

Attenuation relationship of macroseismic intensity in Italy for probabilistic seismic hazard assessment

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Abstract - A statistical analysis of macroseismic intensity decay with distance has been conducted by using the most recent compilation of felt intensities relative to Italian earthquakes. An innovative aspect of the present study has been the attempt to define criteria for the validation of the attenuation relationships considered in their complete probabilistic form by a quantitative comparison with observed macroseismic fields. From the results of our analyses, a new attenuation relationship is proposed for the Italian territory, based on two independent variables only: epicentral intensity and hypocentral distance. Preliminary indications on the existence of differentiated attenuation features of macroseismic intensity over the national territory were gained.

1. Introduction

Empirical rules to evaluate decay of earthquake ground motion with distance play a basic role in seismic hazard studies. When hazard maps are compiled in terms of macroseismic intensity [about 60% of hazard maps described by McGuire (1993)], "attenuation" relationships are required which estimate intensity at the site as a function of epicentral intensity (or maximum intensity) and distance from the macroseismic epicentre (whatever its definition actually is). Macroseismic attenuation to be used for probabilistic hazard estimates has the general form (e.g., Cornell, 1968)

$$\text{prob} [I > I_s | I_S, R_{sS}] = 1 - P(I_s | I_S, R_{sS}) \quad (1)$$

where P is a generic cumulative distribution function (cdf) representing attenuation properties (probabilistic attenuation function), I_s the intensity threshold of interest at the site s , R_{sS} is the

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distance of s from the source S and I_S is the intensity at the source. In general, P can be expected to depend on the specific source S considered or on the specific area under study.

Intensity is a synthetic representation of earthquake effects over a wide area and not a punctual ground shaking measurement. This makes intensity data sensitive to a number of different aspects related both to regional scale features (radiation pattern of seismic sources, energy propagation) and local situation (site geomorphological setting and building vulnerability). In the lack of a direct physical model, the form of P is in general empirically assessed on the basis of a statistical analysis of a set of seismic effects $\{I_{se}\}$ (hereafter “felt intensities”) observed at the sites s during past earthquakes e (for Italy, see the compilations provided by Monachesi and Stucchi, 1997; Boschi et al., 1997, 2000). Though alternative positions are possible (see, e.g., Magri et al., 1994; D’Amico and Albarello, 2003), this analysis is performed in two steps. In the first step, the average μ of the distribution P is assessed by assuming a generic functional dependence F such that

$$\mu = F(I_S, R_{sS}) \quad (2)$$

(loosely speaking, F is commonly defined as “attenuation relation”). As concerns Italy, several functional forms of F have been proposed both on the basis of physical and empirical considerations (e.g., Veneziano, 1986; Grandori et al., 1987; Berardi et al., 1993; Magri et al., 1994; Gasperini, 2001). As a second step, residuals

$$\Delta I_{se} = I_{se} - \mu(I_S, R_{sS}) \quad (3)$$

are examined to assess the form of P . It is commonly assumed (and seldom carefully checked) that

$$P(I_s | I_S, R_{sS}) \approx \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{I_s + 0.5} e^{-\frac{[x - \mu(I_S, R_{sS})]^2}{\sigma^2}} dx \quad (4)$$

i.e., that P is the Gauss distribution function corrected to take into account the discrete character of I_s (see, e.g., Peruzza, 1996; Albarello et al., 2001).

It is a matter of fact that the cdf P is characterized by a quite large variance [actually, variance explained by empirical relationships (2) seldom exceeds 50% of the total]. Such a problem is in many cases under evaluated. Estimates performed for the Italian territory [see, e.g., Albarello et al. (2002)] showed to what extent variance of P dramatically affects hazard estimates. In particular, it can be easily seen that low variances result in low hazard and vice-versa. Thus, a realistic parameterisation of this quantity plays a major role in probabilistic seismic hazard assessment.

In this paper, a new analysis of macroseismic intensity attenuation in Italy is proposed. With respect to previous attempts, the present one relies on the analysis of more recent and extended macroseismic data sets both concerning epicentral information and documented effects at the site. A search for possible biases or incompleteness in the data samples has been performed in

advance but no a priori selection of more “reliable” data has been attempted. In the first part of the paper, a unique attenuation relation F [Eq. (2)] is determined for the whole Italian territory (i.e., the possible dependence of P on the source S or on the area of interest is not considered). In the second part, residuals from this relationship [Eq. (3)] are analysed to establish the form of P . Effectiveness of the adopted parameterisation is then evaluated on the basis of an empirical comparison with felt intensities. In the third part, residuals have been also analysed through a distribution-free approach for a preliminary search of regionalized attenuation patterns.

2. The data set

The CPTI catalogue of Italian earthquakes that occurred since 217 B.C. up to 1992 (Gruppo di Lavoro CPTI, 1999) has been considered in the present analysis both for epicentral and local intensity data [these latter resulting from a combination of information provided by Boschi et al. (1997), Monachesi and Stucchi (1997), macroseismic bulletins of the National Institute of Geophysics and Volcanology]. In particular, each event in the catalogue has been parameterised in terms of epicentral location and epicentral MCS intensity. The latter was preferred to the maximum intensity value since it appears less affected by local effects (see Gasperini and Ferrari, 1995, 1997). A fictitious depth value of 10 km has been attributed to each earthquake in order to avoid possible singularities in resulting attenuation relations (Gasperini, 2001).

Hereinafter, with an epicentral distance D_{sS} (in km), the parameter R_{sS} given by

$$R_{sS} = \sqrt{D_{sS}^2 + 10^2} \quad (5)$$

is used to characterize the hypocentral distance. Frequency distribution of felt intensities as a function of hypocentral distances are reported in Fig. 1.

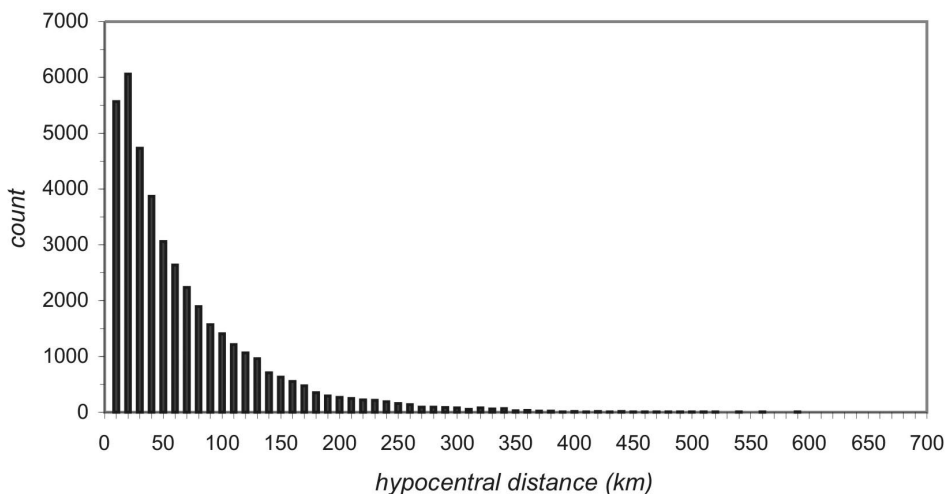


Fig. 1 - Frequency distribution of felt intensities as a function of hypocentral distances [Eq. (5)].

In principle, one could expect that for a given epicentral distance, taking the epicentral intensity value as fixed, the population of felt effects should be symmetrical around an “average” value (i.e., the probabilities that the felt intensity at a site is greater or lower than the average value are equal). This hypothesis allows the safe application of common regression techniques to deduce, from experimental data, the dependence of the average value on hypocentral distance and epicentral intensity. However, some specific features of intensity (e.g. the scarce documentability of lower intensities which could affect the macroseismic field far from the source or the presence of upper and lower bounds to the possible intensity values) could make this assumption unreliable. The presence of such biases, which could affect the statistical analysis, can be revealed by the analysis of the skewness relative to the sample of the attenuation values (epicentral minus felt intensity) as a function of the hypocentral distance.

To perform this analysis, attenuation values have been grouped into classes of hypocentral distance (each 10 km wide) and the sample skewness has been computed for each of them. To reduce interpretative problems, “uncertain” intensity attributions (e.g. VII-VIII), both relative to felt and epicentral intensities, have not been taken into account. Results obtained (Fig. 2) indicate that samples relative to the epicentral area ($R_{sS} \leq 15$ km) are significantly skewed towards positive values. This implies that, in this range of distance, the number of felt intensities lower than the average value is greater than the number of felt intensities higher than the average. Since, in many cases, epicentral intensity is chosen as the maximum intensity or something very near to this value, the presence of such a near-source bias is obvious. Samples relative to small hypocentral distances ($15 < R_{sS} \leq 35$ km) show lower (whether statistically significant) positive skewness. Apparently, no significant bias is present for larger hypocentral distances. This last result indicates that, unlike what emerged in previous studies (e.g. Gasperini, 2001), the effect of “incompleteness” (i.e. the lack of documentation about small seismic effects far from the source) could be not significant.

Since the short-distance bias could affect the following statistical analysis, data relative to sites located at very short hypocentral distances should be discarded. A further reason for tending to reject felt data very close to the epicentre, derives from possible near-source effects that could be responsible for strong lateral heterogeneities of the relevant macroseismic field

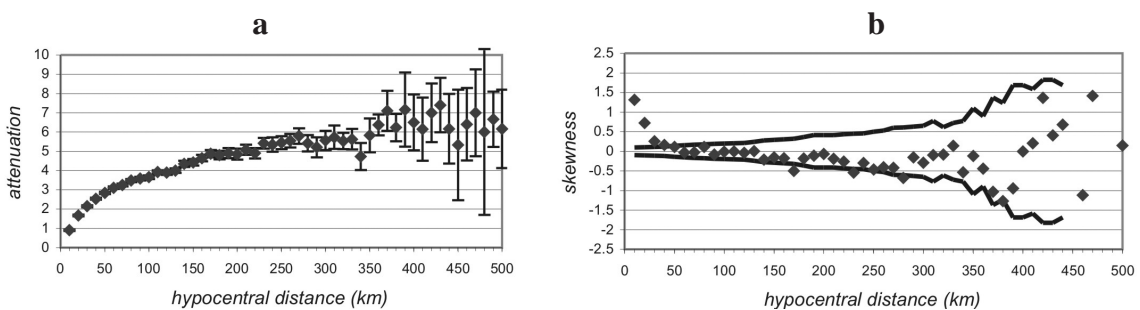


Fig. 2 - a): average attenuation values (epicentral minus felt intensity) with relevant error bars as a function of classes of hypocentral distance (each 10 km wide). “Uncertain” intensity attributions (e.g. VII-VIII), both relative to felt and epicentral intensities, have not been considered. b): skewness analysis relative to attenuation values vs. hypocentral distance classes (see above). Polygons indicate the relevant skewnesses. Continuous lines delimitate the 95% confidence interval computed in the assumption of normality (Rock, 1988).

(Gasperini, 2001). On the other hand, the histogram in Fig. 1 shows that most information is concentrated near the source. Furthermore, in common applications, attenuation relationships are applied without any limitation relative to hypocentral distances. This implies that if near-source data are discarded when attenuation is parameterised, far-field information will dominate making less reliable estimates where major effects are expected. In the present study, a cautious position has been adopted which aims at keeping the data set as large as possible, by only discarding those pieces of information which are more likely affected by sampling problems or by strong asymmetries in the relevant frequency distribution. For this reason, only data with $R_{sS} \leq 15$ km have been rejected. At the other extreme, since the amount of data available above 300 km is negligible (about 1% of the whole sample, see Fig. 1), they have not been taken into account in the following.

The selection reduces the original database from about 41500 to 14870 felt data (i.e., slightly more than 1/3 of the total) relative to 369 earthquakes over a total of 2480 reported in the CPTI catalogue. The resulting data set is anyhow conspicuous enough to perform the following analyses.

3. Statistical analysis

In analogy with the attenuation of instrumental parameters (e.g., Sabetta and Pugliese, 1987) the attenuation relation F [Eq. (2)] is assumed in the form

$$\mu = a + b R_{sS} + c \ln(R_{sS}) + d I_S \quad (6)$$

where, $a-d$ are empirical parameters to be determined by the statistical analysis of data relative to “univocal” intensity attributions (both relative to felt and epicentral intensities) with $15 < R_{sS} \leq 300$ km. This functional form is less exposed to numerical instabilities with respect to other formulations (e.g., Grandori et al., 1987; Peruzza, 1996) and allows us to take into account the documented dependence of attenuation on epicentral intensity (D’Amico and Albarello, 2003). The best fitting parameterisation (in the least squares sense) is

$$\mu = 3.6 - 0.003 R_{sS} - 0.98 \ln(R_{sS}) + 0.705 I_S \quad (7)$$

with a corresponding explained variance of 63%. Just to have a comparison, percentages of variance explained by using the alternative functional forms provided by Berardi et al. (1993) and Gasperini (2001) resulted to be 43.8 and 44.0% respectively. The t-test reveals that all the values relative to the regression parameters are significant at a 0.05 confidence level. Compared to the two above functional forms, Eq. (7) predicts a slower decay of intensity with hypocentral distance, particularly at $R_{sS} > 150$ km (see Fig. 3).

Residuals [Eq. (3)] obtained from the application of Eq. (7) are analysed to define the functional form of the cdf P . Fig. 4 summarizes the results of such analysis. The residual distribution is characterized by a zero average and a standard deviation of 1.071. Standardised

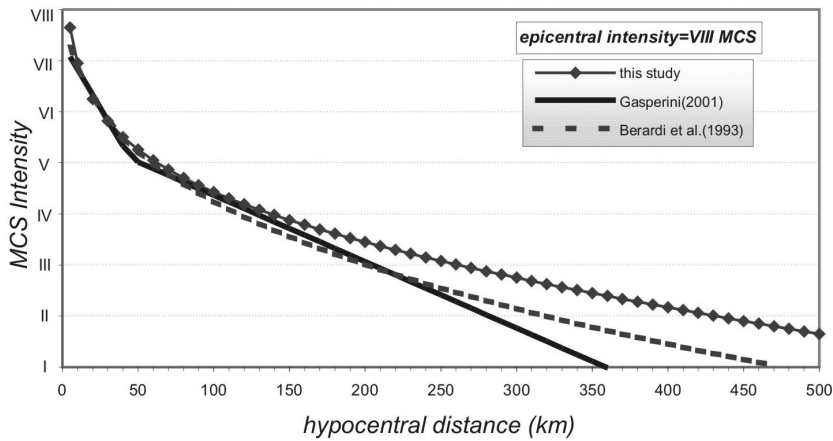


Fig. 3 - Comparison of the attenuation relation in Eq. (7) with those in the forms proposed by Berardi et al. (1993) and Gasperini (2001) with relevant parameters reassessed for the data set considered in this study. Attenuation is computed for an epicentral intensity corresponding to VIII MCS.

skewness and kurtosis (being respectively -0.112 with s.d. 0.020 and 0.357 with s.d. 0.040) are significantly different ($p < 0.05$) from those expected in the case of a normal population. These values imply that the residual distribution has less values than the normal around the average and more values in the negative tail.

For the sake of simplicity and in the hypothesis that discrepancies from Normality, though statistically significant, are not of practical relevance, one can make the assumption that parent distribution of residuals is actually Gaussian. In order to check this possibility, estimated intensities provided by a Gaussian cdf P with the average given by Eq. (7) and standard deviation σ of 1.072 have been compared with the whole set of documented intensities at sites located in the range 15-300 km from the relevant source. This comparison has been performed by using the

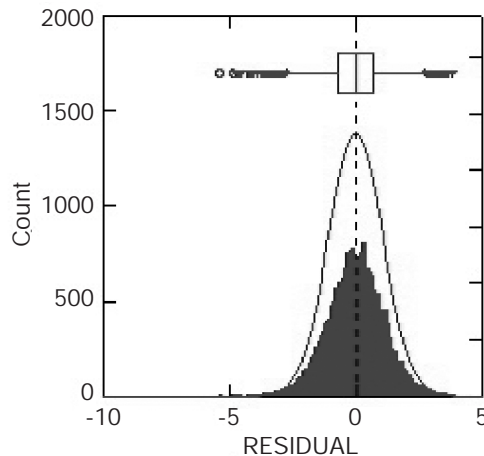


Fig. 4 - Frequency distribution of residuals (observed minus computed) obtained from Eq. (7). In the upper part of the figure, most important features of the sample are summarized in the form of a box and whisker plot (see Rock, 1988). The continuous curve represents the theoretical pattern of the Normal density probability distribution with 0 average and 1.071 standard deviations. Below, the histogram of residuals is reported.

approach described in the Appendix, by considering intensity thresholds ranging from VI to XI MCS. For each intensity threshold I_s , the number N^{obs} of documented felt intensities $\geq I_s$ has been compared with the number N of values expected in the assumption that P is actually Gaussian. In such case, 95% confidence intervals on these values roughly correspond to 2σ . In this comparison, both univocal and uncertain intensity attributions have been considered. Table 1 reports the results of such analysis, which indicate that the considered probabilistic attenuation function tends to overestimate the number of felt intensities for the threshold VI MCS and to underestimate the number of felt intensities for a threshold of VII MCS or higher.

Table 1 - Comparison, for each choice of the intensity threshold I_s in the range VI-XI MCS, between the number N^{obs} of documented felt intensities $\geq I_s$ and the number N of expected felt intensities in the assumption that the probabilistic attenuation function is Gaussian with average given by Eq. (7) and standard deviation 1.072. The columns $2\sigma_{N^{obs}}$ and $2\sigma_N$ indicate the relevant 95% confidence intervals. The comparison concerns all the felt intensities (including uncertain attributions) at sites located in the range 15-300 km from the relevant sources (see Appendix for details).

I_s	N^{obs}	$2\sigma_{N^{obs}}$	N	$2\sigma_N$
VI	12822	36	13587	134
VII	7786	44	7277	114
VIII	3492	36	3086	86
IX	1141	22	953	54
X	332	12	188	26
XI	46	8	20	8

The observed underestimate could be the effect of the asymmetries and of the long negative tail discussed above (see Fig. 4). A possible compensation for this effect could be obtained by modifying the parameterisation of the Gaussian probabilistic attenuation function. In particular, with a trial and error procedure it has been found that a 10% increase in the standard deviation from the original value of 1.072 to 1.25 reduces the observed underestimate (Table 2).

In fact, the proposed correction dramatically reduces the underestimate for intensity thresholds of VII MCS and above without any significant increase of the overestimate relative to the VI MCS threshold which remains of the order of 7%. By adopting a value of 1.25 for the relevant standard deviation, the Gaussian form for the probability attenuation function can be maintained.

Table 2 - Comparison, for each choice of the intensity threshold I_s in the range VI-XI MCS, between the number N^{obs} of documented felt intensities $\geq I_s$ and the number N of expected felt intensities in the assumption that the probabilistic attenuation function is Gaussian with average given by Eq. (7) and standard deviation 1.25 (see caption of Table 1).

I_s	N^{obs}	$2\sigma_{N^{obs}}$	N	$2\sigma_N$
VI	12822	36	13860	142
VII	7786	44	7664	122
VIII	3492	36	3437	92
IX	1141	22	1213	62
X	332	12	300	34
XI	46	8	44	14

4. Search for regional attenuation patterns

The attenuation relation in Eq. (7) can be used as a reference to detect possible regional differences in intensity attenuation. To this purpose, the distribution-free approach proposed by D'Amico and Albarello (1999) has been used. In this approach, residuals obtained by the application of Eq. (3) by using Eq. (7) are analysed as a function of their geographical distribution.

To this purpose, the Italian region has been subdivided into cells of $1^\circ \times 1^\circ$ (in latitude and longitude) with an overlapping of 50% between contiguous cells. The median of residuals in each cell was computed and the hypothesis that the sample is extracted by a population having 0 median was checked by the use of a simple distribution-free test. Significant deviations from this last hypothesis suggest that local attenuation is different from the average one. For this analysis, only "univocal" intensity attributions for sites located in the range 15-300 km from the relevant source have been considered. The results of this analysis are shown in Fig. 5. These

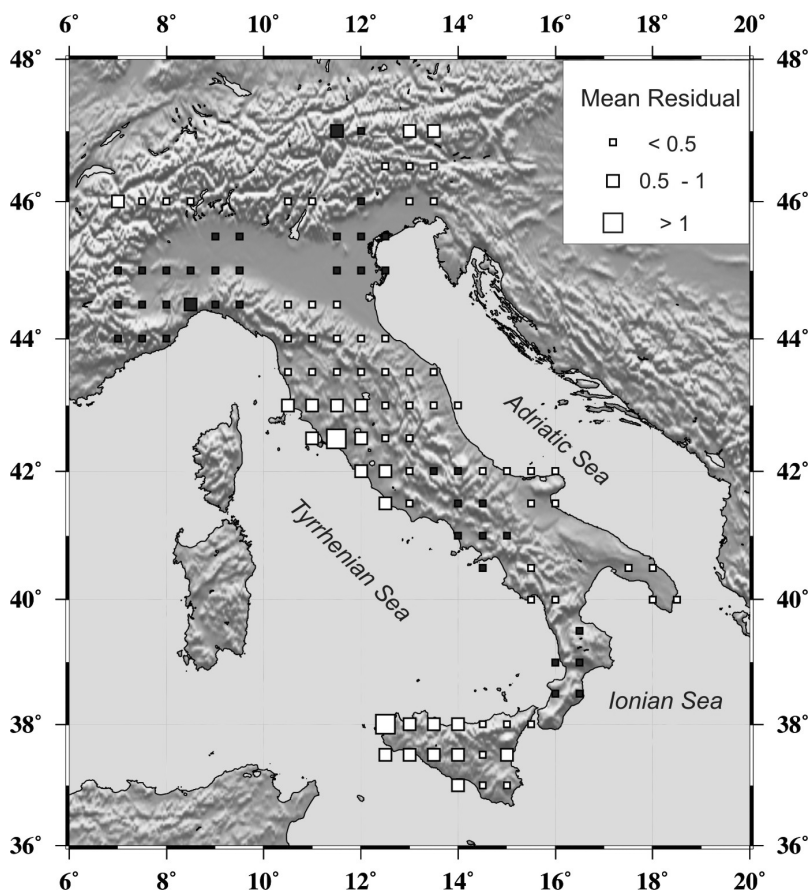


Fig. 5 - Space distribution of residuals of felt intensity (observed-computed) from Eq. (7). Size of squares is proportional to the average of residuals relative to felt data at localities included in a $1^\circ \times 1^\circ$ cell centred in the middle of the square. White and black squares respectively correspond to negative/positive values of the mean residual (i.e. computed site intensities on average higher/lower than observed ones). Only cells characterized by a sample median significantly different from zero are displayed (D'Amico and Albarello, 1999).

show that significant deviations from the average attenuation pattern exist. It can be seen that the application of Eq. (7) generally produces systematic overestimates of felt intensities (i.e. negative mean residuals) along the Tyrrhenian coast of peninsular Italy, in Sicily and in Apulia, with major deviations in southern Tuscany and western Sicily. Small overestimates also characterize the central-northern Apennines and the southernmost part of the southern Apennines. Relatively small (though significant) underestimates, instead, characterize the whole of northern Italy (in particular in the north-western sector), southern Apennines and Calabria.

It is worth noting that, except for southern Tuscany and western Sicily, the average deviations of local patterns with respect to the one of Eq. (7) are less than one half of the relevant standard deviation, whether you consider the empirical (1.072) or the “corrected” one (1.25). This suggests that, except for the two zones cited above, regionalized attenuations (at least as concerns the regional scale considered here) could play just a minor role for the definition of more effective attenuation relationships.

5. Conclusions

The attenuation pattern of intensity in Italy has been parameterised in the form of a probability distribution function to be directly implemented in numerical codes devoted to probabilistic seismic hazard estimates. With respect to previous studies, greater efforts have been devoted to the correct quantification of the variance which characterizes the probabilistic attenuation function. Moreover, a new approach has been developed which allows us to evaluate performances of the considered attenuation function in comparison to data available on macroseismic fields of past earthquakes. The proposed approach also allows us to take into account uncertainties in intensity attributions relative to ill-documented earthquakes.

The empirical parameterisation of the probabilistic attenuation function has been obtained through the statistical analysis of the most recent and extensive compilation of macroseismic data in Italy. This analysis reveals that functional dependence of average attenuation on epicentral intensity and distance from the source allows to explain no more than 63% of the sample variance. The analysis of residuals and the comparison with observed macroseismic fields indicate that, though the shape of the probabilistic attenuation function is slightly different from the Gaussian one, results compatible with observations (at least as concerns intensities \geq VII MCS, i.e. those relevant to hazard studies) can be obtained maintaining the assumption of normality, but enlarging up to 1.25 the relevant standard deviation. This value turns out to be very large with respect to previous estimates, which indicated values generally lower than 1 (e.g., Peruzza, 1996; Gasperini 2001). Since seismic hazard estimates are very sensitive to the variance associated to the attenuation relationship, the application of the parameterisation proposed here will result in a general increase of estimated seismic hazard. According to Cornell (1971) and Brillinger (1982), a quantitative evaluation of this effect can be obtained (see Fig. 6).

It can be debatable if such a high variance is actually representative of uncertainty in the “forecast” of site intensity from epicentral data or if it is an artefact induced by the presence of low quality data in the considered macroseismic database. Indeed, such a large dispersion could

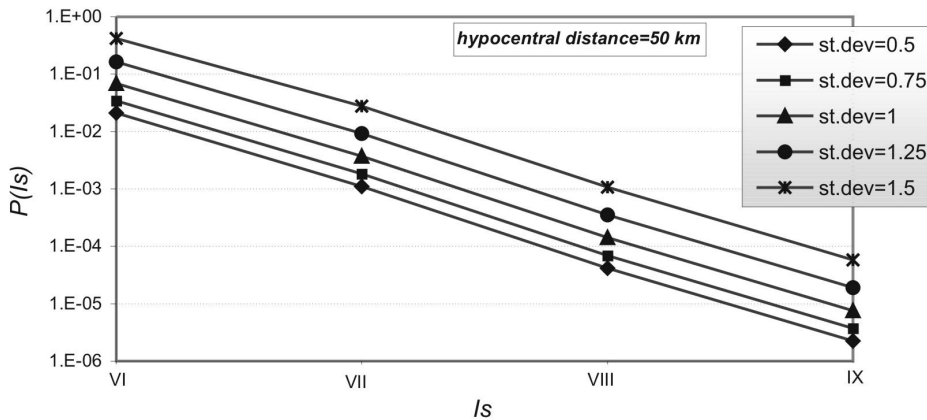


Fig. 6 - Probability of exceedance (exposure time period = 50 yr) of intensity thresholds I_s ranging from VI to IX MCS at a distance of 50 km from a point source, computed for different values of the standard deviation associated to the attenuation relation in the form (7). To this purpose the formulation provided by Cornell (1971) and Brillinger (1982) was used, by using parameters in Eq. (7) and realistic seismicity rates for the Italian situation.

be reduced by an accurate selection of input data, e.g., by excluding the oldest and less reliable information (e.g., Gasperini 2001) or by selecting “representative” events only (e.g., Peruzza 1996). However, empirical relationships obtained by selecting data could be biased towards situations presumed to be “typical” and thus be less representative of all the situations actually possible in the examined area.

Another possibility of reducing the variance is to make explicit the effect of other significant “hidden” variables, such as the “regional” dependence of attenuation patterns. Actually, differentiated attenuation patterns have been hypothesized by Peruzza (1996), who suggested a specific attenuation rule for each seismogenic zone (considering thus a priori that source effects are dominant for attenuation). To avoid such positions which could bias final results, a distribution-free approach and without a priori assumptions about possible source or geostructural effects, has been applied for the preliminary search of regional differences in the attenuation pattern at a 10^2 km scale. This analysis has revealed that significant regional differences seem to exist. However, except for two relatively small areas (southern Tuscany and western Sicily), such differences appear unable to significantly account for the large stochastic dispersion which characterizes observed attenuation patterns.

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Appendix

Observations are constituted by a set of Q earthquakes being the j -th event characterized by the epicentral intensity I_j . For each of these events, there are M localities (being the i -th site located at a distance R_{ij} from the source of the j -th event) where seismic effects have been documented. Through the use of the cumulative distribution function P [Eq. (1)], it is possible to estimate the expected number of these sites \hat{N}_{I_s} that experienced felt intensities $I \geq I_s$. Since such occurrences can be considered as realizations of a Bernoullian stochastic variable, \hat{N}_{I_s} can be computed in the form

$$\hat{N}_{I_s} = \sum_{j=1}^Q \sum_{i=1}^{M_j} \text{prob} [I \geq I_s | I_j, R_{ij}] = \sum_{j=1}^Q \sum_{i=1}^{M_j} H_{ij}(I_s) \quad (\text{A.1})$$

where H is given by

$$H_{ij}(I_s) = \sum_{l=I_s}^{12} p(l | I_j, R_{ij})$$

with p being the density related to the cumulative distribution function P in Eq. (1).

The associated standard deviation is

$$\sigma_{N_{I_s}} = \sqrt{\sum_{j=1}^Q \sum_{i=1}^{M_j} \{H_{ij}(I_s) [1 - H_{ij}(I_s)]\}} \quad (\text{A.2})$$

If epicentral intensities are affected by uncertainty, these formulas have to be modified to take into account such an additional uncertainty. We assume that uncertainty in the intensity attribution is described by a probability density distribution $k(I)$. A possible form for this probability function is the following

$$\left\{ \begin{array}{l} V^{int} = V \\ V^{int} \neq V \end{array} \right. \left\{ \begin{array}{ll} I < V & \rightarrow k(I) = 0 \\ I = V & \rightarrow k(I) = 1 \\ I > V & \rightarrow k(I) = 0 \\ I < V^{int} & \rightarrow k(I) = 0 \\ I = V^{int} & \rightarrow k(I) = 0.5 \\ I = V^{int} + 1 & \rightarrow k(I) = 0.5 \\ I > V^{int} + 1 & \rightarrow k(I) = 0 \end{array} \right. \quad (\text{A.3})$$

being V the value reported in the database to represent the felt intensity and V^{int} the integer part of V . This position makes explicit the interpretation of intermediate intensity values present in the database as expression of uncertain attribution of documented effects to one of the contiguous integer values.

Thus, the probability $G_{ij}(I_s)$ that the j -th event at the i -th site is characterized by felt intensity $\geq I_s$ is given by

$$G_{ij}(I_s) = \sum_{l=1}^{12} [k_j(l) H_{ij}(l)] \tag{A.4}$$

where k_j is defined for each j -th earthquake by (A.3). Thus, (A.1) and (A.2) have to be modified in the form

$$\hat{N}_{I_s} = \sum_{j=1}^Q \sum_{i=1}^{M_j} G_{ij}(I_s) \tag{A.1'}$$

and

$$\sigma_{N_{I_s}} = \sqrt{\sum_{j=1}^Q \sum_{i=1}^{M_j} \{G_{ij}(I_s) [1 - G_{ij}(I_s)]\}} \tag{A.2'}$$

respectively.

These values could be compared with the observed number $N_{I_s}^{obs}$ of sites where felt effects were actually $\geq I_s$. When no uncertainty affects documented intensities, it simply holds that

$$N_{I_s}^{obs} = \sum_{j=1}^Q \sum_{i=1}^{M_j} C_{ij}(I_s) \tag{A.5}$$

with $C_{ij} = 1$ if at the i -th site felt effects were $\geq I_s$ and $C_{ij} = 0$ otherwise. However, when uncertain intensity attributions exist, $N_{I_s}^{obs}$ becomes an aleatory variable with expectation $\hat{N}_{I_s}^{obs}$ and variance $\sigma_{N_{I_s}^{obs}}^2$. To compute these values, one can consider C_{ij} a Bernoullian variable with an associated probability K such that

$$\begin{cases} V^{int} = V & \begin{cases} I_s \leq V & \rightarrow K(I_s) = 1 \\ I_s > V & \rightarrow K(I_s) = 0 \end{cases} \\ V^{int} \neq V & \begin{cases} I_s \leq V^{int} & \rightarrow K(I_s) = 1 \\ I_s = V^{int} + 1 & \rightarrow K(I_s) = 0.5 \\ I_s > V^{int} + 1 & \rightarrow K(I_s) = 0 \end{cases} \end{cases} \tag{A.6}$$

which is analogous to the position (A.3).

In this case we have

$$\hat{N}_{I_s}^{obs} = \sum_{j=1}^Q \sum_{i=1}^{M_j} K_{ij}(I_s) \tag{A.7}$$

with an associated standard deviation

$$\sigma_{N_s}^{obs} = \sqrt{\sum_{j=1}^Q \sum_{i=1}^{M_j} \{k_{ij}(I_s) [1 - K_{ij}(I_s)]\}} \quad (\text{A.8})$$

References

- Albarelo D., Bosi V., Brammerini F., Lucantoni A., Naso G., Peruzza L., Rebez A., Sabetta F. and Slejko D.; 2001: *Carte di pericolosità sismica del territorio Nazionale*. Quad. Geofis., **12**, 1-7.
- Albarelo D., Brammerini F., D'Amico V., Lucantoni A. and Naso G.; 2002: *Italian intensity hazard maps: a comparison of results from different methodologies*. Boll. Geof. Teor. Appl., **43**, 249-262.
- Berardi R., Petrunaro C., Sonetti L., Magri L. and Mucciarelli M.; 1993: *Mappe di sismicità per l'area italiana*. ISMES, Bergamo, Italy, 52 pp.
- Boschi E., Guidoboni E., Ferrari G., Valensise G. and Gasperini P.; 1997: *Catalogo dei Forti Terremoti in Italia dal 461 a.C. al 1990*. ING-SGA, Bologna, 644 pp. and CD-ROM.
- Boschi E., Guidoboni E., Ferrari G., Mariotti D., Valensise G. and Gasperini P.; 2000: *Catalogue of Strong Italian Earthquakes from 461 B.C. to 1997*. ING-SGA, CD-ROM.
- Brillinger D.R.; 1982: *Seismic risk assessment: some statistical aspects*. Earthq. Predict. Res., **1**, 183-195.
- Cornell C.A.; 1968: *Engineering seismic risk analysis*. Bull. Seism. Soc. Am., **58**, 1583-1606.
- Cornell C.A.; 1971: *Probabilistic analysis of damage to structures under seismic loads*. In: Howells D.A., Haigh I.P. and Taylor C. (eds), *Dynamic Waves in Civil Engineering*, J. Wiley, London, pp. 473-493.
- D'Amico V. and Albarelo D.; 1999: *The problem of regionalization in the study of magnitude-intensity correspondences in the Mediterranean area: a distribution-free approach*. Pure Appl. Geophys., **156**, 651-665.
- D'Amico V. and Albarelo D.; 2003: *The role of data processing and uncertainty management in seismic hazard evaluations: insights from estimates in the Garfagnana-Lunigiana area (Northern Italy)*. Natural Hazards, **29**, 77-95.
- Gasperini P.; 2001: *The attenuation of seismic intensity in Italy: a bilinear shape indicates the dominance of deep phases at epicentral distances longer than 45 km*. Bull. Seism. Soc. Am., **91**, 826-841.
- Gasperini P. and Ferrari G.; 1995: *Stima dei parametri sintetici*. In: Boschi E. et al. (eds), *Catalogo dei Forti Terremoti in Italia dal 461 a.C. al 1980*. ING-SGA, Bologna, pp. 96-111.
- Gasperini P. and Ferrari G.; 1997: *Stima dei parametri sintetici: nuove elaborazioni*. In: Boschi E. et al. (eds), *Catalogo dei Forti Terremoti in Italia dal 461 a.C. al 1990*. ING-SGA, Bologna, pp. 56-64.
- Grandori G., Perotti F. and Tagliani A.; 1987: *On the attenuation of macroseismic intensity with epicentral distance*. In: Cakmak A.S. (ed), *Ground Motion and Engineering Seismology*, Elsevier, Amsterdam, pp. 581-594.
- Gruppo di Lavoro CPTI; 1999: *Catalogo Parametrico dei Terremoti Italiani*. ING, GNDT, SGA, SSN, Bologna, 92 pp., Internet web site: <http://emidius.mi.ingv.it/CPTI/home.html>.
- Magri L., Mucciarelli M. and Albarelo D.; 1994: *Estimates of site seismicity rates using ill-defined macroseismic data*. Pure Appl. Geophys., **143**, 4, 617-632.
- McGuire R.K.; 1993: *Computations of seismic hazard*. Ann. Geofis., **36**, 181-200.
- Monachesi G. and Stucchi M.; 1997: *DOM4.1 – an intensity database of damaging earthquakes in the Italian area*. Internet web site: <http://emidius.mi.ingv.it/DOM/>

Peruzza L.; 1996: *Attenuating intensities*. Ann. Geofis., **39**, 1079-1093.

Rock N.M.S.; 1988: *Numerical Geology*. Springer-Verlag, 427 pp.

Sabetta F. and Pugliese A.; 1987: *Attenuation of peak horizontal acceleration and velocity from Italian strong-motion records*. Bull. Seism. Soc. Am. **77**, 1491-1513.

Veneziano D.; 1986: *Seismic hazard analysis for the Friuli region: part III: ground motion models*. Consulting report to Comitato Nazionale per la Ricerca e per lo Sviluppo dell'Energia Nucleare e delle Energie Alternative, Roma, 163 pp.