

## Joint solution of seismicity parameters for seismic source zones through simulation

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**Abstract** - A key issue in seismic hazard modelling is determining seismicity parameters for each source zone in a given model. The three main parameters are the activity rate, the  $b$ -value (slope of the Gutenberg-Richter magnitude-frequency curve), and the maximum magnitude. Procedures for estimating these vary from study to study; in recent years the application of maximum likelihood methods, with or without priors, seems to be the most favoured approach. A new approach is presented here, based on Monte Carlo methods, which solves for all three parameters simultaneously. The conceptual basis is as follows: there exists some “true” set of values for  $a$ ,  $b$  and  $M_{max}$  that governs the long-term occurrence of earthquakes in a zone. Take three values for  $a$ ,  $b$  and  $M_{max}$  at random and use them to generate a synthetic earthquake catalogue, subject to the same historical constraints as the real catalogue. Is the resulting synthetic catalogue similar to the real one? If so, the  $a$ ,  $b$  and  $M_{max}$  values are credible. If not, try again. If one repeats the exercise a very large number of times, one easily builds up a weighted distribution of credible values for  $a$ ,  $b$  and  $M_{max}$  that can be converted directly into a logic tree structure. The method is entirely data-driven, and imposes no preconceived assumptions on the shape of the uncertainty distribution. Also the method tests implicitly whether the Gutenberg-Richter model itself is credible for that data set. If it turns out to be the case that no values for  $a$ ,  $b$  and  $M_{max}$  can provide a good approximation to the observed data, then a different seismicity model is called for.

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## 1. Introduction

Many details of seismic hazard methodology are not widely discussed in the literature; in fact they are often not discussed at all, except in the context of project reports that are difficult to access for the general reader, or the manuals that accompany some seismic hazard software. It is often the case that journal papers on seismic hazard pass over technical issues in order to concentrate discussion on the seismotectonics and the results; while commercial hazard studies, which may contain fuller details on every aspect of the methodology, are usually inaccessible and cannot be referenced.

The purpose of this paper is to discuss certain issues relating to how earthquake recurrence is generally modelled in seismic hazard studies, and to present a new method for estimating the key parameters. It is assumed throughout this paper that earthquake occurrence can reasonably be modelled as a Poisson process within any individual seismic source zone. This assumption is normally made in probabilistic seismic hazard assessment (PSHA). Contrary hypotheses are another topic altogether.

## 2. Magnitude frequency relations in PSHA

Since the PSHA method was first outlined by Cornell (1968), it has been common to characterise seismicity within any source zone in terms of the familiar Gutenberg-Richter (G-R) equation

$$\text{Log } N = a + b M, \quad (1)$$

where  $N$  is the number of earthquakes per year exceeding magnitude  $M$  and  $a$  and  $b$  are constants. The value  $a$  is a measure of the activity rate, while  $b$  is the slope controlling the relative frequency of large earthquakes to small ones.

A problem arises in considering possible uncertainty in the data. For any source zone, the available earthquake data may be restricted to a period of a few hundred years or less, and in general, the lower the seismicity, the more it is the case that the seismicity observed in the historical period may not be representative of the long-term seismicity. There are three options for modelling this uncertainty in the PSHA process:

1. ignore it;
2. use a logic tree approach;
3. model the uncertainty as a continuous function of the modelled parameters.

Option 1 is implicitly adopted in cases where the software used has no facility for addressing such uncertainty. Option 2 is widely used; logic trees were first proposed by Coppersmith and Youngs (1986); a recent example of the application of the method is found in Wahlström and Grünthal (2001). Option 3 is less common, but removes the necessity of making partly subjective decisions about the weighting to be applied to different branches of the logic tree (Musson, 2000). An example of option 3 would be to consider the uncertainty to follow a normal

distribution about the estimated value and to draw many samples from this distribution using Monte Carlo methods. This approach can be mimicked using a logic tree with enough branches. For the purposes of this discussion it will be assumed that the logic tree method is in use.

There are three key parameters that control seismicity recurrence, the  $a$  and  $b$  parameters in Eq. (1), and the maximum magnitude value ( $M_{max}$ ) at which the equation is truncated. Here it is assumed for the purposes of discussion that Eq. (1) is truncated abruptly at  $M_{max}$ ; there are other options available, but these do not materially affect the arguments in this paper.

Existing methods for estimating the  $a$  and  $b$  values and associated uncertainties can be classed under three groups.

The first practice involves mostly the application of expert judgement. Several candidate  $b$  values may be proposed (for example, measured value in the zone, measured value in a larger region containing the zone, measured values from different time periods) and assigned arbitrary weights.

The second practice is to use simple statistical measures of uncertainty on the measured  $a$  and  $b$  values, such as the standard error of the regression.

The third practice is to use a maximum likelihood function to estimate the probability distribution of  $a$  and  $b$  jointly (as in Weichert, 1980; Veneziano and van Dyke, 1985; Johnston et al., 1994). This method is much more rigorous than the previous two.

The estimation of  $M_{max}$  and its uncertainty is usually treated as a separate issue, independent of Eq. (1). Methods for estimating  $M_{max}$  include various statistical measures [see especially Kijko and Graham (1998)], taking the largest historical magnitude and adding a safety margin, using geological criteria (Wells and Coppersmith, 1994; Abrahamson, 2000), extrapolating from largest observed events in similar tectonic environments, and so on. It is not the purpose of this paper to review these methods;  $M_{max}$  is only of concern so far as it influences, or is influenced by, Eq. (1).

The correlation between  $M_{max}$  and the G-R  $a$  and  $b$  values is not intuitive, but exists, if weakly. The argument is as follows. Consider an area with 500 years of observed seismicity, with no earthquakes  $M > 5.5$ . How likely is it that earthquakes up to  $M = 6.5$  are possible, but by chance, none occurred in the historical period? If the activity rate is low and the  $b$  value has a steep slope, the probability of an earthquake  $5.6 < M < 6.5$  is small, so it is reasonable that such an event might not have been observed by chance. If the activity rate is high and the  $b$  value has a gentle slope, the probability of an earthquake  $5.6 < M < 6.5$  in 500 years is much higher, and it becomes unlikely that this did not happen just by chance, therefore it is relatively unlikely that  $M_{max}$  is large. This works both ways; if one assumes that  $M_{max}$  is an independently-determined parameter, a high  $M_{max}$  value tends to lead to estimates of steeper  $b$  values. If one takes the  $b$  value as independent, lower values of  $b$  imply lower values of  $M_{max}$ .

Since one cannot easily affirm which value is independent, the ideal method would determine both jointly. Such a method will now be described (though it can be admitted that since the  $M_{max}$  correlation with  $a$  and  $b$  is usually weak, the effects of ignoring it in practice are not overly significant).

In the following discussion, the objective is to assess the probability distribution of values for  $a$ ,  $b$  and  $M_{max}$  for a single zone in a model. In a full hazard study, the analysis would be repeated for each zone independently. The results are properties of the zones in the model, irrespective of any chosen site for which the hazard is to be assessed.

### 3. Recurrence parameter determination through simulation

Given that, for any seismic source zone, the future occurrence of earthquakes is controlled by the three parameters  $a$ ,  $b$  and  $M_{max}$ , it is assumed that there must be “true” values for these three parameters that represent the long-term recurrence rates over several thousand years. For the sake of simplicity in the discussion that follows, it will be assumed that a study is being conducted in which a single source zone is involved, for which recurrence statistics need to be computed. For a more complex model, the analysis would just be extended to each zone in the model in turn.

The observed seismicity of the zone over the past 500 years is one possible outcome (assuming Poisson behaviour) of the operation of these true values over 500 years. The observed seismicity in the zone over the next 50 years (lifetime of the structure for which hazard is to be assessed) will be another such outcome. If it were possible to know the true values of the long-term parameters, one could compute the true hazard probabilities. Since it is not possible to know the true values, it is necessary to compute the hazard by including the probability that the true values correspond to certain sets of possible values.

The essence of the method described here is to canvas a very large number of possible sets of values ( $a$ ,  $b$  and  $M_{max}$  “triplets”) and examine whether they are credible candidates for the true values. This can be done by forward simulation. Given a particular set of values, what results might have been obtained in the historical period if those were the true values? This can easily be found by making a synthetic earthquake catalogue based on the test values and comparing it to the true catalogue.

At this point it is necessary to raise the question of historical completeness constraints. For any earthquake catalogue, as is well known, one can divide up the length of it into discrete periods according to estimates of completeness with respect to magnitude.

For the illustrative example in this paper, it is assumed for a hypothetical source zone that the completeness is as shown in Table 1. This means that for the first 100 years of the catalogue (starting in 1100 AD) one expects that only very large earthquakes (7.5 or over) would necessarily be reported. For the next 450 years, one can be sure that all earthquakes above 6.0  $M$  are present. And so on. In order to assess recurrence parameters optimally, one needs to use only those events that satisfy each completeness threshold. “Empty” periods can still be used. If there are no earthquakes present  $\geq 6.0 M$  between 1200-1649, this is still useful information that can be used to constrain the results.

The earthquakes that satisfy the completeness constraints can be divided up into magnitude bins and listed as the discrete number of events in each magnitude interval. The aim is to find triplets of  $a$ ,  $b$  and  $M_{max}$  that generate a similar number of events in each magnitude bin, given the same historical constraints.

First, it is necessary to define a parameter space for  $a$ ,  $b$  and  $M_{max}$  which will define the limits of the values to be investigated. This can be as broad as desired. If very wide bounds are used (for example:  $b$  lies between -0.1 and -2.1) then the extreme values will simply not show in the final results if they are unrealistic. If very narrow bounds are used then some realistic values may be excluded. It is possible (and often necessary) to make a test run and then change the parameter space on the basis of the results, better to capture the distribution of probable values.

**Table 1** - Notional completeness periods for an earthquake data set, used in the worked example.

Dates	Completeness
1100-1199	7.5
1200-1649	6.0
1650-1849	5.0
1850-1869	4.5
1870-2000	4.0

The length of each synthetic catalogue that will be generated in the analysis will be the same as the length of the historical catalogue, and it will mimic the historical restrictions of the real catalogue. Thus, working from Table 1, if the historical catalogue starts with a 100 year period in which only events  $\geq 7.5$  are known, then each synthetic catalogue will start with a similar 100 period in which all events  $< 7.5$  are assumed to be lost, and so on for each time period.

Given this information, each synthetic catalogue is constructed from a randomly-chosen  $a$ ,  $b$  and  $M_{max}$  triplet using Monte Carlo simulation and assuming a Poisson process. Events less than the minimum reported magnitude restriction for each “historical” period are discarded. The number of earthquakes in the resulting synthetic catalogue is then counted.

Table 2 shows a worked example using the historical information in Table 1. The numbers of historical events are 7, 6, 3 and 1 in half-magnitude bins starting at  $M = 4$ . Take a set of random values for  $a$ ,  $b$  and  $M_{max}$ . The first random set is  $a = 2.43$ ,  $b = -0.95$ ,  $M_{max} = 6.8$ . A synthetic catalogue is generated for these values, and the results for the first four magnitude bins, when the events are counted, are 2, 1, 4, 0. This is not a good approximation to the historical result. This can be partly due to the fact that the triplet values are not close to the true values, but may also be due to chance factors in one simulation. Since a very large number of simulations will be conducted, this is not a problem.

The second set of values is  $a = 2.97$ ,  $b = -0.97$ ,  $M_{max} = 6.4$ . This time the results are 8, 6, 3, 0, which is a much better fit.

Ideally one would like to generate exactly the historical outcome, but even with only seventeen earthquakes, the number of permutations of the exact magnitude distribution is so large that one has to accept a certain degree of error. In any case, the historical values are not completely certain, as there is some possible error in individual magnitude values than might

**Table 2** - Data set used in the worked example (number of events satisfying completeness thresholds) and the result of two trials with random values for  $a$ ,  $b$  and  $M_{max}$ .

Magnitude	# Events	Trial 1	Discrepancy	Trial 2	Discrepancy
4.0 - 4.4	7	2	5	8	1
4.5 - 4.9	6	1	5	6	0
5.0 - 5.4	3	4	1	3	0
5.5 - 5.9	1	0	1	0	1
6.0 - 6.4	0	0	0	0	0
6.5 - 6.9	0	0	0	0	0
7.0 - 7.4	0	0	0	0	0
7.5 - 8.0	0	0	0	0	0
			12		2

carry them across from one bin to another. Practical experience shows that a 15% tolerance factor is reasonable, in terms of the sum of discrepancies in each magnitude bin. As Table 2 shows, the overall error for Trial 1 is twelve (70.6%) and for Trial 2 is two (11.8%). A further refinement to the method is to require that the number of events match the historical total number of events. In this case, if the historical result is 7, 6, 3 and 1, then 8, 6, 3 and 0 is an acceptable match, but 5, 6, 3 and 1 is not. This improves precision at the expense of extra computing time.

It is assumed in this demonstration that there is no prior information about any of the three parameters, in which case the random values selected are picked from a flat distribution within the predefined limits. Alternatively, the model can easily be modified to include a weighted prior by introducing some other distribution of values (e.g. Gaussian about a given mean value) from which random samples are to be chosen. Furthermore, such an approach can be applied to one, two or three of the parameters, using any combination of distributions and weights.

A further refinement of this method is required to estimate  $M_{max}$  properly. It could be that the total discrepancy is one, but that one corresponds to a magnitude 7 event in the synthetic catalogue where the historical maximum was only 5.5. In such cases, where  $M_{max}$  (synthetic) >  $M_{max}$  (observed), the trial is automatically considered a failure.

The procedure is repeated millions of times if necessary, until a large number (say 5,000) of successes has been registered, and the associated triplet values noted. The distribution of values in the aggregated successes gives the probability distribution for the true recurrence parameters. For practical purposes, some binning of results is necessary. In this example, activity rates have been grouped in bins 0.2 in width,  $b$  values 0.1 in width, and  $M_{max}$  in quarter magnitude units. If one bin contains 36 successful tries, then the probability that these are the true values is  $36/5000 = 0.007$ . In this particular experiment, the best triplet set generated 139 successes, giving a weight of 0.0278 for the triplet  $a = 2.6$ ,  $b = -0.9$ ,  $M_{max} = 5.8$  (incidentally, the best fit values measured by traditional maximum likelihood method from the data at the outset of the experiment were  $a = 2.66$ ,  $b = -0.92$ ). The triplet  $a = 2.0$ ,  $b = -0.7$ ,  $M_{max} = 7.0$ , on the other hand, scored one success and therefore has weight 0.0002.

Note that there is an assumption here that the values that have most success in generating the historical results are the ones most likely to be the true ones. An analogy can be given: consider a situation where you are informed that several dice have been rolled and the result was eleven. You are asked to guess the number of dice. Possible answers range from two to eleven. The best answer is three. Rolling two dice will normally result in lower numbers (eleven is difficult to roll), and rolling more than three dice is likely to result in higher numbers, up to the point where the odds against rolling eleven on eleven dice are 1 in  $3.6 \times 10^8$ . While it is possible to roll eleven on, say, eight dice, it would be a rare event, and it is the nature of rare events that they are, basically, rare. Similarly, it is possible that the observed outcome of the seismicity of the historical period was a freak occurrence (and the true values were a low-scoring triplet) but it is not probable. And PSHA is based on probabilities.

It could also be objected that the historical outcome is only one possible output of the “true” values, which is used in this method to estimate the probability distribution of those values; however, if the historical outcome had been different (as it might well have been) then the same



analysis will yield different results. This neglects the fact that the method takes into account the probability that a significantly different historical outcome could have resulted from the true values. By testing enough triplets to saturate the parameter space, and examining all the possible outcomes, the method elicits the required probability distribution in the end through sheer brute force.

Thus, the weights derived for each triplet value are, in fact, the probabilities that these triplets are the true values.

It may seem strange that one synthetic catalogue only should be generated at each step when a triplet value is tested. Obviously, each triplet could generate an infinite number of possible outcomes. The method works because of the very large number of samples used, and any triplet (or a close approximation) may be sampled many times. An alternative approach, which works just as well, is to progress systematically through triplets within a range of values, and test each 1,000 times, noting how often the outcome approximated to the historical result. The net effect is identical to the procedure outlined above.

In this exercise the total number of triplets assigned non-zero weights was 198. This leads to a logic tree for the zone in question with 198 branches. This may seem a large number, but a modern hazard program on a powerful PC should be able to cope. It is necessary, of course, that the program be capable of handling joint values for  $a$ ,  $b$  and  $M_{max}$ .

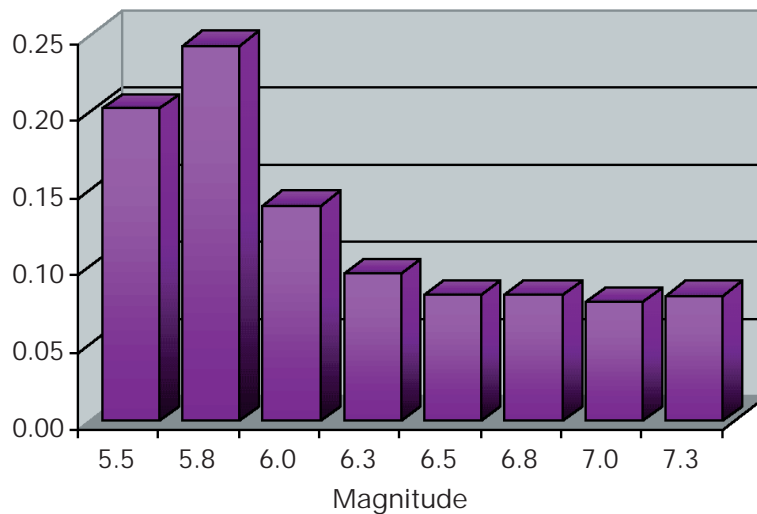
Since it is difficult to present the full three-dimensional results, the probabilities have been deconvolved by extracting the  $M_{max}$  results. In Table 3, the weights for the different  $a/b$  pairs are shown, and Fig. 1 presents the weights for  $M_{max}$ . These have been assessed in bins one quarter of a magnitude unit wide, but the scale shows one decimal place.

#### 4. Discussion

This method has several characteristic features. Firstly, it is the only method using a truncated linear model that determines all three parameters jointly, which has to be considered

**Table 3** - Results of the worked example. The values along the top are the  $a$  values (log number of events above magnitude zero per year) and the values down the left hand side are the  $b$  values. Values in the centre of the matrix are the weight for each  $a/b$  combination for the final logic tree (if  $M_{max}$  is treated separately). The values down the right side and along the bottom are the sum of weights for each  $b$  value and activity rate, allowing one to see the individual distribution of these parameters.

$b \backslash a$	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	weights
-0.7	0.015	0.0006									0.0156
-0.8	0.0494	0.071	0.0288	0.0012							0.1504
-0.9	0.0006	0.0126	0.0592	0.091	0.0458	0.0048					0.214
-1.0		0.0002	0.0008	0.014	0.0676	0.109	0.0484	0.0046			0.2446
-1.1					0.0012	0.0094	0.0672	0.0892	0.0588	0.0032	0.229
-1.2							0.0002	0.0074	0.0546	0.0772	0.1394
-1.3									0.0002	0.0068	0.007
weights	0.065	0.0844	0.0888	0.1062	0.1146	0.1232	0.1158	0.1012	0.1136	0.0872	



**Fig. 1** - Distribution of  $M_{max}$  as a series of probabilities (weights) for values at quarter-magnitude intervals (expressed to one decimal place).

an advantage even if the  $M_{max}$  correlation is not strong. It has always been possible to determine all three parameters jointly using an asymptotic model, such as the double-truncated exponential model, but the author's experience of such models is that the resulting  $M_{max}$  values tend to be highly controlled by the largest observed value in the catalogue.

Secondly, it is very much data driven and objective in nature. The principal human intervention is in setting the limits for the values of the parameter triplets within which random values will be taken. This can be done as an iterative procedure. There is no setting of priors, which might be considered to be introducing a subjective judgement. On the other hand, the absence of priors means that if the data are very poorly constrained, then one has to live with a high degree of uncertainty in the results (it can be argued that this is realistic). There is still the issue of the decisions made as to the historical completeness periods; this is an inevitable source of subjectivity, discussed in Musson (2002).

Thirdly, this method is unique in that it determines also the number of branches required in the logic tree to capture the full uncertainty (subject to the coarseness of the aggregation). If the results are highly constrained to a narrow range of values, the logic tree that is output will have relatively few branches.

Fourthly, the method also provides a test of whether the G-R model is actually appropriate. In some cases it will be found to be more or less impossible to generate successful results for any value triplets, even after millions of tries. This is an indicative warning that a linear model is simply not an adequate fit to the data. In such a case, forcing a linear fit will arguably invalidate the hazard results, because the hazard model will be such that, since it cannot reproduce the historical outcome, it is unlikely that the future outcome will correspond to it either.

Given such an indication, one should consider applying some other model, of which a characteristic earthquake model is most likely to be the best alternative. The principle of the method shown here can easily be adapted to other seismicity models besides the G-R.



As for disadvantages, there are two. Firstly, even using a tolerance factor, for large models in active seismic areas the calculation of the results can be time-consuming. Increasing the tolerance factor speeds up the computation at the expense of increasing the diversity of the results. The basic shape of the results, and the central values, are not affected. The value of 15% proposed and used in the example is derived from experience as being an expedient value in the trade-off between running time and precision of results.

The second is that the results are subject to small fluctuations due to chance. In Fig. 1, the last column is very slightly higher than the preceding one, which is an artefact.

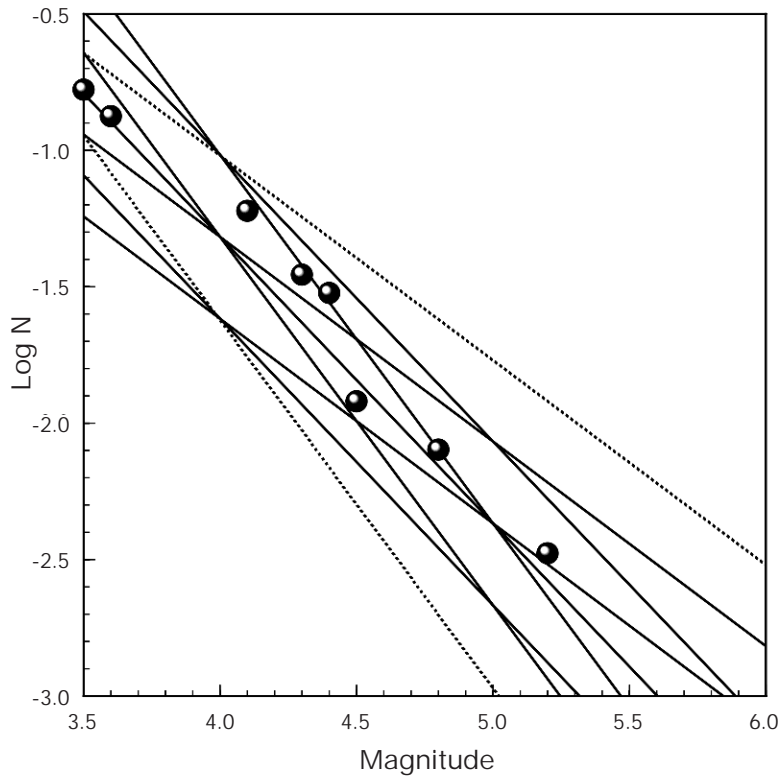
The shape of the  $M_{max}$  distribution curve in Fig. 1 is typical, and is similar to what would be obtained from a maximum likelihood assessment with no prior distribution. Both this method and the maximum likelihood approach seek to answer the question, "If there is no earthquake greater than  $x$  in  $y$  hundred years, how likely is that to be a chance occurrence?"

It will be noted in Table 3 that the results are subject to a marked diagonal pattern, indicating a very strong correlation between  $a$  and  $b$ . This requires some comment, as it relates to a topic not often discussed.

The reason for the correlation is evident, and the strength of it is due to the fact that the  $a$  values are expressed in terms of Eq. (1), i.e.  $a$  is the log of the number of earthquakes per year exceeding zero magnitude. In such a case, any variation of  $b$  is anchored around an  $a$  value located on the  $y$ -axis itself. Keeping  $a$  constant and changing  $b$  means that increasing (steepening) the  $b$  value causes the recurrence curve to underpredict all the data points, while decreasing the  $b$  value has the opposite effect. In order to increase  $b$  and still have the curve intersect the data, it is necessary to increase  $a$  as well.

The situation is not so extreme if, as is so often the case with hazard programs descended from EQRISK (McGuire, 1976), the activity rate is expressed not as in Eq. (1), but as the absolute number of events per year exceeding the hazard minimum magnitude value (which we can call  $M = 4$  for the sake of argument; in general practice values range from 3.7 to 5.0). In this case, there is still a correlation between  $a$  and  $b$ , but it is weaker, because the fulcrum about which the recurrence curve rotates as  $b$  varies is located at  $M = 4$  and not  $M = 0$ , and therefore the curve stays closer to the data. It is still the case that ignoring the correlation will lead to pathological cases if higher activity rates are combined with lower  $b$  values, or vice versa, a point made by Budnitz et al. (1999) in connection with a study by Bernreuter et al. (1989). This is shown in Fig. 2. Other studies could be adduced where this correlation has not been taken into account, with undesirable consequences. References in the general literature to this correlation existing are rare, even though it is well known to expert hazard practitioners from experience.

In Table 4, the results in Table 3 are recalculated using 4.0 as the base magnitude for the activity rate (still expressed in logarithms). There is still a significant diagonal element, although it is not as strong as in Table 3. It will also be noted, comparing the two tables, that in Table 3 the value of  $b$  is well constrained but  $a$  is not, while in Table 4,  $a$  is well constrained but  $b$  is not. Table 3 is perhaps an extreme case, but this trade-off occurs over the whole range of magnitudes at which activity rate may be expressed, and the shape of the uncertainty in either parameter is a function of this value.

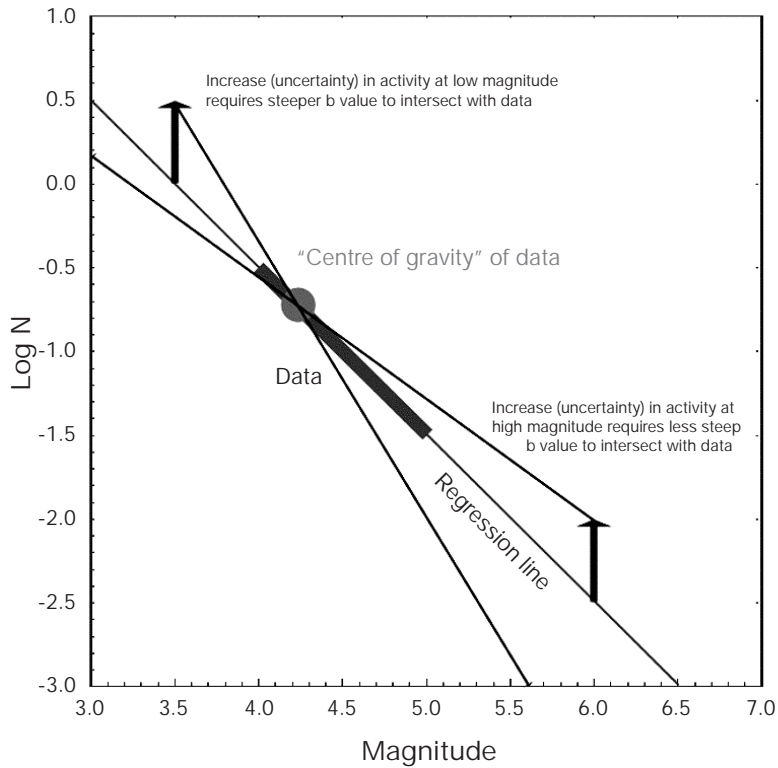


**Fig. 2** - Sample G-R plot showing the reality of three variations each on activity rate and slope, applied independently. The two lines marked with dots are particularly degenerate cases, and do not intersect with the data at all.

As a matter of interest, if the activity rate is calculated as the number of earthquakes per year exceeding  $M = 5$ , the correlation reverses itself, and a higher activity rate requires a lower (less steep)  $b$  value. The fact is, that for any earthquake data set, there exists a sort of centre of gravity in the G-R plot, which can be denoted as  $M_{mid}$ . In this case  $M_{mid}$  is about 4.3. The further is the magnitude at which activity rate is calculated from 4.3, the stronger will be the correlation, and the direction of the correlation depends on whether the value is larger or smaller than 4.3 (see Fig. 3). If the activity rate is calculated at the centre of gravity value, then the correlation disappears altogether, and  $a$  and  $b$  can be treated as independent variables. This can sometimes be useful to know. For instance, in cases where a logic tree is not used, but  $a$  and  $b$

**Table 4** - As Table 3, but the results have been calculated for the log number of events above magnitude 4.0.

$b \backslash a$	-1.6	-1.4	-1.2	-1.0	-0.8	-0.6	weights
-0.7	0.0002	0.0022	0.0284	0.0412	0.0032		0.0752
-0.8	0.0002	0.0048	0.0376	0.0714	0.0076		0.1216
-0.9	0.0002	0.0034	0.0522	0.098	0.0178	0.0004	0.172
-1.0	0.0004	0.0038	0.043	0.1168	0.0274		0.1914
-1.1		0.0028	0.0404	0.1114	0.034	0.0004	0.189
-1.2		0.0018	0.0308	0.089	0.0262	0.0004	0.1482
-1.3		0.0014	0.0172	0.0618	0.0222		0.1026
weights	0.001	0.0202	0.2496	0.5896	0.1384	0.0012	1



**Fig. 3** - Illustration of why the direction of the *a/b* correlation changes as a function of the magnitude at which activity rate is expressed.

are sampled from distributions, ensuring that recurrence relationships are calculated with respect to the centre of gravity point allows one to ignore the *a* and *b* correlation with impunity. Also, measuring the activity rate at this value minimises the uncertainty in *a*, but maximises the uncertainty in *b*.

Table 5 is an extension of Table 4, and addresses the issue of the sensitivity of the results to the number of trials. This table shows the mean, the standard deviation of the mean, and the confidence level of the mean at a significance of 5%, for each parameter, as the result of the analysis of 100, 250, 500, 1000 and 5000 successes. The actual values of the parameters are quite stable, although unsurprisingly, the confidence improves as the number of analysed values increases. Note that these are the overall mean values, and different from the values of the

**Table 5** - Mean values for parameters (compare to Table 4), as obtained with different numbers of successes. The standard deviation and the 95% confidence in the mean is also shown.

N	b value			a value			<i>M</i> <sub>max</sub>		
	mean	sd	Conf	mean	sd	Conf	mean	sd	Conf
100	-1.012	0.123	0.025	1.054	0.164	0.033	6.189	0.593	0.118
250	-1.010	0.120	0.015	1.033	0.173	0.022	6.181	0.562	0.070
500	-1.015	0.119	0.010	1.018	0.177	0.016	6.159	0.551	0.048
1000	-1.026	0.123	0.008	1.008	0.174	0.011	6.142	0.553	0.034
5000	-1.030	0.123	0.003	1.016	0.174	0.005	6.154	0.559	0.015

highest scoring single triplet - the difference being akin to the difference between mean and modal values. Table 4 is a better description of the distribution of probabilities of different values of  $a$  and  $b$  than is using a mean and standard deviation. The number of actual trials needed to obtain the preferred number of successes, as mentioned before, is a function of the tolerance level and the extent to which the data are “well-behaved” with respect to the model. With the test data set used in this study, if only two misfit values are tolerated, the total number of trials needed is roughly four times greater than if three misfits are allowed. If only one misfit is allowed, the number needed is six times greater again. For different data sets, the proportions may vary.

## 5. Conclusions

This paper has demonstrated a method for computing the parameters of earthquake recurrence for seismic hazard zones as a series of triplet values with associated probabilities that these values are the unknowable true values that govern long-term seismicity in the zone. These probabilities are also the weights that should be used in a logic tree formulation of the uncertainty in the hazard parameters. The method generates the entire part of the logic tree concerning recurrence, objectively determining the number of branches required to capture fully the uncertainty in the parameters for the zone in question.

This method is useful to apply, therefore, in cases where it is desired to incorporate the full uncertainty in these parameters, and where it is desired to maximise the objectiveness of the procedures used.

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