# A study about the aftershock sequence of 27 December 2003 in Loyalty Islands

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***ABSTRACT*** Investigating both aftershock behavior and a wide spectrum of parameters may find the key to better explain the mechanism of seismicity as a whole. In particular, the purpose of this work is to highlight some methodological aspects related to the observation of possible anomalies in the temporal decay of aftershock sequence. We have focalized our work mainly on the analysis of the Loyalty Islands seismic sequence of 27 December 2003 by means of some seismic parameters that showed remarkable variations during the aftershocks decay. The magnitude of the mainshock was $M = 7.3$. We also made a comparison with the sequence that happened in New Guinea on 29 April 1996, with mainshock magnitude $M = 7.5$. The daily temporal series of the shocks observed can be considered a sum of a deterministic contribution and of a stochastic contribution. If the decay can be modeled as a non-stationary Poissonian process where the intensity function is equal to $n(t) = K \cdot (t+c)^{-p} + K_1$, the number of aftershocks in a small time interval $\Delta t$ is the mean value $n(t) \cdot \Delta t$, with a standard deviation $\sigma = \sqrt{n(t) \cdot \Delta t}$. We observe that, before the occurrence of large aftershocks, there are some $\Delta/\sigma$ values that can be considered as seismic anomalies. The data concerning the temporal series, checked according to completeness criteria, come from the web site of NEIC-USGS data bank.

## 1. Introduction

About a third of the earthquakes detected all over the world are aftershocks, whose distinguishing feature is clustering in space and time. Their temporal distribution often follows a regular trend, first observed by Omori (1894). The aftershocks are very frequent immediately after the mainshock, but the frequency decreases rapidly, with a power law decay following the Omori law $n(t) = n_1 \cdot t^{-p}$, where $p$, the decay parameter, is nearly 1 and $n_1$ is related to the number of shocks on the first day (Pasquale, 1985). The aftershocks principally tend to involve fault segments near the mainshock. The mainshock increases the probability of following shocks by a triggering action (Matsu'ura, 1986).

## 2. Aftershock temporal series: some theories and models

An earthquake big enough to cause damage will likely be followed by several aftershocks of considerable size in a short time span. Earthquakes located within a characteristic distance from the main event can be considered aftershocks. This distance is usually considered equal to one or two lengths of the fault segment related to the mainshock. There are some empirical relations that allow to evaluate, in a more accurate way, the length of the zone fractured by the mainshock when the magnitude of this event is known. One of these, relating to the fractured fault segment length $L$ to the seismic event surface wave magnitude $M_S$ was found by Utsu (1969):

$$\lg_{10} L = 0.5 \cdot M_s - 1.8 \tag{1}$$

The main parameters of aftershock sequences are: the spatial distribution, the total number of aftershocks and the temporal decay rate of the aftershock sequence. Aftershocks are frequent immediately after the mainshock and decay, approximately, as the inverse of the elapsed time in the Omori (1894) law. The magnitudes of the aftershocks generally follow the Gutenberg-Richter (G-R) relation (Gutenberg and Richter, 1954):

$$\lg_{10} N(M) = a - b \cdot M \tag{2}$$

where $N$ is the number of shocks with magnitude equal or greater than $M$, while $a$ and $b$ are characteristic parameters for the sequence.

The sequence examined is described by the modified Omori formula (Utsu, 1961):

$$n(t) = K(t+c)^{-p}$$

where $n(t)$ is the frequency distribution, $K$, $c$ and $p$ are constants. The original Omori (1894) law was found empirically and considers $p$=1 and $c$=0. The number of events will rapidly decrease immediately after the mainshock, until it is lost within the background seismicity of the examined area. It has also been observed that sometimes a large aftershock occurs in an area near to the mainshock rupture zone. This aftershock is followed by its own aftershocks, called “secondary aftershocks”. Moreover, occasionally anomalous aftershock activity decreases or increases are observed. So the temporal decay is a matter of study since it gives information about the seismogenic process and the physical conditions of the source region.

From a comparison between the maximum likelihood fit and the temporal decay, Page (1968) states that if the fracture process can be considered stochastic then there will be random fluctuations of the experimental data (number of shocks) around the theoretical value predicted by the decay law. The number of aftershocks $N$ in a small time interval $\Delta t$ follows a Poissonian trend with a mean $n(t) \cdot \Delta t$ and a standard deviation $\sqrt{n(t) \cdot \Delta t}$ .

The confidence interval, at about 99%, for $N$ is given by:

$$\left[ n \cdot \Delta t - 2.5 \cdot \sqrt{n \cdot \Delta t}; n \cdot \Delta t + 2.5 \cdot \sqrt{n \cdot \Delta t} \right].$$

Matsu’ura (1986) plots the cumulative number of aftershocks versus the “frequency linearized time”, $\tau$. In this way, the fit of the Omori (1894) formula to the temporal features of the sequences is checked. The aim of her work is to point out the anomalies that can occur during an aftershock sequence. The frequency linearized time is defined as:

$$\tau = \int_0^t n(s) \cdot ds,$$

where the integral represents the cumulative number of aftershocks using the parameters estimated in $n(t)=K(t+c)^{-p}$. The cumulative number of aftershocks increases linearly with $\tau$ for sequences that are well represented by the modified Omori formula (Utsu, 1961). During the occurrence of a decay anomaly, i.e. a decrease or increase of activity, the cumulative number vs. $\tau$ fluctuates around the straight line that represents the Omori formula for the period under study.

This anomalous change in the $p$ value of the modified Omori formula could give useful information about a likely large aftershock occurrence. By studying an aftershock sequence in real time, it would be possible to observe an anomaly immediately after its beginning, to notice the quiescence and recovery stages and then to evaluate the probability of a forthcoming large aftershock (Matsu'ura, 1986).

### 3. Data and analysis

The study is focalized on the Loyalty Islands seismic sequence that happened on 27 December 2003, $S_1$. In the following part it an other seismic sequence, $S_2$ (enclosed here as a comparison), that happened on 29 April 1996 in New Guinea (Caccamo *et al.,* 2005) is reported.

We calculated "a priori" the dimensions of the area involved using the Utsu (1969) empirical Eq. (1). We acquired data in a square with sides at a 3$L$ distance from the mainshock epicenter and in the first 10 days starting from the mainshock. Using these data, we calculated the "barycenter" of the aftershock sequence (D'Amico *et al.,* 2004); the barycenter coordinates are calculated as an arithmetical mean of the latitude and longitude values of the shocks:

$$B_{LAT} = \frac{\sum_{i=1}^{n} Lat_i}{n}; B_{LON} = \frac{\sum_{i=1}^{n} Lat_i}{n} \quad (3)$$

where $Lat_i$ and $Lon_i$ are, respectively, the latitude and the longitude of the *i-th* aftershock epicenter, and $n$ is the number of aftershocks with a magnitude $M \geq 4.0$.

From the point of view of the completeness, the first 10 days are representative of a sequence, because about 70% of the aftershocks occurs within this initial period. In this sequence, the aftershocks that occurred in the first 10 days are greater than 89% of the total aftershocks. The final rectangular sector is centered on the sequence barycenter and has its sides at a distance from this point, in terms of latitude and longitude, equal to 1.5$L$.

The evaluation of the completeness threshold of a data set is made using the G-R diagram (Fig. 1). In Fig. 1a the G-R diagram for $S_1$ is shown, Fig. 1b shows the same diagram for $S_2$. The

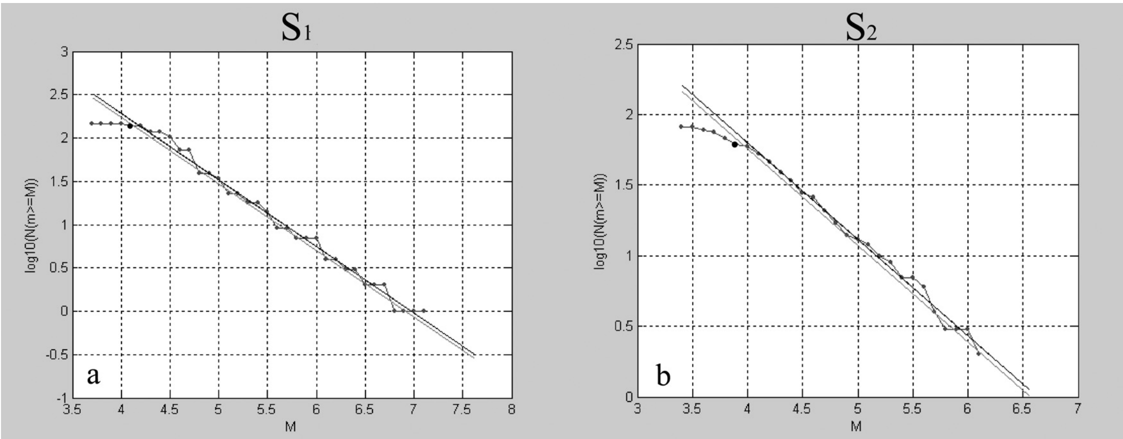


Fig. 1 - G-R relation, computed in the first ten days: a) for the Loyalty Islands seismic sequence ($S_1$) on 27 December 2003 and b) for the New Guinea seismic sequence ($S_2$) on 29 April 1996. From here we can see (biggest black dots) the minimum magnitude of completeness $M_c$.

point at which the observed data move away from the theoretical straight line, generally, indicates the lowest magnitude at which we can consider the data set complete. To objectively establish the minimum magnitude of completeness of a sequence, the criterion used by us was to calculate the

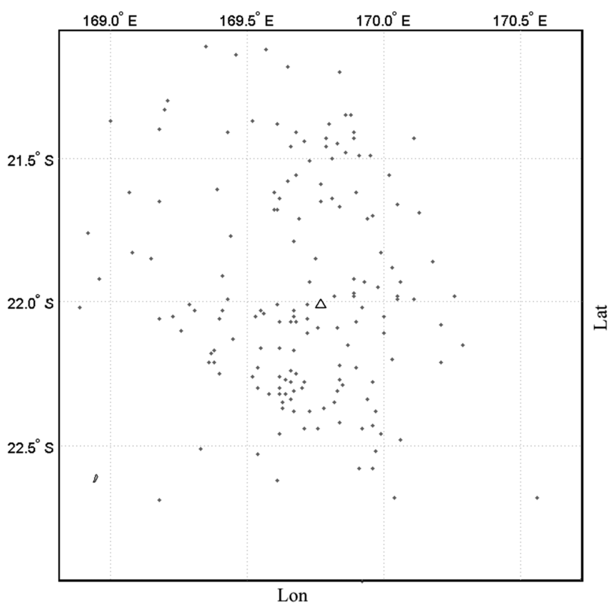


Fig. 2 - Loyalty Islands seismic sequence. The map is centered on the coordinates (triangle) of the *M* 7.3 mainshock on December 27, 2003: 22.01°S, 169.77°E. The epicentral distribution of events with magnitude $M \geq 4.0$ in the first ten days since the occurrence of the mainshock is shown. Latitude range: 23.92°S to 20.1°S; longitude range:167.86°E to 171.68°E. Depth range: 0 km to 70 km.

straight line regression (dark lines, Fig. 1) for aftershocks occurring during the first 10 days, and to consider the first point (biggest black dots in Fig. 1) that, starting from the lower magnitude values (top of the G-R plot, Fig. 1), is below a parallel straight line (light lines, Fig. 1). If $y$ and $Y$ represent, respectively, the cumulative number of events in the dark and the light line, so the light line is obtained by $Y= y$-10%$y$. In this case, the magnitude of completeness $M_c$ is 4.1 for the Loyalty Islands seismic sequence, where as it is $M_c$=3.9 for the New Guinea seismic sequence, computed with data related to the epicentral distribution for events with magnitude $M$≥1 reported on the web site of NEIC-USGS data-bank (http://neic.usgs.gov/neis/epic/) in the first 10 days after the mainshock and in a square centered on the mainshock (magnitude $M$=7.3 for $S_1$) at 22.01°S, 169.77°E. In Fig. 2, the epicentral distribution of events for $S_1$ with magnitude $M$≥4.0 in the first ten days since the occurrence of the mainshock (latitude range: 23.92°S to 20.1°S is reported; longitude range: 167.86°E to 171.68°E. Depth range: 0 km to 70 km). The sides of this square are 1.5 fault lengths ($L$=70 km) from the mainshock epicenter, with $L$ estimated by the Eq. (1) of Utsu (1969). The depth of earthquakes is less or equal to 70 km.

In Fig. 3, the data used to obtain the temporal trends related to the parameters shown in Figs. 4, 5, 6 and 7 are reported. The map is centered on the coordinates (black dot) of the "barycenter": 21.94°S, 169.69°E. The epicentral distribution of events of $S_1$ with magnitude $M≥Mc$ in a period $D$=71 days since the occurrence of the mainshock is shown.

Fig. 4 shows a cumulative number of aftershocks per day for $S_1$. It is possible to notice an increase of seismicity on the eighth day.

Fig. 5 shows cumulative energy of $S_1$. Such value can be expressed as,

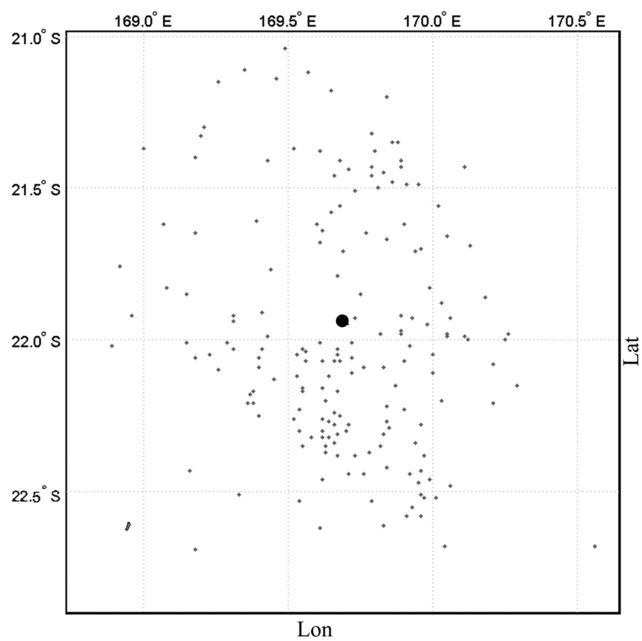


Fig. 3 - Loyalty Islands seismic sequence. The map is centered on the coordinates (black circle) of the "barycenter": 21.94°S, 169.69°E. The epicentral distribution of events with magnitude $M≥M_c$ in a period $D$=71 days since the occurrence of the mainshock is shown. Latitude range: 22.9°S to 20.98°S; longitude range: 168.73°E to 170.65°E. Depth range: 0 km to 70 km.

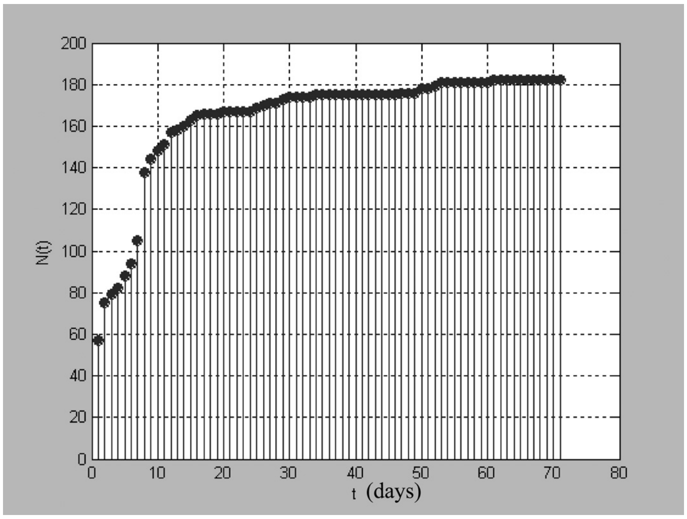


Fig. 4 - Cumulative number of shocks (relative to the epicentral map in Fig. 3) per day during the duration of the Loyalty Islands seismic sequence on December 27, 2003 (71 days).

$$E_n = \sum_{i=1}^{n} \sum_{j=1}^{m_i} e_{ij} \qquad n = 1,...,71,$$

where $e_{ij}$ represents the energy of *j*-th aftershock in the *i*-th day, $m_i$ the number of aftershocks in the *i*-th day and *n* is the temporal length, in days, of the sequence. The empirical magnitude-energy relation used is:

$$\log_{10} E = 1.5M + 4.2,$$

with energy *E* in Joule (Gutenberg and Richter, 1942, 1956; Udìas, 1999). The energy involved in the decay process of $S_1$ was about 2.57x1015 J. From this we have not noticed significant anomalies during the decay process of the sequence.

In Figs. 6 and 7, the variations of *p* and *b* values for $S_1$ are shown (Gutenberg and Richter, 1954), where only the central graphs represent the parameter variations, the others represent the interval confidence at 95% level. The *b*- value is related to the stress distribution and to homogeneity of the source region (Mogi, 1962). The *p*-value is related to the temporal trend of shocks (Matsu'ura, 1986). Their variations after ten days, from the occurrence of the mainshock, are within the interval confidence.

Finding the anomalies of the temporal trend occurring in the seismic sequences before a large aftershock, is one of the purposes of this work. These anomalies are thought to be seismic activity variations. The possible explanation for this phenomenon could be found in terms of fracture of an asperity related to the large aftershock. The fracture of neighboring asperities, initiated after the mainshock of a particular asperity, presumably due to the redistribution of the strain energy accumulated within an asperity, released by the mainshock, results in the enhancement of the

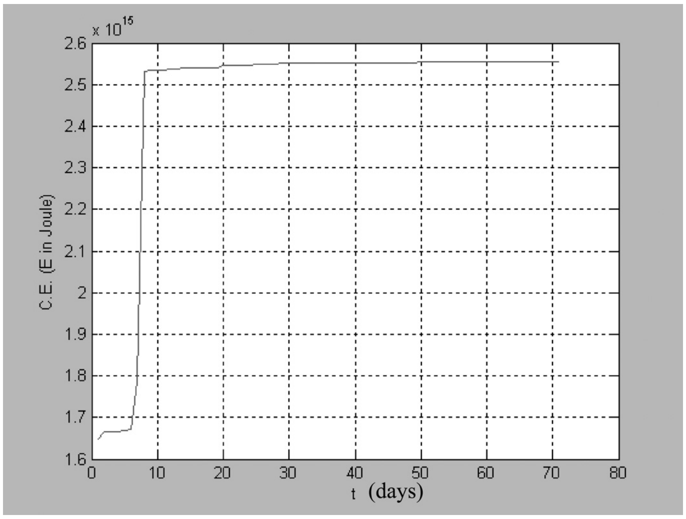


Fig. 5 - Cumulative energy of the sequence shocks (the relative epicentral map is reported in Fig. 3). The energy involved in the decay process was about 2.57x1015 J.

stress concentration around the nearest neighboring intact asperities (Lei, 2003).

For the temporal duration *D* of the seismic sequence, $D=n_1+n_2$, where $n_1$ is the number of days equal to the number of aftershocks in the first 24 hours starting from the occurrence of the mainshock, and $n_2$ is the number of days, after $n_1$, that reach and include 10 days in which there are not earthquakes. This is an “a priori” hypothesis to cut, the sequence appropriately. We defined this hypothesis because we often noticed that at the end of the sequence, the number of

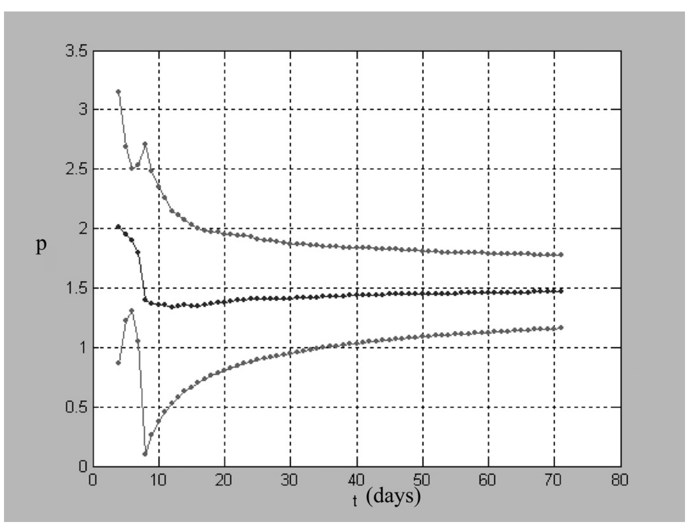


Fig. 6 - Variations of *p*-value during the decay process of the Loyalty Island seismic sequence on December 27, 2003 (the relative epicentral map is reported in Fig. 3). Central graph represents the parameter variation, while the other graphs represent the interval confidence at 95% level. This value is related to the temporal trend of shocks.

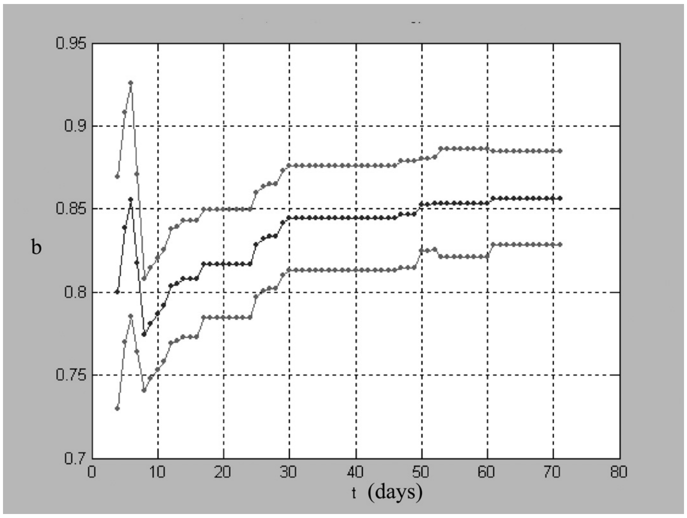


Fig. 7 - Variations of *b*-value during the decay process of the Loyalty Islands seismic sequence on December 27, 2003 (the relative epicentral map is reported in Fig. 3). Central graph represents the parameter variation, while the other graphs represent the confidence interval at 95% level. This value is related to the stress distribution and to homogeneity of the source region.

aftershocks is equal to zero, considering those that have a magnitude greater than 4. We noted this also for other sequences, that we are studying (Utsu *et al.*,1995).

We suppose that, if the decay generally follows a classical Omori formula (hyperbolic law), the number of aftershocks will be mixed with the background seismicity when we consider a number of days equal to the number of shocks of the first day.

In mathematical terms, the observed temporal series of the shocks per day can be considered as a sum of a deterministic contribution and a stochastic contribution. These contributions are, respectively, the aftershock decay with a power law, i.e. the modified Omori formula (Utsu, 1961), and the random fluctuations around a mean value represented by the above-mentioned power law.

Since the decay phenomenon can be modeled as a non-stationary Poissonian process, where the intensity function is equal to $n(t) = K\cdot(t+c)^{-p}$ (Page, 1968; Matsu'ura, 1986), the number of aftershocks in a time interval $\Delta t$ is, as a mean value, $n(t)\cdot\Delta t$, with a standard deviation $\sigma = \sqrt{n(t)\cdot\Delta t}$ . Considering, however, a daily sampling of the aftershocks ($\Delta t$=1 day), the expected number of shocks per day is then $n(t)$ with a standard deviation $\sigma = \sqrt{n}$. We may expect that the random fluctuations around the mean value $n(t)$ will be within a $2.5\sigma$ range (that represents approximately 99%): in particular, data with

$$\left| n_{obs} - n_{calc} \right| > 2.5\sigma,$$

have an occurrence of probability less than about 1%. We find the anomalies by estimating the ratios of the absolute values of differences between the observed temporal trend and the theoretical trend, that is the fit, and the standard deviations of the theoretical values.

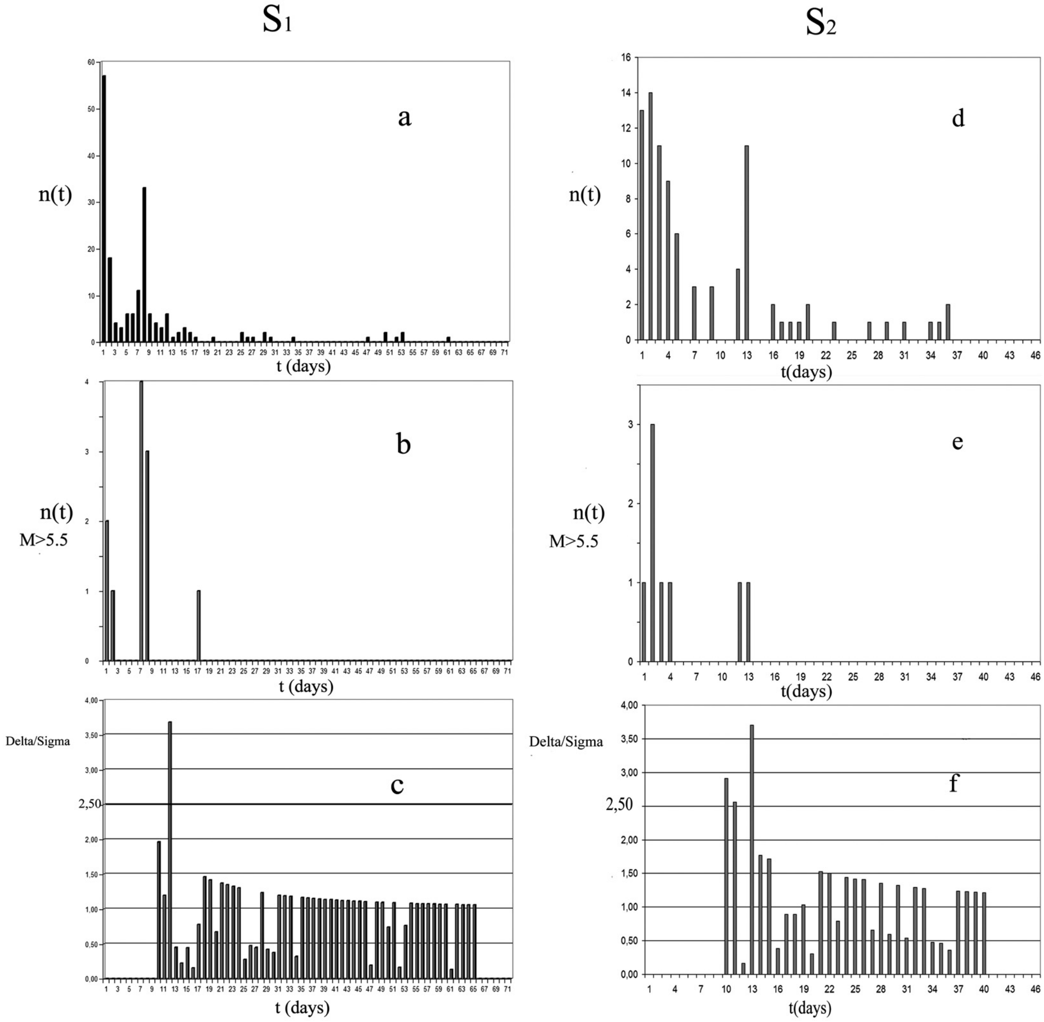


Fig. 8 - a) Temporal trend *n(t)*, number of shocks per day, for the events with a magnitude $M \geq M_c$, where $M_c$=4.1, of the Loyalty Islands seismic sequence ($S_1$); b) temporal trends for the shocks with magnitude $M$>5.5 for $S_1$; c) $\Delta/\sigma$ values versus time *t*, excluding the first 10 days for $S_1$. There is one $\Delta/\sigma$ value exceeding the threshold: $\Delta/\sigma$=3.68 on the 12th day with relative shock with magnitudes $M$=5.8 (plot 8b); d) Temporal trend *n(t)*, number of shocks per day, for events with magnitude $M \geq M_c$, where $M_c$=3.9, of the New Guinea seismic sequence ($S_2$); e) temporal trends for the shocks with magnitude $M$>5.5 for $S_2$; f) $\Delta/\sigma$ values versus time t, excluding the first 10 days for $S_2$. There are 3 $\Delta/\sigma$ values exceeding the threshold: $\Delta/\sigma$=2.91 on the 10th day, $\Delta/\sigma$=2.56 on 11th day and $\Delta/\sigma$=3.70 on the 13th day, with shocks with magnitudes $M$=6.3 and $M$=5.6 on the 12th and 13th day respectively (plot 8e). The temporal scales of the three plots a), b) and c) and of the three plots d), e) and f) are the same, with zero time at the mainshock occurrence.

The complete series of the real data is

$$\{n_{obs}(t_j)\} \quad \text{with} \quad j=1,\ldots,D.$$

Hypothesizing that an anomaly occurs several days before a large aftershock, depending on its magnitude, we introduced a constant shift $s$=6. The predicted theoretical value of the series, obtained by the extrapolation, is

$\{n_{calc}(t_k)\}$ with $k=h+s,\ldots,D,$

and $h=2\cdot v$, where $v=2$ is the number of unknown variables, $k$ and $p$, of the function we use here:

$$n(t) = k \cdot t^{-p} + k_1. \qquad (4)$$

The generic element of this series, $n_{calc}(t_g)$, is obtained by using the subset $\{n_{obs}(t_e)\}$ with $e=1,\ldots,g\text{-}s$ (excluding the last six data from the elaboration).

The differences between the calculated data and the observed data, $\Delta(t_k)$, are obtained by

$$\Delta(t_k) = \left|n_{obs}(t_k) - n_{calc}(t_k)\right|.$$

When $\Delta(t_k) > 2.5 \cdot \sqrt{n_{obs}(t_k) \cdot \Delta t}$, we considered this as an "anomaly".

Synthetically, as explained above, the first point of the extrapolation is relative to $t_k$=10 days. We considered the term $K_1$ [Eq. (4)] to take into account the background seismicity. This parameter should be typical of the seismicity of an area. To evaluate it, we should consider several interseismical periods of the area. We assumed $K_1$=1 in this work, since the aftershock decay is a discrete phenomenon thus the function we are considering here stretches asymptotically to 1 (Utsu *et al.,*1995; Caccamo *et al.*, 2005).

The software for the calculations is Matlab, allowing us to solve problems in "non-linear least squares", with Newton-like optimization techniques (Dennis, 1977; More, 1977; Coleman and Li., 1994, 1996) suitable for the modified Omori formula, which is a power law. Because of the non-linearity of the problem [*n(t)* is a power law], the fit is made using non-linear curve fitting methods. In particular, these methods are based on optimization algorithms like the interior-reflective Newton method (Coleman and Li, 1994, 1996; Dennis, 1977).

Using these data the temporal trend *n(t)*, number of shocks per day for $S_1$ and $S_2$, can be calculated as shown in Figs. 8a and 8d. The three plots of Fig. 8, for $S_1$ and $S_2$ respectively, have the same temporal scale. In Figs. 8b and 8e shocks with magnitude $M$>5.5 are shown. In Figs. 8c and 8f the $\Delta/\sigma$ values versus time $t$ in days are shown, excluding the initial 10 days. Regarding to $S_1$ there is one $\Delta/\sigma$ value exceeding this threshold (2.5 as before described ): $\Delta/\sigma$=3.68 on the 12th day with shock with magnitude $M$=5.8 17th day (Figs. 8b and 8c). Regarding $S_2$ there are 3 $\Delta/\sigma$ values exceeding the threshold: $\Delta/\sigma$=2.91 on the 10th day, $\Delta/\sigma$=2.56 on 11th day and $\Delta/\sigma$=3.70 on the 13th day, with shocks with magnitudes $M$=6.3 and $M$=5.6 on the 12th and 13th day respectively (Figs. 8e and 8f).

## 4. Conclusions

It is observed that, several days before the occurrence of large aftershocks, it is possible to point out the anomalies in the decay (represented in Figs. 8c and 8f), considered as deviations from the mean value *n(t)* greater than a 2.5 quantity, that, likely, are not randomlike fluctuations but probably precursors.

The data concerning the temporal series, checked according to completeness criteria, come from the NEIC-USGS data bank (http://neic.usgs.gov/neis/epic/). From the scientific point of view, this kind of analysis offers new horizons to the research about seismic anomalies and precursors.

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