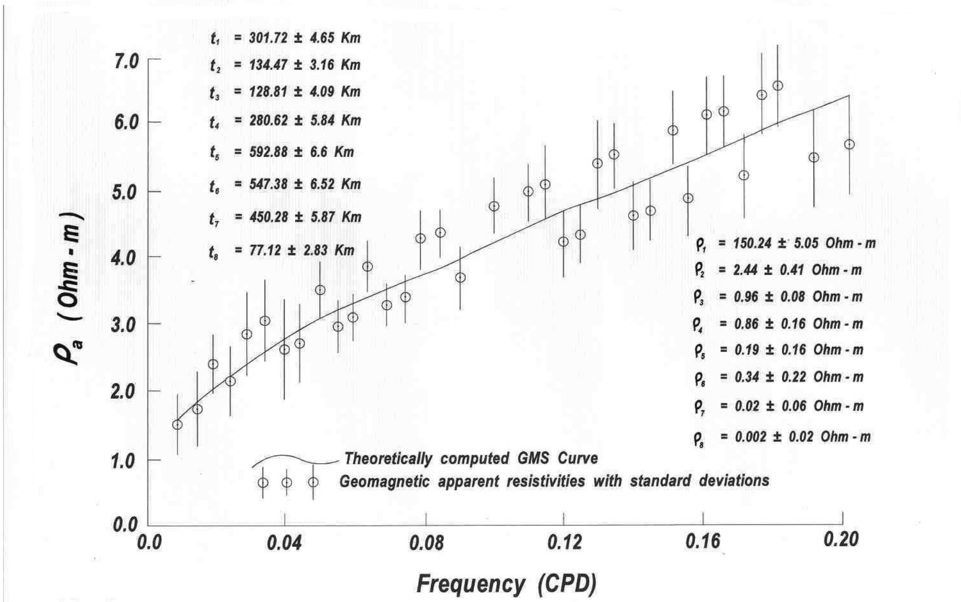

Fig. 3 - Geomagnetic responses (apparent resistivities) and the corresponding standard deviations, best-fit curve shown against the resolved frequencies and the estimated parameters along with the respective total estimation errors.

involved in the estimations of the parameters already mentioned. We have also not taken cognizance of the estimated shallow resistivity distribution within the depth span of 436.19±7.81 km for the poor resolutions involving ρ_1 and ρ_2 estimates.

For the best-fitting conductivity model of Banks (1969, Fig. 19), estimated resistivity at about 400 km depth gives a value of 1 ohm·m, which continues for about 200 km. Subsequently, at a depth of 500 km or more, the resistivity gets leveled off, thereafter resistivity continues to decrease monotonically, albeit rather slowly. Parker (1970) obtained a decrease of deeper resistivity starting from a 300 to 700 km depth as based on Banks (1969) low frequency data. Parker's (1970) resolution spread having been in excess of a 300 km, considerable uncertainty crops up in assigning the depth of resistivity lows. However, resistivity lows at 400 and 600 km depths caused by mineral phase changes mentioned earlier, are covered by Parker's (1970) resistivity lows.

The abrupt resistivity lows at the two depth levels of 400 and 600 km (approx.) are well known seismic discontinues believed to have been caused by prominent mineral phase changes taking place in the mantle (Rokityanski, 1982). Abrupt resistivity lows around the 400 and 700 km depths were also noted by Utada *et al.* (2003) while estimating electrical conductivity in the mid-mantle beneath the North Pacific region. Estimated resistivity values at the mentioned depth levels by Utada *et al.* (2003) closely follow the already well established trends already discussed.

Olsen (1998), using the C-response values of geomagnetic observations arrived at a conductivity structure, which shows the upper mantle has remarkably little structure with a monotonic decrease of resistivity from 100 ohm·m at a depth of 200 km to about 0.7 ohm·m below the depth of 1000 km. Based on the spherical harmonic analysis of solar daily variations,

Table 3 - Parameter correlation coefficient matrix of the estimated parameters.

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7	ρ_8	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
1.0	0.0	-0.01	0.01	0.02	0.02	0.03	0.02	0.01	0.02	0.03	0.05	0.03	0.01	0.02	0.01
	1.0	-0.49	0.17	-0.03	0.02	0.05	0.02	-0.01	-0.01	0.0	-0.01	-0.05	0.02	0.04	0.03
		1.00	-0.38	0.14	0.00	0.1	0.08	-0.48	-0.18	0.03	0.04	0.0	0.0	0.03	0.02
			1.00	0.04	0.40	0.01	0.01	0.11	0.03	-0.02	-0.12	-0.1	0.04	0.02	0.02
				1.00	0.24	0.02	0.01	0.01	-0.01	-0.03	-0.08	-0.08	0.02	0.04	0.03
					1.00	-0.01	-0.01	0.06	0.02	0.01	0.09	0.09	0.02	0.04	0.02
						1.0	0.02	0.01	0.02	0.011	0.04	0.07	0.03	0.02	0.08
							1.0	0.04	0.03	0.06	0.07	0.01	0.04	0.02	0.02
								1.0	-0.03	0.04	-0.02	-0.02	0.06	0.04	0.03
									1.0	0.06	0.04	0.01	0.06	0.04	0.03
										1.0	0.03	-0.01	0.02	0.03	0.04
											1.0	-0.04	0.06	0.04	0.07
												1.0	0.03	0.02	0.02
													1.0	0.01	0.06
														1.0	0.03
															1.0

Schmucker (1999) found the well established transition in mantle resistivity between 400 and 800 km depths from values close to 100 ohm·m to values below 1 ohm·m. The mentioned transition in resistivity, according to Schmucker (1999), could take place at 500 and 750 km depths in a single step around 600 km or with a smooth decent starting at 400 km ending below 800 km.

The resistivity low in the present work starts at a higher depth of 436.19±7.81 km (approx.). The depth level of two abrupt resistivity lows at 436.19±7.81 km and 565.0±11.9 km approximately, closely follow the postulated well known mineral phase changes at 400 and 600 km depths of the mantle.

Thus, the deeper resistivity-depth profile of the Earth, in general, shows a region of high resistivity (poorly resolved) covering the upper several hundred kilometers, leading to a zone of low resistivity which continues for about 1000 km or so, resistivity remains ill defined from 1000 km down.

5. Conclusions

The performance of the GMS inversion study in the light of the findings is satisfactory. Out of the sixteen parameters, defining the layer thicknesses and resistivities of the Earth model, four parameters, namely; ρ_3, ρ_4, t_3, and t_4 have been estimated with good resolutions. The rest of the parameters, namely; ρ_1, ρ_2, ρ_5, ρ_6, ρ_7, ρ_8, t_1, t_2, t_5, t_6, t_7 and t_8 elude adequate estimations. Thus, inadequate frequency bandwidth coupled with noise contaminated data are principally responsible for non estimations of ρ_1, ρ_2, ρ_5, ρ_6, ρ_7, ρ_8, t_1, t_2, t_5, t_6, t_7 and t_8 parameters.

Improper parameter estimates in respect of ρ_3, ρ_4, t_1 and t_2 are most likely due to sharp high frequency cutoff imposed at 0.2 CPD. Non-convergences of parameters ρ_5, ρ_6, ρ_7, ρ_8, t_5, t_6, t_7 and

t_8 and the resultant improper resolutions, leading to non estimations in respect of the aforestated parameters are most probably attributable to sharp low frequency cutoff imposed at 0.01 CPD, drastically reducing the skin depth as mentioned earlier.

If it were possible to extend the lower frequency limit beyond 0.01 CPD by incorporating the 11-year sun spot variation, perhaps, a deeper conductivity structure of the Earth would have been adequately deciphered. However, attempts at retrieving the 11-year sun spot variation by Eckhardt *et al.* (1963) were in the negative.

The solution obtained by stochastic inversion in general is not unique. It is evident from the fact that models satisfying the data belong to Hilbert space, which includes all the possible models. Such models cannot be unique for the given data, as only a finite collection of observed data is available (Backus and Gilbert, 1967). The properties of the Earth, on the other hand, are likely to be infinitely variable if viewed on a sufficiently finer scale. In other words, an infinite dimensional vector in an abstract infinite dimensional parameter space fits the exact description of the model.

Notwithstanding the aforestated inherent shortcomings, the stochastic inversion utilized herein has been successful in estimating the vertical resistivity distribution of the Earth for about 410 km, commencing at a depth of around 436 km from the Earth's surface, using the band-limited and noise contaminated geomagnetic continuum data.

Acknowledgements. Authors are grateful to Sri. A. K. Agrawal, Retd. Dy. Director General (Geophysics) for scrutinizing this paper. Authors express their indebtedness to the reviewers for their excellent suggestions which helped in improving the quality of the presentation. Kind permission to publish this paper by Director General, Geological Survey of India is thankfully acknowledged.

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