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Polarization analysis on three-component seismic data

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Abstract - The acquisition of three-component high-density seismic data in WARR surveys, aimed at the two or three-dimensional modelling of buried structures, makes it possible to apply statistical-analysis techniques to estimate the propagation parameters of waves. The availability of the three components allows the use of the information about the state of the seismic wave polarization in order to filter the signal, reducing uncorrelated noise and separating kinematically equivalent seismic phases. In order to study polarization properties of seismic waves in WARR data, a correction of the polarization angle estimate (obtained by the SVD of the covariance matrix of three-component data) was performed. Therefore, it is possible to remove the bias produced by the superposition of a non-polarized component of the wavefield to a possible linearly polarized one, and reduce the estimate variance. An interesting target in seismic profile analysis consists in the simultaneous determination of kinematic and polarization properties of the signal. This procedure is addressed to the estimation of two fundamental attributes of seismic waves: apparent slowness and polarization angle. This method provides good results only when applied to data with a high S/N ratio. Therefore, it should be improved to extend it to low S/N data.

1. Introduction

Traditional techniques in polarization property analysis are usually based on the eigenanalysis of the covariance matrix of three-component signals, generally one vertical and two horizontal. In this approach, the estimated eigenvalues, calculated in proper time windows, are used to calculate a gain function, linked to the rectilinearity degree of the wavefield (Flinn, 1965; Montalbetti and Kanasewich, 1970; Vidale, 1986; Jackson et al., 1991; de Franco et al., 2001). Actually, the proposed gain function turns out valid if one of the estimated eigenvalues is much larger than the others. This condition is met when the wavefield polarization is linear and

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the S/N ratio is high inside the space-time analysis windows.

A feature of the seismic signal which may prove quite helpful in the de-noising stage is its larger coherence than noise both in terms of intensity and space extension. Such a coherence affects, in a different measure, all kinematic and dynamic parameters of the wavefield, including polarization properties. Therefore, it seems useful to design stacking and coherence filtering techniques for three-component data, using the information on the wave polarization state. Some techniques aimed at the de-noising of the three components have already been proposed by Jurkevics (1988) and Kennet (2000). Such techniques generally provide good results when applied to seismological data in order to automatically detect the first arrivals of P- and S-phases. On the contrary, the tests conducted on WARR profiles have led to unsatisfactory results, because of the low signal-to-noise ratio S/N that often characterizes them as well as the complexity originating from the temporal proximity of the numerous seismic phases which are to be interpreted, in the typical offset intervals of crustal seismic surveys.

A linear-polarization angle estimator is proposed here which considers the average noise properties, in order to correct the estimates and decrease their variance.

2. The polarization angle estimator

If the investigation direction is perpendicular to 2D structures, it is possible to analyse, at the same time, the vertical and the longitudinal components separately from the transversal one.

In the theoretical case of noise-free data, the two-component covariance matrix J will depend only on the signal S:

$$J = \begin{bmatrix} S_{xx} & S_{xy} \\ S_{xy} & S_{yy} \end{bmatrix}, \tag{1}$$

where y and x stand for vertical and longitudinal components, respectively. Thus, the solutions of the characteristic equation of the eigenvalue problem $\lambda^2 - (S_{xx} + S_{yy})\,\lambda + S_{xx}\,S_{yy} - S_{xy}^2 = 0$ are

$$\begin{aligned} \lambda_1 &= S_{xx} + S_{yy}\,, \\ \lambda_2 &= 0. \end{aligned} \tag{2}$$

In the analysis of noisy data, the two-component covariance matrix is also related to noise and the solutions to the characteristic equation are

$$\begin{aligned} \lambda_{1N} &= S_{xx} + S_{yy} + N, \\ \lambda_{2N} &= N, \end{aligned} \tag{3}$$

where N represents the noise perturbation.

The mutual dependence between the eigenvalues relative to a noise-free signal and those relative to a signal polluted with noise is evident by considering their sum and their product

$$S_N = \lambda_{1N} + \lambda_{2N} = \lambda_1 + \lambda_2 + 2N,$$
$$P_N = \lambda_{1N}\,\lambda_{2N} = \lambda_1\,\lambda_2 + N\,(S_{xx} + S_{yy} + N) + N_{xy}^2 - 2S_{xy}\,N_{xy}. \quad (4)$$

Consequently, the corrected estimates of the polarization angle can be obtained by

$$\hat{\lambda} = \frac{S_N - K_1}{2} \pm \frac{\sqrt{(S_N - K_1)^2 - 4\,(P_N - K_2)}}{2}\,, \quad (5)$$

where

$$K_1 = 2N,$$
$$K_2 = N\,(S_{xx} + S_{yy} + N) + N_{xy}^2 - 2S_{xy}\,N_{xy}. \quad (6)$$

An average value for the parameter K_1 can be estimated within areas of the seismic section where no phase arrivals are definitely present. Whilst the parameter K_2 depends both on noise and signal energy for any time window, its average value does not represent a significant estimate of the parameter which should be used for each window position.

If the scope of the analysis is locating and characterizing linearly polarized phases on a seismic section, a possible strategy consists in forcing the assumption of linear polarization, by imposing that the smallest between the corrected eigenvalue estimates be equal to zero. Under this assumption, $N = N_{xx} + N_{yy}$ and $K_2 = P_N$.

In this case the estimated polarization angle is

$$\vartheta = \arctan \frac{(T_{xx} + T_{yy}) + \sqrt{(T_{xx} + T_{yy})^2 - 4\,(T_{xy} - N_{xy})}}{2\,(T_{xy} - N_{xy})}. \quad (7)$$

If a linearly polarized phase is present in the window, the estimates of its polarization vector will be less affected by noise and will prove quite stable along its correlation band. If noise predominates in a certain area of the section, polarization angle estimates will turn out rapidly variable, both along time and offset axes. Statistical algorithms capable of separating the coherent from the incoherent component of polarization vectors can be thus designed. In Fig. 1 the vectors weighted by their variances are shown.

The comparison between the results achieved by traditional algorithms and the corrected ones, applied to synthetic WARR sections with different *S/N* ratios, highlighted the usefulness of such corrections performed on the estimators.

The application of the corrected algorithm to WARR sections, recorded in the frame of the CROP MARE II Project, proved quite useful to define the *P*- or *S*-nature of delayed phases. In Fig. 2 an example of its application to the M27 profile is shown.

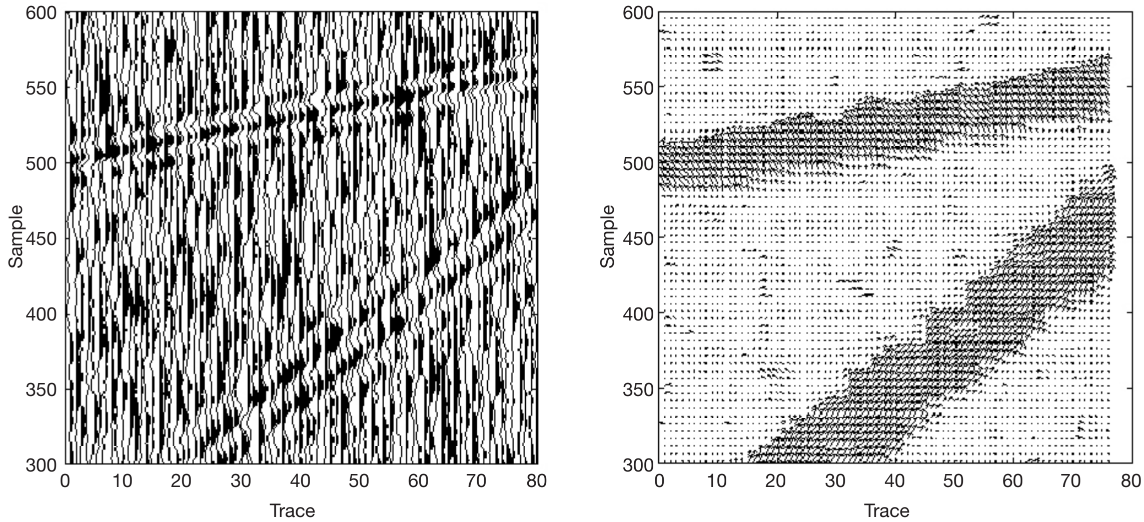

Fig. 1 - Polarization vectors weighted by their variance in a synthetic section.

3. Slowness and polarization

An important application of the study of the polarization properties of a seismic wavefield consists in the separation of the latter into components characterized by a constant apparent velocity and polarization direction within a small offset interval. This procedure represents a de-noising technique. The separation can be obtained with the simultaneous analysis of dynamic and polarization properties of three-component signals (Cho and Spencer, 1992; Richwalski et al., 2000, 2001).

The procedure is addressed to the determination of two fundamental attributes of seismic waves: apparent slowness and polarization angle. In the plane-wave assumption, these

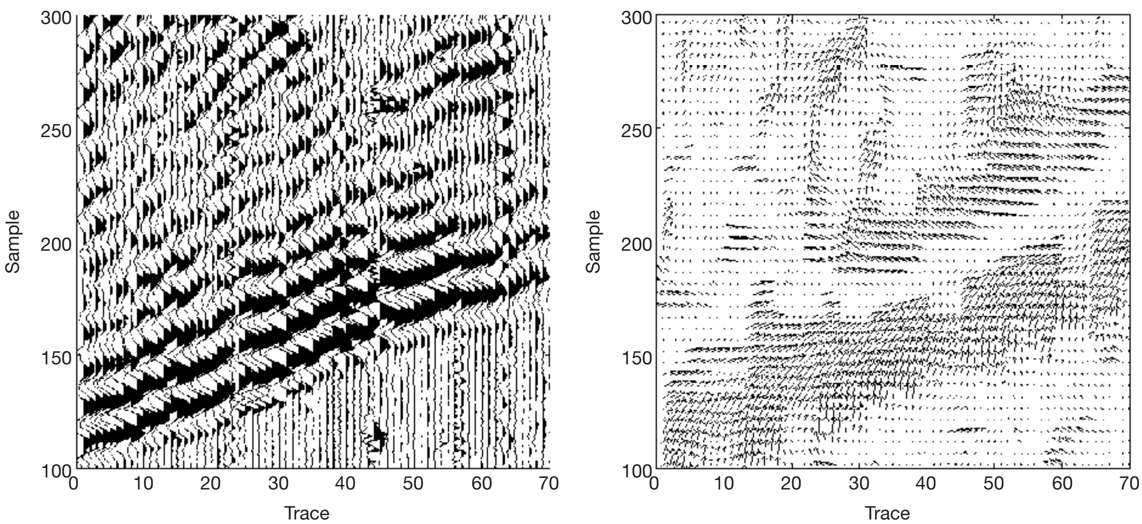

Fig. 2 - Application of the polarization angle estimator to the M27 profile (CROP MARE II Project).

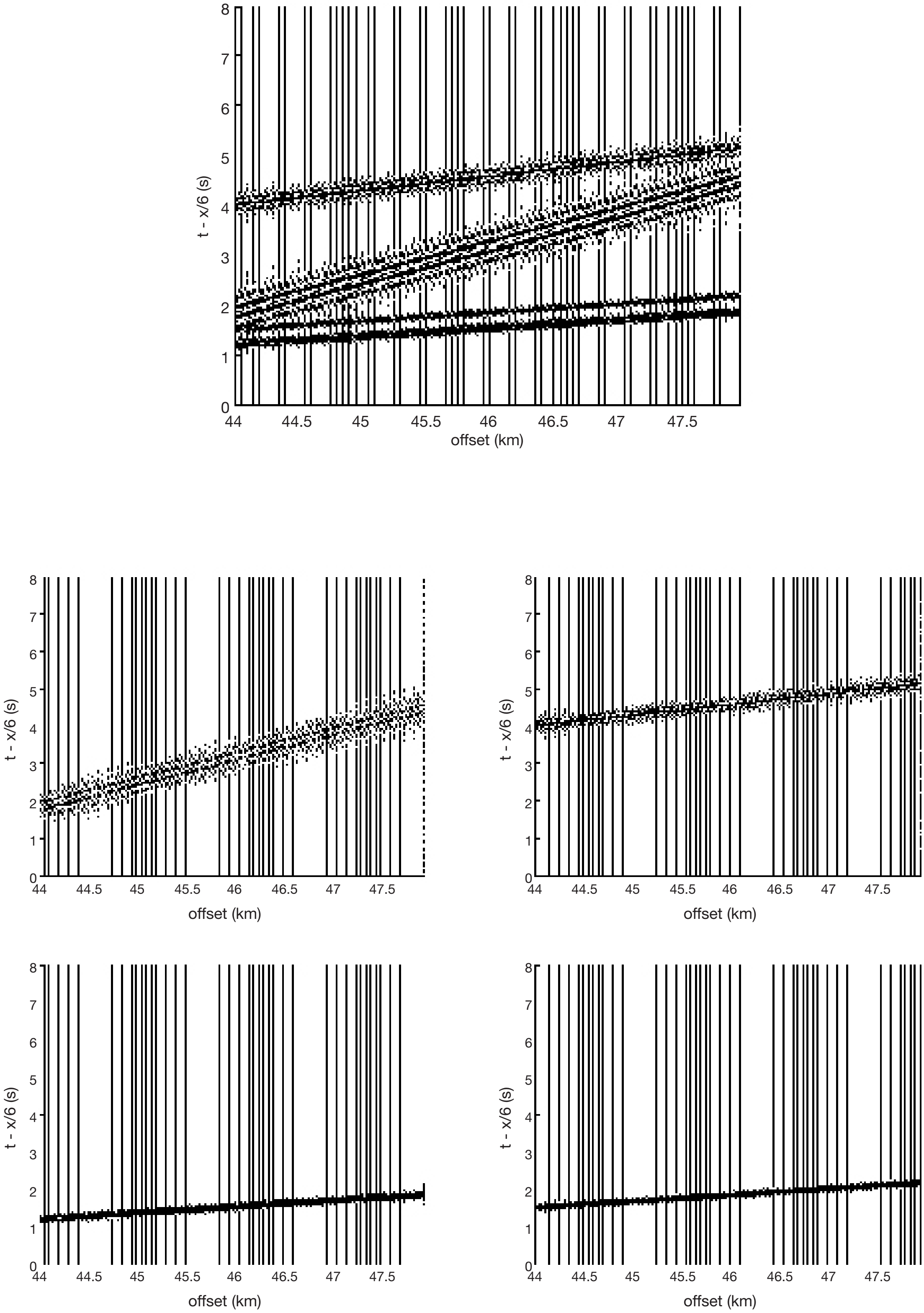

Fig. 3 - Waveform reconstruction by using the separation filter (Richwalski et al., 2000, 2001) relative to the synthetic section in the uppermost panel.

parameters can be found by the eigen-analysis of the transfer matrix *Y*, defined as the linear operator that, applied to the *n-th* trace Fourier transform, returns that of the $n + 1$-*th* trace,

$$U^{n+1} = YU^{n}. \tag{8}$$

In the presence of random noise, the algorithm reliability is obtained by the mean of the transfer matrix in the frequency domain. The so-estimated parameters can thus be used for the construction of a propagation matrix *P* (Richwalski et al., 2000, 2001), which allows to reconstruct the signal produced by a certain number of phases **w** among those detected within the analysis window, starting from the traces spectra **u**, as

$$\mathbf{u} = P\mathbf{w}. \tag{9}$$

By the generalised inverse in the least-squares sense, the operator *P* is capable of reconstructing the spectra of the single wavelets for each window position,

$$\mathbf{w} = (P^{*}\, P)^{-1}\, P^{*}\, \mathbf{u}. \tag{10}$$

The original arrival times are reconstructed using the central trace of the window analysis. In this way, a simple projection and no phase shift are applied to the data. The algorithm requires only the estimate of the number of phases present in the analysis window, but the synthetic tests showed that also this technique provides excellent results as long as the *S*/*N* ratio is high (Fig. 3), but turns quickly unstable as this ratio begins to decrease.

4. Conclusions

The analysis of polarization properties of seismic waves turns out useful in the processing stage, because the information about the polarization mode of the wavefield is used to implement gain functions which will be applied to the data. While in the interpretation stage, this information facilitates the determination of the physical nature of the highlighted phases, which is crucial for the design of the velocity model as well as of the discontinuities.

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