The role of the interference polynomial in the Euler deconvolution algorithm

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ABSTRACT The Euler deconvolution method belongs to the most popular semi-automated interpretation techniques in gravimetry and magnetometry. During its application on real data, it was realised that it can be performed only with the additional constant term B (background term), because of the regional field components. In the proposed contribution, the idea of the interference polynomial from the Werner deconvolution is introduced into the Euler deconvolution algorithm. Its role can be seen on two levels: a) it helps to deal with the influence of neighbouring anomalies, b) only thanks to this modification is it possible to obtain real solutions for the correct value of the structural index of a contact structure in gravimetry ($N = -1$). On the other hand, the proposed modification amplifies the instability of the method and either the smoothing of the input field or the stabilisation of its gradient evaluation must be realised.

1. Introduction

The interpretation outputs from potential field methods play an important role in geophysical exploration [e.g. the introduction of independent information into the process of PreSDM in reflection seismics; e.g. Anderson *et al*. (2002)]. The Euler deconvolution method belongs to the most discussed interpretation methods in potential fields over the last ten years. It uses the Euler homogeneity theorem in an effective way, appling it on functions that describe the anomalous potential field (gravitational, magnetic) of simple sources. Various authors from the 1950's and 1960's (Smelie, 1956; Hood, 1965) have recognized that direct problems, for a variety of simple sources in gravimetry and magnetometry, are described by means of relatively simple rational functions, which are homogeneous in the sense of Euler's theorem. For a homogeneous function $f(x,y,z)$ (3D-problem) or $f(x,z)$ (2D-problem) the interpretation equation can be written in the form:

$$\left(x-x_0\right)\frac{\partial f}{\partial x}+\left(y-y_0\right)\frac{\partial f}{\partial y}+\left(z-z_0\right)\frac{\partial f}{\partial z}=-Nf, \qquad \text{(3D-problem)} \qquad (1)$$

$$\left(x-x_0\right)\frac{\partial f}{\partial x}+\left(z-z_0\right)\frac{\partial f}{\partial z}=-Nf, \qquad \text{(2D-problem)} \qquad (2)$$

where x, y and z are the coordinates of the point, where the potential function f (potential or its higher derivatives) is defined; x_0, y_0 and z_0 are the coordinates of the source; N is the so-called structural index (sometimes the abbreviation *SI* is used). The role of the value N during the application of the method is very important, it describes the type of source, whose contribution

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Table 1 - Values of the structural index *N* for various anomalous bodies in gravimetry and magnetometry.

elementary body model (source type)	number of infinite dimensions	N used in magnetometry	N used in gravimetry
sphere	0	3	2
pipe (vert. cylinder)	1 (z)	2	1
horizontal cylinder	1 (x-y)	2	1
dike (sheet)	2 (z and x-y)	1	0
sill	2 (x and y)	1	0
contact	3 (x, y and z)	0	-1

is recognised in the interpreted data. Thompson (1982) defined it as a measure of the rate of change with distance of the potential function - but this property definition holds only for point or line sources, this will be discussed later in the text. The values of *N* were derived and published by various authors (e.g.: Thompson, 1982; Reid *et al*., 1990; Stavrev, 1997; Yaghoobian *et al*., 1992), a summary of the sources most used is presented in Table 1. This contribution focuses on the classical algorithm, where the value of *N* is assumed to be known, but various approaches on how to get its value directly during potential field data interpretation also exists (e.g. Fedi and Florio, 2002).

It follows that from the values presented a relationship between the value *N* and the number of infinite dimensions of the model body exists, as well as a constant shift between the values used for the magnetic and gravity fields. These properties were generalised by Stavrev (1997). The negative value of the index for the contact model in gravimetry comes directly from this generalisation and it may look a little bit strange in the light of Thompson's definition. It would express a fact that the field is growing with the distance from the body and it is often interpreted in such a way (e.g. Geosoft, 1994; FitzGerald *et al*., 2004). This is, in my opinion, a "trap" of this mentioned definition. Being really rigorous, we have to mention that also the sill and dike models have a strange index in gravimetry - zero (which would describe, when the field is not changing with distance). Thompson's definition holds only for functions f, which can be described by rational functions of type $1/(r^N)$ - a pole, dipole and lines of poles and dipoles (sphere, vertical and horizontal cylinder), as was mentioned at the beginning of this text. For the expression of the direct problem of more complicated bodies (bodies of a sheet form bodies, step, contact), we use functions of the arctan( ) and/or ln( ) type. It is very interesting that the derivations of the *N* value for these bodies give in general values of 0 or even -1. So, in the scope of these facts, it is more correct to speak about *N* only as a structural index. Here it is very important to mention that the gravity field of the contact model is not strictly homogenous in the sense of the Euler theorem (Pasteka, 2001).

2. Introduction of the interference polynomial

During the realisation of the method, experts have recognised that the method based on the numerical solution of the Eq. (1) [or Eq. (2)] in a moving window gives appropriate results only for well-separated anomalies, without any interference or influence of regional components of the anomalous field. This fact has inspired Thomspon (1982) to the introducing the *B* term (so called "background" term) into the right-hand side of the Eqs. (1) and (2):

$$\left(x-x_0\right)\frac{\partial f}{\partial x}+\left(y-y_0\right)\frac{\partial f}{\partial y}+\left(z-z_0\right)\frac{\partial f}{\partial z}=-N\left(f-B\right), \qquad \text{(3D-problem)} \qquad (3)$$

$$\left(x-x_0\right)\frac{\partial f}{\partial x}+\left(z-z_0\right)\frac{\partial f}{\partial z}=-N\left(f-B\right), \qquad \text{(2D-problem)} \qquad (4)$$

The Thompson's (1982) idea can be enlarged: instead of a constant term, we can take into account a polynomial of a higher degree:

$$\left(x-x_0\right)\frac{\partial f}{\partial x}+\left(y-y_0\right)\frac{\partial f}{\partial y}+\left(z-z_0\right)\frac{\partial f}{\partial z}= \qquad (5)$$

$$=-Nf+A_0+A_1x+A_2y+A_3xy+A_4x^2+A_5y^2+A_6x^2y+A_7xy^2+A_8x^3+A_9y^3+\ldots$$

$$\left(x-x_0\right)\frac{\partial f}{\partial x}+\left(z-z_0\right)\frac{\partial f}{\partial z}= \qquad (6)$$

$$=-Nf+A_0+A_1x+A_4x^2+A_8x^3+\ldots$$

where A_0, A_1, … are coefficients of the interference polynomial.

The idea of introducing this polynomial was adopted from the well known Werner deconvolution (Werner, 1953; Hartman *et al.*, 1971). This method is built on the recognition of the anomalous effect of a dike in the profile magnetic field ΔT [Hartman *et al.* (1971), Eq. 1, p. 902). The introduction of a polynomial of higher degree, together with the magnetic field, in the interpretation equation [Hartman *et al.* (1971), Eq. 5, p. 904], helped to improve the stability and clustering of the solutions. The role of the polynomial was to "describe" the effects of close overlapping (interfering) anomalies and, therefore, it was named the interference polynomial. The introduction of such a polynomial into the Euler deconvolution algorithm (instead of a constant background term) should improve the clustering of the solutions in the case of several closed sources. In Fig. 1, the reader can see the Euler depth estimations for three thick dikes (with relatively intensively overlapping/interfering anomalous fields). Solutions, with an adopted polynomial of 2 degrees, give better focused clusters of estimates and also the depth information is more correct (e.g. in Fig. 1a: the case of the right-hand side edge of the central dike). In general, we can see that the depth solutions for the dike models are slightly deeper (Figs. 1a, 1d) than the real position of their upper edges. This is caused by the fact that the structural index 1 was derived for thin dikes (the central and right-hand positioned dikes in Fig. 1 have greater widths by comparison to the depth of their upper boundaries) and for a thicker sheet a lower value of the structural index would be more appropriate. It is important to mention that by comparison to the Werner deconvolution, in the case of the Euler deconvolution, the polynomial was added only to the right-hand side of the interpretation equation.

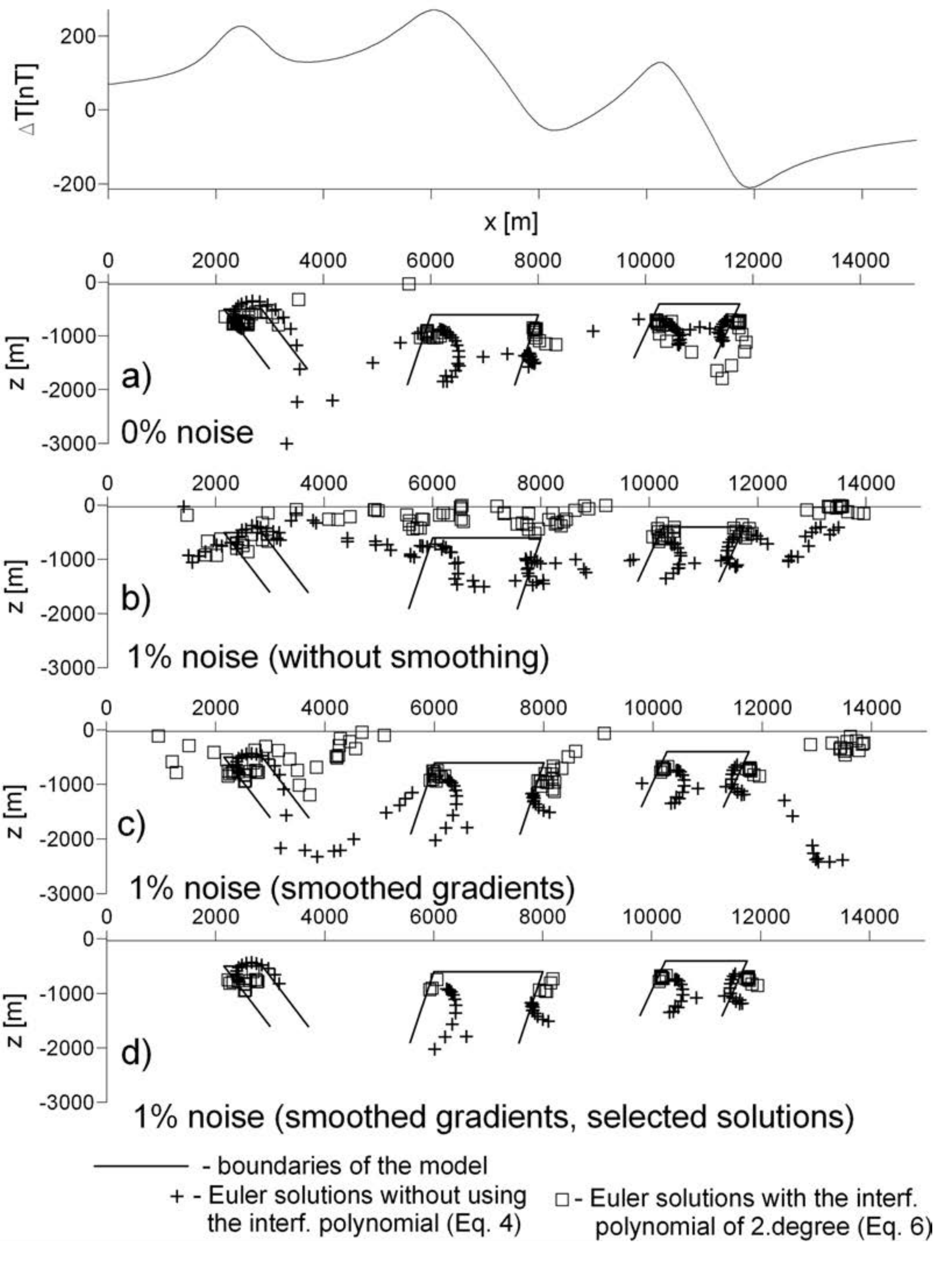

Fig. 1 - Comparison of Euler depth estimates for the modelled magnetic field of three dikes (used structural index $N = 1$) using the algorithm with and without the interference polynomial: a) input data – synthetic ΔT field without additional noise, b) input data – synthetic ΔT field with additional 1% white noise (solutions without smoothing the input gradients), c) input data – synthetic ΔT field with additional 1% white noise (solutions with smoothed input gradients by means of regularisation algorithm) and d) only selected solutions from the case (c) with the standard deviations of the depth estimates lower than 20% of the maximum one. The size of the used moving window was equal for all presented results: 15 profile points.

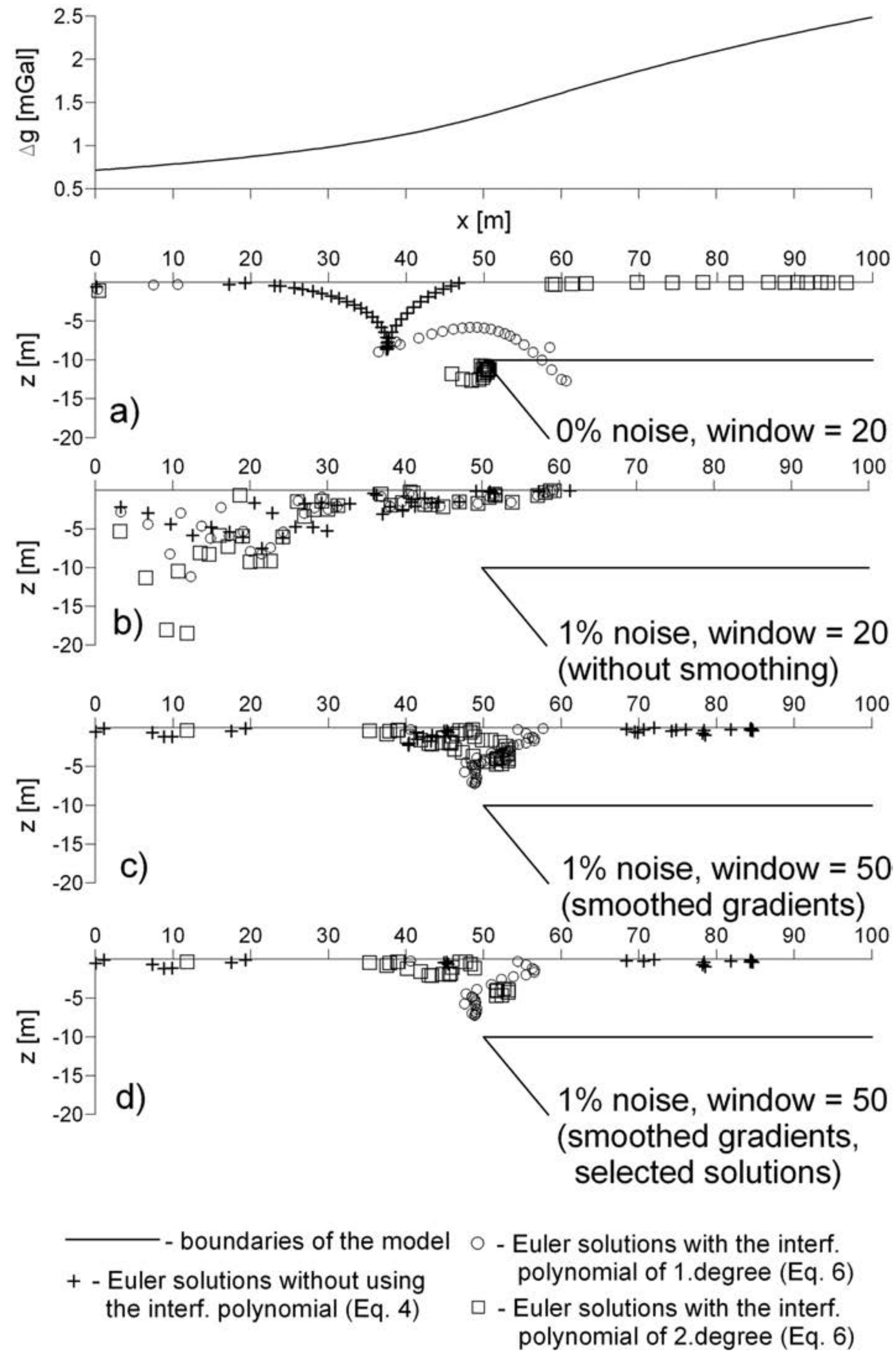

Fig. 2 - Euler depth estimates for the modelled gravity field of an inclined contact (used structural index $N = -1$) using the algorithm with and without the interference polynomial: a) input data – synthetic Δg field without additional noise, b) input data – synthetic Δg field with additional 1% white noise (solutions without smoothing the input gradients), c) input data – synthetic Δg field with additional 1% white noise (solutions with smoothed input gradients by means of regularisation algorithm) and d) only selected solutions from the case (c) with the standard deviations of the depth estimates lower than 20% of the maximum one. The size of the used moving window was for the cases (a) and (b) 20 profile points and for (c) and (d) 50 profile points.

Later on, during the realisation of model studies, it was realised that when we use the negative value N = -1 for the recognition of the contact or step structure in gravity data (Pasteka, 2001), we have to adopt the interference polynomial with higher orders (from model studies, it follows that the first or second order degree gives best results). This fact is displayed in Fig. 2a, where the results of the application of the Euler algorithm with various degrees of the adopted interference polynomial (0, 1 and 2 degree) are shown. It can be clearly seen that the correct focusing of the solutions at the position of the upper edge of the contact is completed, when we adopt the interference polynomial of 2 degree (for higher degrees the solutions became more unstable and defocused - not shown in Fig. 2).

The form of the interpretation equation with the interference polynomial unifies various forms of this equation: the first simple version [Eqs. (1) and (2)], without the "background" B term on the right-hand side (e.g. Hood, 1965), where $A_0 = A_1 = \ldots = 0$; Thompson's version with the B term [Eqs. (3) and (4)], where A_0 = NB and $A_1 = A_2 = \ldots = 0$ and in the end also the modified equation for the contact structure in magnetometry (Reid *et al.*, 1990):

$$\left(x - x_0\right)\frac{\partial f}{\partial x} + \left(y - y_0\right)\frac{\partial f}{\partial y} + \left(z - z_0\right)\frac{\partial f}{\partial z} = A \qquad (7)$$

where A is a constant, incorporating amplitude, strike and dip factors. Here we can define it by comparing with Eq. 5: $A_0 = A$, $A_1 = A_2 = \ldots = 0$ and of course $N = 0$.

The very important fact, occurring during the generalisation of the Euler deconvolution algorithm by means of the introduction of the interference polynomial is unfortunately a growth of the instability of the interpretation equation. Simple tests with randomly distributed (white) noise, added to the synthetic data (Figs. 1b and 2b) have shown that a very small level of it (only 1%) has caused great problems to the method – the solutions became unfocused and shallower (accompanied by a large amount of erroneous solutions). Also the standard deviation of the depth estimates (obtained from the LSQ-algorithm) and condition number [or singularity ratio, e.g. FitzGerald *et al*. (2004)] increase their values with the increasing order of the interference polynomial (Tables 2 and 3). The utilisation of standard deviations of the depth estimates is well known in geophysical literature and was introduced by Thompson (1982). The condition number (singularity ratio) is an important stability parameter, obtained from the SVD algorithm (e.g. Press *et al*., 1986); its higher values detect a numerical instability of the solved system of linear equations [discussed and utilised by Mushayandebvu *et al*. (2003), FitzGerald *et al*. (2004)], connected often with erroneous solutions. A preliminary smoothing of the input data, stabilisation of the derivative evaluation or the system of linear equations solution became inevitable. As it can be seen in Figs. 1c and 2c, the introduction of smoothed-regularised derivatives (Pasteka and Richter, 2002) has made the focusing of the solutions much better, but there are still great errors – e.g. the false clusters approximation with the x-coordinates 4300 m and 13600 m in Fig. 1c or the shallow depths of the best clustered solutions in Fig. 2c (error close to 40%). When we do not display all the solutions, but only those whose standard deviation depth estimate is lower than 10-20% of the maximum one (in Figs. 1d and 2d this level was 20%; in Figs. 3a and 3b it was 5%), we get better focused results (closer to the real positions of the

Table 2 - Median values of the standard deviation of the depth solution (obtained from LSQ-algorithm) and condition number (obtained from SVD-algorithm) for the 2D Euler deconvolution of synthetic magnetic data from Fig. 1a with various degrees of the introduced interference polynomial.

degree of the interference polynomial	median value of the standard deviation of the depth solution [m]	median value of the condition number [ ]
0	3.5	$19.1\ 10^{1}$
1	6.5	$7.8\ 10^{5}$
2	16.6	$4.1\ 10^{10}$

Table 3 - Median values of the standard deviation of the depth solution (obtained from LSQ-algorithm) and condition number (obtained from SVD-algorithm) for the 2D Euler deconvolution of synthetic gravity data from Fig. 2a with various degrees of the introduced interference polynomial.

degree of the interference polynomial	median value of the standard deviation of the depth solution [m]	median value of the condition number [ ]
0	24.8	$2.7\ 10^{2}$
1	326.8	$6.7\ 10^{4}$
2	982.9	$1.1\ 10^{7}$

sources). This rule is very close to that of Cooper (2004), based on the acceptation of the first 5-10% of the solutions that fit the Euler interpretation equation. The approach presented here (i.e. to take the solutions, which have the lower standard deviations) gave good results during model studies and practical data applications, where sources with similar values of structural indices are expected. This rule often gives better results, than the ratio of the standard deviation to the obtained depth estimate, as is often done in various software implementations of the method, (e.g. Geosoft, 1994).

As a typical example of the application of the modified Euler deconvolution algorithm (with the adopted interference polynomial) in combination with the rule of the lower standard deviation (of the depth estimate) acceptation an UXO-detection magnetometry dataset interpretation was selected. Grid of high-definition ΔT field (square of 20 x 20 m, x-spacing: 0.1 m, y-spacing: 0.1 m) from the Zelezna Studienka site (SW-Slovakia) shows several isolated "strong" anomalies (Fig. 3), typical for the occurrence of unexploded ordnance objects (UXO), but also several smaller anomalies, connected with fragments of exploded objects (the locality was an old German ammunition deposit from the World War II). After realisation of the 3D Euler deconvolution with the interference polynomial adoption, an analysis of the clustering for various levels of standard deviation of the depth estimate was realised. It was found that an acceptance level of 5% from the maximum standard deviation gives well focused clusters in the case of the most important anomalies (higher than the threshold of 5 nT). It can be seen that the clusters of solutions are better focused in the case of a 2nd degree polynomial adoption (Fig. 3b) than in the case of the classical "B-term" version of the algorithm (Fig. 3a). The depths obtained were, with a 10-15% error, in agreement with the real positions (varying from 0.2 to 1.1 m) of excavated objects (mainly complete 81 mm mortars were found).

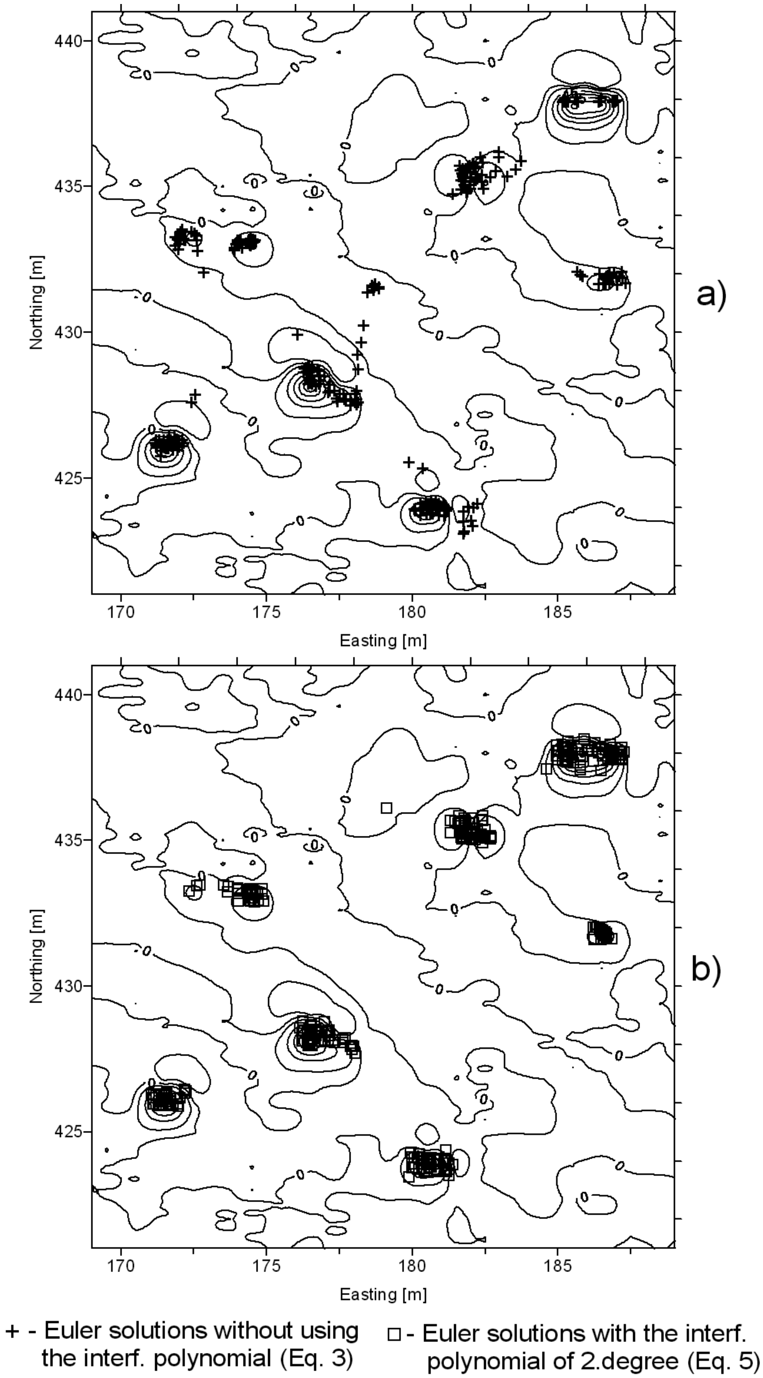

Fig. 3 - Results of the 3D Euler deconvolution algorithm applied on high-definition magnetometry grid of 20 x 20 m size (ranging from -14.24 to 58.05 nT) (used structural index N = 3): a) results for the constant background term (classical algorithm); b) results for the adopted 2nd degree interference polynomial. The size of the used moving window was 5 x 5 grid nodes. Calculations were realised with smoothed input gradients by means of regularisation algorithm. In both cases solutions with standard deviations of the depth estimation lower than 5% of the maximum one have been displayed.

3. Conclusions

Introduction of the interference polynomial into the Euler deconvolution algorithm has shown its benefits and traps. From the theoretical point of view it is the only known method (for the author), on how to get numerically correct solutions for the contact structure in gravimetry (when the input field is Δg). The solutions obtained without the interference polynomial create serious "artifacts" and their positions are often completely wrong (e.g. Fig. 2a – horizontal position was approx. 38 m instead of 50 m). On the other hand, the introduction of the interference polynomial has caused a dramatic increase in the instability of the whole method. Simple model results with a 1% level of randomly distributed (white) noise have shown that the results of the method are erroneous and defocused (Figs. 1b and 2b). Improvement of the situation can be achieved by introducing the smoothed-regularised derivatives (Pasteka, Richter, 2002) and a minimum level of standard deviation in the depth estimate for accepted solutions (Figs. 1d, 2d, 3). Results from the application to practical data (high-definition UXO-magnetometry) shows good properties of such an approach – better focused clusters of solutions have been obtained for a higher degree of the adopted interference polynomial. Despite these encouraging results, a more detailed analysis of this instability and a search for other stable algorithms should be realised in the future.

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