Coupling Biot wave equation with de la Cruz and Spanos boundary conditions for multi-layered porous systems

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ABSTRACT A large portion of the attenuation that arises in heterogeneous porous media is associated to the mesoscopic fluid flows resulting from energy partitioning at discontinuities. Traditional approaches have quantified these effects considering the Open Pore and the Natural Boundary Conditions. In the present work, a formulation that couples a more thorough prescription of boundary conditions with Biot poroelastic theory by means of domain decomposition techniques is proposed. Additionally, a set of boundary equations for the simulation of seismic waves in unbounded poroelastic media is developed. The two sets are given by systems of PDE's that provide a solution for particle velocities and pressures. These formulations provide a different approach for assessing the attenuation associated to mesoscopic flows for multilayered porous systems.

1. Introduction

Seismic attenuation is important in many areas of research and development such as in the exploration of natural resources, civil engineering, non-destructive evaluation of materials, medical imaging, etc. The reason is two-fold: for focusing amplitudes and fluid detection (Quiroga-Goode, 2001).

Seismic waves undergo amplitude attenuation and velocity dispersion due to several causes. Within the context of the Biot homogeneous poroelastic theory, the attenuation is associated to the equilibration of pore pressure gradients arising among the peaks and troughs of the propagating waves that results in the diffusion of viscous fluids. The magnitude and behavior of the predicted dissipation however, are not realistic for typical sedimentary porous rocks (White, 1975; Bourbie *et al.*, 1987).

This attenuation scenario is substantially changed when heterogeneities are considered because now local flows are induced, in additional to the global ones, in response to the pressure gradients generated by the wave among heterogeneous regions of varying compressibilities. These range from lithological variations (e.g., mixtures of sands and clays) with a single fluid saturating all the pores to a single uniform lithology saturated in mesoscopic patches by two immiscible fluids (e.g., air and water). The mechanism is termed mesoscopic flow (Carcione *et al.*, 2000, 2003; Pride *et al.*, 2004). It is also called macroscopic squirt-flow (Quiroga-Goode, 2002a) because it is analogous to, but an up-scaled version of, the more widely known squirt-flow attenuation mechanism that occurs at broken contacts and/or microcracks in the grains (Norris, 1993; Gelinsky and Shapiro, 1995; Gurevich *et al.*, 1997). It has been demonstrated that these wave-induced mesoscopic flows represent a major source of intrinsic dissipation in the surface seismic frequency band (Carcione *et al*., 2003; Pride *et al*., 2004). They are predicted naturally in the Biot heterogeneous formulation, i.e., without making ad hoc assumptions in the derivation of the theory (Quiroga-Goode and Carcione, 1997; Quiroga-Goode, 2002a; Carcione *et al*., 2003; Pride *et al*., 2004). In the presence of sharp angular discontinuities, they are seen to produce vortexes (Quiroga-Goode, 2002a) that appear to induce permanent residual deformation (Quiroga-Goode, 2002b).

An additional portion of attenuation is related to the interference of energy fluxes that occur between the various wave modes among the heterogeneous media (White, 1975; Quiroga-Goode and Carcione, 1997). The magnitude of these dissipation mechanisms depends on two main factors. The first is related to the contrast of the bulk mechanic properties and also to the size of the various heterogeneous regions compared to the fluid diffusion skin depth (Norris, 1993; Gurevich *et al.*, 1998). The second factor, which is the main focus of the present work, depends on how the surfaces separating dissimilar porous media are defined. This has been subject of controversy (Gurevich, 1993).

A simplistic configuration of the surface boundary that separates porous media has been widely used to quantify the wave-induced mesoscopic flows arising in finely layered systems (Norris, 1993; Gelinsky and Shapiro, 1995; Gurevich *et al.,* 1997, 1998). The corresponding boundary conditions are the so-called 'Open Pore Conditions' (OPC), which were derived from Biot equations (Deresiewicz and Skalak, 1963; Gurevich and Schoenberg, 1999). In these, the solid velocity vector, the normal component of relative fluid velocity vector, the normal and the tangential components of the bulk stress and the pore fluid pressure are assumed to be continuous across the boundary, regardless if the hydraulic connectivity between the dissimilar porous media allows for full or partial viscous flows. This implies, however, that pores of both media are completely aligned or connected at the boundary, a condition that in realistic cases may not be satisfied.

There is yet another boundary configuration, called the "Natural Boundary Conditions" (NBC: Deresiewicz and Skalak, 1963) similar to OPC except that pressure drop ∇p is proportional to $k\,\dot{w}_n$ to honor the partial fluid flows that arise from an imperfect pore alignment at the contact between dissimilar porous media. The normal component of relative velocity is denoted $\dot{w}_n$ and k corresponds to a coefficient of resistance that controls the interface hydraulic permeability.

Therefore, as a significant portion of seismic attenuation in porous media is associated to viscous flows across inhomogeneities (White, 1975; Dutta and Ode, 1983; Gurevich *et al*, 1998) it is important to study the effect of other types of boundary configurations than those given by the conventional approaches (NBC or the OPC) implied in the heterogeneous formulation of Biot. de la Cruz and Spanos (1989) proposed a different description of the surface discontinuity that holds between two porous media. In their boundary description, the fluid component of one porous medium can be acted upon by both the solid and fluid components of the other porous medium, and vice versa. In addition, porosity is allowed a dynamical role, which is absent in OPC and in NBC.

Within this context, the first goal in this work is to develop a formulation that couples Biot dynamic equations of poroelasticity with de la Cruz and Spanos (1989) boundary conditions for multiple interfaces (DLCSC). Heuristic approaches for implementing boundary conditions in single-phase media directly discretize the boundary equations (Alterman and Karal, 1968; etc.) incurring thus in numerical inaccuracies. In this paper however, the coupling is accomplished rigorously via domain decomposition techniques. The ultimate purpose of this formulation, which will be the subject of future work, is to quantify the amount of P-wave dissipation caused by the cumulative effect of multiple discontinuities and then compare these effects to those obtained by assuming OPC and NBC.

A full description of a boundary value problem must include all boundaries in the model, internal and those external that limit the size of the numerical model. Therefore, the second objective of the present work is to develop an effective means of simulating infinite media. This, again, is accomplished via domain decomposition.

The layout of this work is as follows. The next section presents the Biot poroelastic wave equation followed by its decomposition into incoming and outgoing waves. Since DLCSC provide the solution of eight scattered waves in the more general case, four of which are diffusive, the next section deals with the simplification of the boundary conditions. The following two sections deal with the numerical implementation of the internal and external boundary conditions using domain decomposition. In section 5, the subject of numerical accuracy is emphasized as related to the physical processes taking place in the neighborhood of heterogeneities.

2. Biot theory

Biot (1962a, 1962b) derived the dynamic equations that describe the coupling between seismic wave propagation and fluid diffusion in heterogeneous porous media. As the goal of this work is to develop a formulation that can be used to quantify the attenuation only for compressional waves, shear waves can be disregarded. This is accomplished by setting solid rigidity to zero. Thus, for a compressional source, normal incidence and large distances compared to wavelength, only dilatational deformations are implied.

The velocity-pressure formulation of the Biot wave equation in the low-frequency range can be expressed in the following matrix form,

$$\frac{\partial \boldsymbol{V}}{\partial t} = \boldsymbol{A}\boldsymbol{V} + \boldsymbol{B}\frac{\partial \boldsymbol{V}}{\partial x} + \frac{\partial \boldsymbol{S}}{\partial t} \tag{2.1}$$

where

$$\boldsymbol{V} = [v, q, p, p_f]^T \tag{2.2}$$

corresponds to the unknown vector in which the v's and q's correspond to the solid and fluid (relative to the solid) particle velocities, and p and p_f to the bulk and fluid pressures, respectively. The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$

$$\boldsymbol{A} = \begin{bmatrix} 0 & G_{12} & 0 & 0 \\ 0 & G_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 & \gamma_{22} & -\gamma_{12} \\ 0 & 0 & -\gamma_{12} & -\gamma_{11} \\ v-H & -C & 0 & 0 \\ -C & -M & 0 & 0 \end{bmatrix}, \tag{2.3}$$

contain the elements of

$$\boldsymbol{G} = \Theta H \frac{1}{\det \Theta} \frac{\eta}{\kappa} \begin{bmatrix} 0 & -\rho_f \\ 0 & -\rho \end{bmatrix} , \tag{2.4}$$

where H represents the damping matrix,

$$\boldsymbol{M} = \begin{bmatrix} -H & C \\ -C & M \end{bmatrix} , \tag{2.5}$$

is the bulk modulus matrix and $\gamma_{i,j}$, $i, j = 1,2$ correspond to the elements of the inverse of the mass matrix

$$\Theta = \begin{bmatrix} m & -\rho_f \\ \rho_f & -\rho \end{bmatrix}, . \tag{2.6}$$

The acoustic coefficients of the bulk modulus matrix

$$H = [K_m^{-1} - K_s^{-1} - \phi(K_s^{-1} - K_f^{-1})]K^{-1}, \tag{2.7}$$

$$C = (K_m^{-1} - K_s^{-1})K^{-1}, \tag{2.8}$$

$$M = K_m^{-1}K^{-1}, \tag{2.9}$$

with

$$K = \phi K_m^{-1}(K_f^{-1} - K_s^{-1}) + K_s^{-1}(K_m^{-1} - K_s^{-1}), \tag{2.10}$$

where K_s, K_m and K_f are the bulk moduli of the solid, matrix and fluid, respectively, and ϕ is the effective porosity. Moreover, η is the dynamic fluid viscosity and κ corresponds to the global permeability and $\rho = (1 - \phi)\rho_s + \phi\rho_f$ is the composite or bulk density, with ρ_s and ρ_f the solid and fluid densities, and $m = \alpha\rho_f/\phi$ with α being the tortuosity, a dimensionless parameter that depends on the pore geometry.

The source vector, denoted by

$$\boldsymbol{S} = [0, 0, s, s_f]^T \tag{2.11}$$

considers three cases (Carcione and Quiroga-Goode, 1995). The first is a bulk source, where the energy is partitioned between the two phases such that the relation between the solid and fluid source strengths is $(1/\phi) - 1$ (Hassanzadeh, 1991). This means that $s = s_f$. A solid source, in the second case, assumes $s_f = 0$. And third, a fluid volume injection: $s = \phi s_f$.

3. Wave decomposition

The formulae derived in the following sections for the computational implementation of boundary conditions are based on domain decomposition techniques. Domain decomposition was developed for vector initial-boundary value hyperbolic problems (Gottlieb *et al.*, 1982), generalized within the context of fluid dynamics (Thompson, 1990), and later applied for the solution of surface motions in elastodynamic problems (Kosloff *et al.*, 1990; Carcione, 1991).

The central concept in domain decomposition is that hyperbolic systems can be decomposed into outward going and incoming waves all of which travel with well-defined velocities. At any boundary, the incoming waves depend on the solution exterior to the computational volume and thus, they must be computed from the appropriate boundary conditions by solving an inverse problem. As the outgoing characteristics have their behavior defined entirely by the solution at and within the boundary, their representation does not depend on the boundary conditions. Thus, they can be obtained directly from the governing equations.

The wave characteristic equation corresponding to Eq. (2.1) can be written as

$$-\boldsymbol{R}^{-1}\frac{\partial \boldsymbol{V}}{\partial t}+\mathcal{H}+\boldsymbol{R}^{-1}\boldsymbol{D}=0, \qquad \boldsymbol{D}=\boldsymbol{A}\boldsymbol{V}+\frac{\partial \boldsymbol{S}}{\partial t}, \tag{3.12}$$

or

$$-\frac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{R}\mathcal{H}+\boldsymbol{D}=0 \tag{3.13}$$

where

$$\mathcal{H}\equiv \varDelta\boldsymbol{R}^{-1}\frac{\partial \boldsymbol{V}}{\partial x} \tag{3.14}$$

is the characteristic vector describing $1D$ wave propagation in the positive and negative x directions. The matrix

$$\varDelta=\boldsymbol{R}^{-1}\,\boldsymbol{B}\boldsymbol{R}\,, \tag{3.15}$$

is diagonal and contains the real eigenvalues λ_i, i = 1,..., 4 of $\boldsymbol{B}$ as the system is hyperbolic. The eigenvalues give the velocities of all the characteristics modes propagating in the medium. They are the solution to

$$\det(\boldsymbol{B}-\lambda\boldsymbol{I})=0\,, \tag{3.16}$$

which, in terms of the elements of the bulk modulus and mass matrices, are given by

$$\lambda_1=-V_s=\left[\frac{U-(U^2-4\det\boldsymbol{M}\,\det\Theta)^{1/2}}{2\det\boldsymbol{M}}\right]^{1/2},\ \lambda_2=V_s \tag{3.17}$$

$$\lambda_3=-V_f=\left[\frac{U+(U^2-4\det\boldsymbol{M}\,\det\Theta)^{1/2}}{2\det\boldsymbol{M}}\right]^{1/2},\ \lambda_4=V_f \tag{3.18}$$

where

$$U=2\rho_f\,C-\rho\,M-H\,m \tag{3.19}$$

Biot heterogeneous equations provide the solution of two compressional modes of propagation. The first is the commonly known fast wave (with subscript f) with phase velocities

λ_i, i = 3, 4 (negative and positive directions, respectively) in which the solid matrix and the saturating fluid of the porous medium move in phase. The other mode corresponds to the so-called 'slow wave' (with subscript s) with phase velocities λ_i, i = 1, 2 (negative and positive directions, respectively) which has solid and fluid components moving out of phase. As the slow wave is the result of a fluid diffusion process, it is strongly dissipative in the low frequency range. For inviscid fluids at low frequencies, or for viscous fluids at high frequencies, it propagates as a true wave with a phase velocity approaching asymptotically to that of the fluid permeating the porous matrix.

The columns of the matrix $\boldsymbol{R}$ are the *right* eigenvectors of $\boldsymbol{B}$ and the rows of R^{-1} its *left* eigenvectors. The characteristics $\mathcal{H}$, computed from Eq. (3.14), are, after some algebra

$$\mathcal{H}_{1,2} = \pm\, \Xi_s \left[V_s \left(-\det \boldsymbol{M} \left(\partial_t q + \gamma_{11} \frac{\eta}{\kappa} q \right) + V_f^2 \left(H\, \partial_x p_f - C\, \partial_x p \right) \right) + \det \boldsymbol{M} \left(-\gamma_{11}\, \partial_t p_f - \gamma_{12}\, \partial_t p - V_f^2\, \partial_x q \right) \right] \tag{3.20-21}$$

$$\mathcal{H}_{3,4} = \pm\, \Xi_f \left[V_f \left(-\det \boldsymbol{M} \left(\partial_t q + \gamma_{11} \frac{\eta}{\kappa} q \right) + V_s^2 \left(H\, \partial_x p_f - C\, \partial_x p \right) \right) - \det \boldsymbol{M} \left(-\gamma_{11}\, \partial_t p_f - \gamma_{12}\, \partial_t p - V_s^2\, \partial_x q \right) \right] \tag{3.22-23}$$

with

$$\Xi_i = \frac{\gamma_{12} \det \boldsymbol{M} - C V_i^2}{2 \det \boldsymbol{M} \left(C\gamma_{11} + \gamma_{12} H \right) \left(V_f^2 - V_f^2 \right)}, \quad i = s,\ f. \tag{3.24}$$

where $\mathcal{H}_1$ and $\mathcal{H}_3$ take positive sign and $\mathcal{H}_2$ and $\mathcal{H}_4$ negative. The procedure for decomposing the governing equations into outgoing and incoming waves is essentially the same for the 3-D case since all terms involving z and y derivatives are carried along passively and do not contribute in the analysis. They can be lumped together in matrix $\boldsymbol{D}$ (Thompson, 1990).

4. Boundary conditions

Given that the choice of a boundary configuration between two porous media has an important effect on seismic attenuation (related to viscous flows across interfaces), it will prove useful to test DLCSC as it appears to be more representative than the conventional approaches which consider NBC or OPC.

The subject of macroscopic boundary conditions in porous media has been controversial. Nevertheless, most theoretical and numerical studies have considered only OPC, as explained before. A main departure is found in DLCSC, in which it is concluded that the solid and relative fluid particle velocities are not the quantities required to be continuous, as in OPC and NBC, but the center of mass velocity, as is explained below. In a discussion by Gurevich and Lopatnikov (1995) it is argued however, that the boundary conditions above do not hold in some limiting cases. We find nevertheless, that their argument does not stay close to the physics since one of the basic premises, namely that porous materials are dynamical objects, is violated in their discussion.

The kinematic condition pertaining to the boundary conditions of de la Cruz and Spanos (1989) is given by

$$\boldsymbol{v} \cdot \boldsymbol{n} = \boldsymbol{v}' \cdot \boldsymbol{n} \,, \tag{4.25}$$

where

$$\boldsymbol{v} = \frac{\phi\rho_f \boldsymbol{v}_f + (1-\phi)\rho_s \boldsymbol{v}_s}{\rho} \,, \tag{4.26}$$

is the average mass velocity and $\boldsymbol{n}$ is the unit normal. From here on, primed quantities denote those in the transmission medium. Fluid and solid particle velocities correspond to $\boldsymbol{v}_f$ and $\boldsymbol{v}_s$ and $\rho = \phi\rho_f + (1-\phi)\rho_s$ is the effective density, where ρ_f and ρ_s are the densities in the fluid and solid respectively. Porosity corresponds to ϕ. In this equation, all the 0's could be dropped since $\boldsymbol{v}_f$ and $\boldsymbol{v}_s$ are first order. Other conditions are

$$\phi\sigma_{ik}^{f} n_k = \sigma_{ik}^{\prime f} n_k \phi\phi'\beta + \sigma_{ik}^{\prime s} n_k \phi(1-\phi'\beta) \tag{4.27}$$

$$(1-\phi)\sigma_{ik}^{s} n_k = \sigma_{ik}^{\prime f} n_k \phi'(1-\phi\beta) + \sigma_{ik}^{\prime s} n_k (1-\phi-\phi'+\phi\phi'\beta) \tag{4.28}$$

$$\phi'\sigma_{ik}^{\prime f} n_k = \sigma_{ik}^{f} n_k \phi\phi'\beta + \sigma_{ik}^{s} n_k \phi'(1-\phi\beta) \tag{4.29}$$

where σ_{ik}^{s} and σ_{ik}^{f} correspond to the stress tensors in the solid and the fluid, respectively. The manner of transition from one porous medium to an other is found to have effects on the resulting boundary conditions (de la Cruz and Spanos, 1989). That is, the effects depend on whether it is the solid or fluid phases of one of the porous media which is in contact with the solid or fluid phases of the other porous medium, and vice versa. These effects are summarized by the alignment parameter β, which has the following range of values

$$0 \le \beta \le \frac{1}{\phi} \quad for \quad \phi > \phi', \tag{4.30}$$

and

$$0 \le \beta \le \frac{1}{\phi'} \quad for \quad \phi' > \phi \,, \tag{4.31}$$

In the general case, where solid and fluid phases of one porous medium are 'randomly' aligned relative to the solid and fluid portions of the other porous medium: $\beta = 1$.

The remaining boundary conditions involve the tangential components $\boldsymbol{v}_t$ and $\boldsymbol{v}_t'$ which can be represented by the macroscopic no-slip condition

$$\boldsymbol{v}_t = \boldsymbol{v}_t' \,. \tag{4.32}$$

A plane analysis based on the previous relations would show the presence of four scattered waves in each medium for each compressional and shear waves in the most general case (de la Cruz and Spanos, 1989); two dilatational and two shear waves, two of which possess a diffusive character (slow waves). One of those waves is compressional and the other is shear. In the Biot theory, the slow shear wave fails to appear because $\sigma^f_{ik} = -p_f \delta_{ik}$, i.e., that of non-viscous fluids. In the Biot theory, the viscosity appears as a Darcian force (de la Cruz *et al.*, 1992). Since, for ideal fluids, no boundary conditions can be imposed on the tangential components of the velocities, Eq. (4.28) must be dropped.

Then, the previous boundary conditions specialize to

$$\frac{\rho_f}{\rho} q_z + v_s = \frac{\rho_f^{'}}{\rho'} q_z^{'} + v_s^{'} , \tag{4.33}$$

obtained by substituting ρ_s from the equation for the composite density, and the velocities in the solid and in the fluid relative to the solid $q = \phi(v_f - v_s)$. Moreover:

$$p_f = \phi' \left[\beta(1+\phi') - 1\right] p_f^{'} - (1-\phi'\beta)\tau_{zz}^{'} \tag{4.34}$$

$$(1-\phi)(\tau_{zz} + \phi p_f) = \phi'(-\phi - \phi' + \phi\beta + \phi\phi'\beta) p_f^{'} + (1 - \phi - \phi' + \phi\phi'\beta)\tau_{zz}^{'} \tag{4.35}$$

$$p_f^{'} = \phi \left[\beta(1+\phi) - 1\right] p_f - (1-\phi\beta)\tau_{zz} \tag{4.36}$$

where $\tau_{ij} = \sigma^s_{ij} - \phi p_f \delta_{ij}$ (Biot, 1956) was used. The bulk hydrostatic stress tensor corresponds to τ_{ij}.

4.1 Boundary treatment

A rigorous approach to numerically implementing boundary conditions should consider domain decomposition (Kosloff *et al.*, 1990), rather than simply applying (dis)continuity conditions directly into a numerical scheme since it may lead to instabilities and thus inaccuracies (Alterman and Karal, 1968). This is particularly crucial in cases dealing with vector equations (Gottlieb *et al.*, 1982) and especially where the physics of the equations relate to more than one scale ranges, i.e., different resolution attributes. As will be seen below, both conditions apply to seismic wave propagation in poroelastic media. The numerical implementation of a boundary includes two general cases. Thus, it distinguishes whether the boundary is joining another domain with different material properties (either single or multiphase rheologies and/or different resolution attributes) or a semi-infinite space.

Let the model be composed of two semi-infinite porous half-spaces (or sub-domains) in contact. The extension to a multilayered system is explained in section 5. For normal incidence, there are mode conversions from fast to slow waves and vice versa. Let x increase towards the transmission medium. Then, the wavefield scattered by the boundary separating the two sub-domains is given by the outgoing characteristics in the incidence medium, which are propagating with speeds $-V_s$ and $-V_f$. They are represented by $\mathcal{H}_i$, i = 1, 3 (for $\lambda_i < 0$). Whereas in the transmission medium, they are the outgoing characteristics $\mathcal{H}_i^{'}$, i = 2, 4 (for $\lambda_i^{'} > 0$) travelling with speeds V_s' and V_f'. These scattered waves are determined from the boundary conditions, as will be shown next. On the other hand, the incoming $\mathcal{H}_i$, i = 2, 4 (for $\lambda_i > 0$) and incoming $\mathcal{H}_i'$, i = 1, 3 (for $\lambda_i' < 0$) characteristics in the incidence and transmission media respectively, are obtained from Eqs. (3.20-3.23).

The boundary conditions Eqs. (4.33) - (4.36) are reformulated as a function of the wave characteristics $\mathcal{H}$. This is achieved by substituting the boundary conditions Eqs. (3.33) - (3.36) into the wave characteristic equation Eq. (4.12). The results so obtained, not shown here for conciseness, are then expressed in matrix form as $\boldsymbol{Dx} = \boldsymbol{y}$. The elements of the scattering matrix $\boldsymbol{D}$ and those of vector $\boldsymbol{y}$ are given in the Appendix. The vector of unknowns $\boldsymbol{x}$, representing the scattered waves, is

$$\boldsymbol{x} = [\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_2', \mathcal{H}_3, \mathcal{H}_4']^T. \tag{4.37}$$

The scattered waves Eq. (4.37) are obtained by solving the previous system for $\boldsymbol{x}$, using Cramer's rule, and substituted into Eq. (2.13) together with the remaining characteristics $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_2'$, $\mathcal{H}_3$, $\mathcal{H}_4'$ [Eqs. (3.20) - (3.23)] to give the relations to be used at the boundary separating the two porous media. The result is, after some algebra,

$$\frac{\partial v}{\partial t} = -\frac{\eta}{\kappa}\gamma_{12} q + \frac{\left(V_f + V_s\right)^2 - \gamma_{22} H - \gamma_{11} M}{4\left(V_f + V_s\right)}\frac{\partial v}{\partial x} + \frac{-\gamma_{22} C + \gamma_{12} M}{2\left(V_f + V_s\right)}\frac{\partial q}{\partial x} + \frac{\gamma_{22}}{2}\frac{\partial p}{\partial x} - \frac{\gamma_{12}}{2}\frac{\partial p_f}{\partial x}$$
$$+\frac{\det \boldsymbol{M}\,\gamma_{12} - C V_s^2 + M\left(\gamma_{11} C + \gamma_{12} H\right)}{C\left(\det \boldsymbol{M}\,\gamma_{12} - C V_s^2\right)} V_s \mathcal{H}_1 + \frac{\det \boldsymbol{M}\,\gamma_{12} - C V_f^2 + M\left(\gamma_{11} C + \gamma_{12} H\right)}{C\left(\det \boldsymbol{M}\,\gamma_{12} - C V_s^2\right)} V_f \mathcal{H}_3, \tag{4.38}$$

$$\frac{\partial q}{\partial t} = -\frac{\eta}{\kappa}\gamma_{11} q + \frac{\gamma_{11} C + \gamma_{12} H}{2\left(V_f + V_s\right)}\frac{\partial v}{\partial x} + \frac{\left(V_f + V_s\right)^2 + \gamma_{22} H + \gamma_{11} M}{4\left(V_f + V_s\right)}\frac{\partial q}{\partial x} - \frac{\gamma_{12}}{2}\frac{\partial p}{\partial x} - \frac{\gamma_{11}}{2}\frac{\partial p_f}{\partial x}$$
$$-\frac{\gamma_{11} C + \gamma_{12} H}{\det \boldsymbol{M}\,\gamma_{12} - C V_s^2} V_s \mathcal{H}_1 - \frac{\gamma_{11} C + \gamma_{12} H}{\det \boldsymbol{M}\,\gamma_{12} - C V_f^2} V_f \mathcal{H}_3, \tag{4.39}$$

$$\frac{\partial p^{new}}{\partial t} = \frac{1}{2}\left[\frac{\partial p^{old}}{\partial t} + \frac{1}{2}\frac{\left(V_f + V_s\right)^2 - \gamma_{22} H - \gamma_{11} M}{V_f + V_s}\frac{\partial p}{\partial x} + \frac{\gamma_{11} C + \gamma_{12} H}{V_f + V_s}\frac{\partial p_f}{\partial x}\right]$$
$$-\frac{\det \boldsymbol{M}\,\gamma_{11} + H V_s^2}{\det \boldsymbol{M}\,\gamma_{12} - C V_s^2}\mathcal{H}_1 - \frac{\det \boldsymbol{M}\,\gamma_{11} + H V_f^2}{\det \boldsymbol{M}\,\gamma_{12} - C V_f^2}\mathcal{H}_3, \tag{4.40}$$

$$\frac{\partial p_f^{new}}{\partial t} = \mathcal{H}_1 + \mathcal{H}_3 + \frac{1}{2}\left[\frac{\partial p_f^{old}}{\partial t} - \frac{\left(CV_f^2 - \gamma_{12}\det \boldsymbol{M}\right)\left(CV_s^2 - \gamma_{12}\det \boldsymbol{M}\right)}{\left(V_f + V_s\right)\left(\gamma_{11}C + \gamma_{12}\det \boldsymbol{M}\right)}\frac{\partial p}{\partial x}\right.$$

$$\left. + \frac{\left(V_f + V_s\right) + \gamma_{22}H + \gamma_{11}M}{2\left(V_f + V_s\right)}\frac{\partial p_f}{\partial x}\right]. \tag{4.41}$$

The superscripts "new" and "old" refer to before and after the numerical solution of the PDE's (see Carcione, 1991).

4.2 Simulation of an infinite medium

Due to the limited computer power and memory available, the computational domain must be artificially bounded. Thus, the remaining (external) boundaries for the two half-space model must be specified. Different choices comprise Dirichlet and Neumann boundary conditions. These however produce reflections that can cause interference if they arrive within the area of analysis. A more convenient alternative is given by non-reflecting conditions to damp all scattered energy that can be reflected back into the model (from $\pm\infty$). This can be achieved by cancelling the incoming wave modes by means of either averaging the solutions for a Neumann and Dirichlet problems (Smith, 1974) or via the characteristics approach, as is derived below.

In the incidence and in the transmission media, the characteristics corresponding to the waves coming from $\pm\infty$ are given by $\mathcal{H}_i$, i = 2, 4 and $\mathcal{H}_i'$, i = 1, 3, respectively, must then be cancelled. The equations giving the characteristics associated with incoming waves in the incidence medium, i.e., waves coming from $-\infty$, are, after some algebra

$$\mathcal{H}_2 - \frac{\partial}{\partial t}\left[\frac{\det \boldsymbol{M}\,\gamma_{12} - C\,V_s^2}{2V_s\det \boldsymbol{M}\left(\gamma_{11}C - \gamma_{12}H\right)\left(V_f^2 - V_s^2\right)}\left\{V_s\left(\det \boldsymbol{M}\,\gamma_{11} + HV_f^2\right)p_f - \det \boldsymbol{M}\left(\gamma_{12}C + \gamma_{11}M\right.\right.\right.$$

$$\left.\left.\left. - V_f^2\right)q + V_s\left(\det \boldsymbol{M}\,\gamma_{12} - CV_f^2\right)p - \det \boldsymbol{M}\left(\gamma_{11}C + \gamma_{12}H\right)v\right\}\right] + \frac{\det \boldsymbol{M}\,\gamma_{12} - CV_s^2}{2V_s\left(\gamma_{11}C + \gamma_{12}H\right)\left(V_f^2 + V_s^2\right)}$$

$$\left[\left(\gamma_{12}C + \gamma_{11}M - V_f^2\right)\gamma_{11} + \left(\gamma_{11}C + \gamma_{12}H\right)\gamma_{12}\right]\frac{\eta}{\kappa}q = 0 \tag{4.42}$$

$$\mathcal{H}_4 + \frac{\partial}{\partial t}\left[\frac{\det \boldsymbol{M}\,\gamma_{12} - C\,V_f^2}{2V_f\det \boldsymbol{M}\left(\gamma_{11}C - \gamma_{12}H\right)\left(V_f^2 - V_s^2\right)}\left\{V_f\left(\det \boldsymbol{M}\,\gamma_{11} + HV_s^2\right)p_f - \det \boldsymbol{M}\left(\gamma_{12}C + \gamma_{11}M\right.\right.\right.$$

$$\left.\left.\left. - V_s^2\right)q + V_f\left(\det \boldsymbol{M}\,\gamma_{12} - CV_s^2\right)p - \det \boldsymbol{M}\left(\gamma_{11}C + \gamma_{12}H\right)v\right\}\right] - \frac{\det \boldsymbol{M}\,\gamma_{12} - CV_f^2}{2V_f\left(\gamma_{11}C + \gamma_{12}H\right)\left(V_f^2 + V_s^2\right)}$$

$$\left[\left(\gamma_{12}C + \gamma_{11}M - V_s^2\right)\gamma_{11} + \left(\gamma_{11}C + \gamma_{12}H\right)\gamma_{12}\right]\frac{\eta}{\kappa}q = 0. \tag{4.43}$$

Imposing constant amplitudes in time to the incoming characteristics, so that the time derivative terms in Eqs. (4.42) and (4.43) are cancelled is equivalent to suppressing the incoming waves in the incidence medium. This can be done by choosing,

$$\mathcal{H}_2 = +\frac{C V_s^2 - \det \boldsymbol{M}\, \gamma_{12}}{2V_s\left(V_f^2 + V_s^2\right)\left(\gamma_{11} C - \gamma_{12} H\right)}\left[\left(\gamma_{12} C + \gamma_{11} M - V_f^2\right)\gamma_{11} + \left(\gamma_{11} C + \gamma_{12} H\right)\gamma_{12}\right]\frac{\eta}{\kappa}, \tag{4.44}$$

$$\mathcal{H}_4 = -\frac{C V_f^2 - \det \boldsymbol{M}\, \gamma_{12}}{2V_f\left(V_f^2 + V_s^2\right)\left(\gamma_{11} C - \gamma_{12} H\right)}\left[\left(\gamma_{12} C + \gamma_{11} M - V_s^2\right)\gamma_{11} + \left(\gamma_{11} C + \gamma_{12} H\right)\gamma_{12}\right]\frac{\eta}{\kappa}. \tag{4.45}$$

Therefore, the system $\partial \boldsymbol{V}/\partial t$ of PDE's to be used at the artificial boundary of the model to cancel the waves coming from $-\infty$ is obtained by substituting the incoming $\mathcal{H}_i$, i = 2, 4 Eqs. (3.44) and (3.45) and the outgoing characteristics $\mathcal{H}_i$, i = 1, 3 Eqs. (3.20) - (3.22) into Eq. (3.13). The result is, after some algebra.

$$\frac{\partial v^{new}}{\partial t} = \frac{1}{2}\left(\frac{\partial v^{old}}{\partial t} \pm \frac{\left(V_f + V_s\right)^2 - \gamma_{22} H - \gamma_{11} M}{2\left(V_f + V_s\right)}\frac{\partial v}{\partial x} \pm \frac{-\gamma_{22} C + \gamma_{12} M}{V_f + V_s}\frac{\partial q^{old}}{\partial x}\right), \tag{4.46}$$

$$\frac{\partial q^{new}}{\partial t} = \frac{1}{2}\left(\frac{\partial q^{old}}{\partial t} \pm \frac{\gamma_{11} C + \gamma_{12} H}{V_f + V_s}\frac{\partial v}{\partial x} \pm \frac{\left(V_f + V_s\right)^2 + \gamma_{22} H + \gamma_{11} M}{2\left(V_f + V_s\right)}\frac{\partial q}{\partial x}\right). \tag{4.47}$$

$$\frac{\partial p^{new}}{\partial t} = \frac{1}{2}\left(\frac{\partial p^{old}}{\partial t} \pm \frac{\left(V_f + V_s\right)^2 - \gamma_{22} H - \gamma_{11} M}{2\left(V_f + V_s\right)}\frac{\partial p}{\partial x} \pm \frac{\gamma_{11} C + \gamma_{12} H}{V_f + V_s}\frac{\partial p_f}{\partial x} \pm \frac{-\det \boldsymbol{M}\, \gamma_{11} + H V_f V_s}{V_f V_s\left(V_f + V_s\right)\left(\gamma_{11} C + \gamma_{12} H\right)}\right.$$
$$\left.\left[\gamma_{11}\left\{\gamma_{12} C + \gamma_{11} M + V_f V_s + \det \boldsymbol{M}\left(V_f + V_s\right)^2 \gamma_{11}\right\} + \gamma_{12}\left(\gamma_{11} C + \gamma_{12} H\right)\right]\frac{\eta}{\kappa} q\right), \tag{4.48}$$

$$\frac{\partial p_f^{new}}{\partial t} = \frac{1}{2}\left(\frac{\partial p_f^{old}}{\partial t} \pm \frac{\left(C V_f^2 - \gamma_{12} \det \boldsymbol{M}\right)\left(C V_s^2 - \gamma_{12} \det \boldsymbol{M}\right)}{\left(V_f + V_s\right)\left(\gamma_{11} C + \gamma_{12} H\right)\det \boldsymbol{M}}\frac{\partial p}{\partial x} + \frac{\det \boldsymbol{M}\, \gamma_{12} + C V_f V_s}{V_f V_s\left(V_f + V_s\right)\left(\gamma_{11} C + \gamma_{12} H\right)}\right.$$
$$\left[\gamma_{11}\left\{\gamma_{12} C + \gamma_{11} M + V_f V_s + \det \boldsymbol{M}\left(V_f + V_s\right)^2 \gamma_{12}\right\} + \gamma_{12}\left(\gamma_{11} C + \gamma_{12} H\right)\right]\frac{\eta}{\kappa} q$$
$$\left.\pm \frac{\left(V_f + V_s\right)^2 + \gamma_{22} H + \gamma_{11} M}{2\left(V_f + V_s\right)}\frac{\partial p_f}{\partial x}\right). \tag{4.49}$$

By symmetry considerations, the system of PDE's to be used at the artificial boundary of the model in the transmission medium to cancel waves coming from $+\infty$ is similar to previous equations except for a change in sign in some of the terms (denoted by ±). Therefore, the same set of Eqs. (3.46) and (3.49) are used except that the positive and negative signs are taken at the artificial boundaries in the incidence and in the transmission medium, respectively.

5. A numerical approach

The resolution attributes of heterogeneous porous systems span several orders of magnitude in both scales: time and space. Some of these are related to slow-wave attenuation via diffusion of viscous fluids (squirt-flow and the interaction of energy fluxes between wave modes). Both mechanisms occur in the neighborhood of heterogeneities and range from millimeters to centimeters and micro to milliseconds. The others are related to seismic wave propagation, which implies wavelengths in the order of 10's and 100's of meters and seconds.

Therefore, as the only reason to consider poroelasticity is for the attenuation processes that are originated at and near heterogeneities, the numerical discretization should be fine enough in their neighborhood to track all small-scale physical phenomena described above. However, a numerical method must be able to resolve all scales not only accurately but also efficiently.

Numerical investigations using finite difference and finite element approximations in porous media (Hassanzadeh, 1991; Zhu and McMechan, 1991; Dai *et al*., 1995; Han *et al*, 1998; etc.) have attempted to incorporate poroelastic attenuation effects into their results. However, since the spatial discretization intervals considered in those works was much larger than the resolution required to map the diffusive processes causing attenuation, it is not clear whether their results are representative.

The Chebyshev pseudospectral method can be a suitable candidate for evaluating the spatial derivatives accurately and efficiently. In this approximation, the spatial derivatives are obtained by analytically differentiating the N^{th}-order Chebyshev polynomial

$$T_N(x) = \cos[N \operatorname{arc}\cos(x)] \qquad (5.50)$$

at the extremal points

$$x_i = \cos\left(\frac{i\phi}{N}\right), \quad i = 0,\ldots,N. \qquad (5.51)$$

which correspond to the grid points where the numerical solution is sought. The grid points $i = 0, \ldots, N$ make up the whole domain and $i = 0$ and $i = N$ correspond to the boundaries, which can be internal or external (artificial).

The relevant aspect of the Chebyshev approximation for the spatial derivatives is two fold. First, it possesses spectral-like resolution for a band-limited source. The second advantage is implied in Eq. (4.51): the discretization interval within a domain decreases in size as grid points get closer to the boundaries, i.e., as $i \to 0$ and $i \to N$. This means that the neighborhood surrounding a discontinuity, at which the scattering takes place (including viscous diffusion of fluids and interference fluxes), will be finely sampled and thus well resolved. On the other hand, that part of the model away from the boundaries is sampled coarsely, but enough to represent the propagation of seismic waves adequately.

The extension to a multi-layered model can be constructed by joining domains. The numerical solution in the interior of each domain and at their physical boundaries is obtained by the solution of Eq. (2.1) and by Eqs. (3.38) and (3.41), respectively. At the artificial edges of the computational model, Eqs. (3.46) and (3.49) replace Eqs. (3.38) and (3.41). The numerical implementation and results will be presented in a future work.

Summary

In porous heterogeneous media, a large portion of seismic wave attenuation arises at and in the neighborhood of discontinuities. This is due to energy partitioning from the incident field into the scattered field, part of which is represented in terms of mesoscopic fluid flows and also to the interference fluxes of energy between wave modes, in the incidence and in the transmission media. Traditional approaches to study these effects have considered the OPC or the NBC. In this work, I derive a formulation to couple a more realistic kind of boundary condition (de la Cruz and Spanos, 1989) with the Biot (1956, 1962a, 1962b) wave equation. Heuristic approaches for implementing boundary conditions directly discretize the boundary equations incurring thus in numerical inaccuracies. In this paper however, the coupling is accomplished rigorously via domain decomposition techniques. In addition, I implement a set of non-reflecting conditions for the simulation of seismic waves in unbounded poroelastic media within the context of Biot theory of poroelasticity.

The two sets are given by systems of PDE's that provide a solution for particle velocities and pressures in a heterogeneous poroelastic model. The ultimate purpose for deriving these formulations is to quantify, in a future work, the amount of attenuation induced in a multi-layered system and compare it to the more conventional approaches that consider OPC and NBC.

Special emphasis is placed on accuracy and efficiency to resolve the diffusive processes that cause the attenuation of seismic waves. As a consequence, the Chebyshev pseudospectral method is suggested as a candidate for obtaining a numerical solution.

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Appendix

The elements of the scattering matrix $\boldsymbol{D}$ are given by

$$D_{11} = \frac{f_1 - f_3(\rho_f/\rho)V_s}{f_7}, \qquad D_{12} = D_{11}^{'}, \qquad D_{13} = \frac{f_2 - f_3(\rho_f/\rho)V_f}{f_6}, \qquad D_{14} = \frac{f_2^{'}}{f_6^{'}},$$

$$D_{22} = \frac{f_4^{'}(1-\phi') - f_7^{'}\phi'^2}{f_7^{'}}, \qquad D_{24} = \frac{f_5^{'}(1-\phi') - f_6^{'}\phi'^2}{f_6^{'}} + \mathcal{H}_1^{'}\,\frac{f_4^{'}(1-\phi') - f_7^{'}\phi'^2}{f_7^{'}},$$

$$D_{31} = \frac{(f_4 + f_7\phi)(1-\phi')}{f_7}, \qquad D_{32} = -(1-\phi)\left(f_4'(1-\phi') - f_7'\phi'^2\right),$$

$$D_{33} = \frac{(f_5 + f_6\phi)(1-\phi')}{f_6}, \qquad D_{34} = -(1-\phi)\left(f_5'(1-\phi') - f_6'\phi'^2\right),$$

$$D_{41} = f_4(1-\phi) - f_7\phi^2, \qquad D_{43} = f_5(1-\phi) - f_6\phi^2, \qquad D_{21} = D_{23} = D_{42} = D_{44} = 1.$$

The elements of the vector of unknowns $\boldsymbol{y}$ correspond to:

$$y_1 = -\frac{f_2}{f_6}\mathcal{H}_4 - \left(\gamma_{12} + \gamma_{11}\frac{\rho_f}{\rho}\right)\frac{\eta}{\kappa}q + \left(\gamma_{12}' + \gamma_{11}'\frac{\rho_f'}{\rho'}\right)\frac{\eta'}{\kappa'}q' + \frac{-f_2' + f_3'\left(\rho_f'/\rho\right)V_f'}{f_6'}\mathcal{H}_3' + \frac{-f_1 + f_3\left(\rho_f/\rho\right)V_s}{f_7}\mathcal{H}_2$$
$$+\frac{-f_1' + f_3'\left(\rho_f'/\rho\right)V_s'}{f_7'}\mathcal{H}_1', \qquad \text{(A1)}$$

$$y_2 = \mathcal{H}_2 + \mathcal{H}_4 + \frac{f_5'\left(1-\phi'\right) - f_6'\phi'^2}{f_6'}\mathcal{H}_3', \qquad \text{(A2)}$$

$$y_3 = \mathcal{H}_4\left(f_5 + f_6\phi\right)\left(1-\phi'\right) + \mathcal{H}_2\left(f_4 + f_7\phi\right)\left(1-\phi'\right) - \mathcal{H}_3'\left(1-\phi\right)\left(f_5' - f_5'\phi' - f_6'\phi'^2\right)$$
$$-\mathcal{H}_1'\left(1-\phi\right)\left(f_4' - f_4'\phi' - f_7'\phi'^2\right), \qquad \text{(A3)}$$

$$y_4 = \mathcal{H}_1' + \mathcal{H}_3' + \frac{f_5\left(1-\phi\right) - f_6\phi^2}{f_6}\mathcal{H}_4 + \frac{f_4\left(1-\phi\right) - f_7\phi^2}{f_7}\mathcal{H}_2, \qquad \text{(A5)}$$

where

$$f_1 = C\gamma_{12} + M\gamma_{11} - V_s^2, \quad f_2 = C\gamma_{12} + M\gamma_{11} - V_f^2, \quad f_3 = C\gamma_{11} + H\gamma_{12}, \quad f_4 = \det\boldsymbol{M}\ \gamma_{11} + HV_s^2,$$

$$f_5 = \det\boldsymbol{M}\ \gamma_{11} + HV_f^2, \quad f_6 = \det\boldsymbol{M}\ \gamma_{12} + CV_f^2, \quad \textit{and} \quad f_7 = \det\boldsymbol{M}\ \gamma_{12} + CV_s^2.$$

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