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Shallow water tidal determination from altimetry - the M_4 constituent

O. B. ANDERSEN

National Survey and Cadastre, Geodesy, Copenhagen, Denmark

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Abstract. With the long duration of accurate satellite altimetric measurements from the TOPEX/POSEIDON and the ERS satellites (ERS-1 and ERS-2) it is interesting to investigate if satellite altimetry can provide more accurate tidal models on continental shelves. The call for increasing accuracy in shelf regions, calls for inclusion of more than just semi-diurnal and diurnal constituents in future global ocean tide models. This is because a considerable part of the tidal variability on the shelves is caused by shallow water constituents. One example is the M_4 constituent, which exceeds 50 centimetres at several locations on the northwest European shelf. So far satellite altimetry has not been considered for resolving these constituents, due to its accuracy and coarse ground-track resolution. Reliable empirical estimates of the major shallow water tide constituent called M_4, can be obtained from TOPEX/POSEIDON, by combining along-track and crossover observations. Estimates are presented in this study and compared with tide gauges and existing hydrodynamic models.

1. Introduction

High-accuracy observations from the TOPEX/POSEIDON (herein after referred to as T/P) and to a lesser extent the ERS satellites have vastly improved our knowledge about most diurnal and semidiurnal ocean tide constituents (Andersen et al., 1995). In the deep ocean present global ocean tide models from T/P altimetry agree within 2-3 cm (Shum et al., 1997; Smith and Andersen, 1998).

On the continental shelves major differences are still found between these global ocean tide models, and different models seem to perform better in certain regions. It is considerably more difficult to model tides in shallow water, because the tidal range is usually larger, and the tidal waves are much more complex in coastal regions. The spatial pattern of the tidal waves is scaled

Corresponding author: O.B. Andersen; Kortog Matrikelstyrelsen, Geodesy, DK 2400, Copenhagen NV, Denmark; phone: +45 3587 5255; fax: +45 3587 5052; e-mail: oa@kms.dk

down over the shelves, first of all, because the tidal wavelength is proportional to the square root of the depth. Similarly, resonance or near resonance responses add to the complexity of the tidal pattern and produce some of the world's largest tidal amplitudes like in the Irish Sea, the bay of Mt. St. Michel, and in the Gulf of Maine / Bay of Fundy.

This investigation is limited to investigating the possibility of determining shallow-water tidal constituents on the northwest European shelf only. Only the estimation of the major shallow water tidal constituent - the M_4 is investigated here. The northwest European shelf region was preferred to other shelves due because it has very a profound M_4 shallow water constituent that is larger than the major diurnal constituents (K_1 and O_1) in most places. The northwest European shelf is located between 48° and 61° latitude, and 12° W and 12° E longitude.

The largest shallow-water constituent in the northwest European shelf region is by far the M_4 constituent. This constituent should be resolvable from altimetry, as the averaged amplitude exceeds 8 cm. The amplitude actually exceeds 50 centimetres in some local areas like the bay of Mt St. Michel on the French coast in the English Channel, and in the Severn Estuary on the British coast.

Improvement of the technique used for global ocean tide modelling in the deep ocean will be described. Most recent global ocean tide models are based on sub-sampled altimetric observations or observations being averaged over boxes (i.e. 3° latitude by 3° longitude) which is adequate for ocean tide modelling in the deep ocean. However, shallow-water constituents have such short and varying wavelengths, that this approach will not provide adequate resolution. An approach combining tidal estimates obtained in along-track normal points with tidal estimates obtained at crossover locations was used instead. Finally, the altimetric derived M_4 constituent is compared with a high-resolution hydrodynamic shelf-tide model used for operational storm surge warning.

2. Amplitudes and alias periods of shallow-water constituents

Tidal variations in sea level can be described using amplitude and phase (A_i , g_i) or sine and cosine coefficients (U_i, V_i) like

$$\sum_{i=1}^{i=n} A_i \cos(\sigma_i t - g_i) = \sum_{i=1}^{i=n} U_i \cos(\sigma_i t) + V_i \sin(\sigma_i t), \tag{1}$$

where g_i is the phase lag of the tide at Greenwich, and F_i is the angular speed of the i'th constituent (Knudsen, 1993; Pugh, 1987). Here a summation over the n largest astronomical tidal constituents has been performed. In the deep ocean the dynamics are linear, and the tide is adequately described using a number of diurnal and semidiurnal constituents.

In shallow water the dynamics become nonlinear, and the tidal spectra appears more complicated. The nonlinear distortions causes compound and overtides to appear within the diurnal, semidiurnal, quarter-diurnal and even higher constituent bands. Compound and over-tides are normally called shallow-water tides, as they are caused by the nonlinear distortions of the major

Table 1 - Major quarter diurnal shallow-water constituents on the northwesten European Shelf.

Constituents	Angular speed (origin)	Angular (°/day)	Alias period T/P (days)	Alias period ERS (days)	Amplitude of Flather model (cm) (1)
MN_4	M_2+N_2	1378.17	244.40	3193.71	2.47
M_4	M_2+M_2	1391.23	31.05	135.06 (3)	8.19
MS_4	M_2+S_2	1415.61	1088.60 (4)	94.48 (2)	5.17
MK_4	M_2+K_2	1417.59	219.76	195.67	1.44
S_4	S_2+S_2	1440.00	29.37	∞(5)	0.55

(1) The average amplitude on the shelf is computed for depths of less than 200 metres from the shelf model by Flather.
(2) same as the ERS alias period for M_2 (95 days),
(3) inseparable from ERS alias period for Q_1 (133 days),
(4) inseparably from inter-annual variation in sea level at three years period (1106 days),
(5) S_2 is frozen in the orbit for the sun-synchronous ERS satellites. Consequently, all shallow water constituents generated from S_2 alone will be frozen in the orbits as well (S_4, S_6 etc.)

astronomical tidal constituents (i.e. M_2, S_2, K_1) in shallow water. The non-linearities can conveniently be expressed as simple harmonic constituents with angular speed being multiples, sums or differences of the frequencies of the well known astronomical constituents (i.e. M_2 and S_2).

A list of the largest quarter or fourth diurnal shallow water constituents, their angular speeds, alias periods and amplitudes are shown in Table 1. Amplitudes were calculated by averaging the amplitudes of the 12-km Flather NW European Shelf model (48° < N < 62°, -12°< 8 < 13°) for depths less than 200 metres. The Flather NW European shelf model is a 12-km (0.11° latitude by 0.16° longitude) model covering latitudes within 48° N and 62° N, and longitudes within 12° W and 13° E. The model is used operationally for coastal flood forecasting in the United Kingdom (UK) and is thus optimized to predict water levels on UK coasts. It contains most diurnal and semidiurnal constituents, and has 26 shallow-water constituents. Open boundary forcing is based on the (Flather, 1981) NEA model results and tide gauges around the shelf. The shallow-water constituents in the model are generated from the major constituents, as the model is fully nonlinear (Flather, 1976). As this is the only purely hydrodynamic tide model covering the whole of the northwestern European Shelf this model will in the following be used for comparison with the T/P empirical model.

The first satellite to consider for resolving shallow-water tides would naturally be the two ERS European space agency satellites. Observations from more than six years are now available and the spatial sampling is roughly 3.5 times that of the joint NASA/CNES TOPEX/POSEIDON satellite. Due to the sampling interval of the satellites being much longer than the tidal period, the tidal constituents are observed as having much longer periods (i.e. Andersen (1994, 1995)). The sampling period is a function of the orbit characteristics, and as seen from Table 1 the alias periods are problematic because of its sun-syncronous orbit. Consequently, observations from the ERS satellites cannot be used in this investigation.

Observations from the T/P satellite will be used instead. Most important shallow-water constituents are well separated from each other as well as from the major astronomical semidiurnal and diurnal constituents from T/P altimetry. Especially the M_4 constituent to be considered in this investigation has an alias period of 31 days which is neatly separated from the alias periods of the major four diurnal and semidiurnal constituents (62, 59, 50, 87, 173, 46, 89, 69 days for M_2, S_2, N_2, K_2, K_1, O_1, P_1 and Q_1), respectively (Andersen and Knudsen, 1997).

3. Shallow-water tidal prediction

The altimetric observations were analysed using a combination of the “response method” (Munk and Cartwright, 1966) for remaining semidiurnal and diurnal constituents, supplemented by harmonic analysis methods for the M_4 constituent. The reason for considering both residual diurnal and semidiurnal constituents along with the M_4 constituent is because the sea-level variations still have large contributions from imperfections in the applied ocean tide model. This results in the following formalism relating the observational sea-level heights $h(t)$ to the unknown parameters:

$$h(t) = \sum_{m=1}^{2} \sum_{k=-K}^{K} \left[u_k a^m (t - \Delta k) + v_k b^m (t - \Delta k) \right] + U_{M4} \cos(\sigma_{M4} t) + V_{M4} \sin(\sigma_{M4} t) \qquad (2)$$

Here, the $a^m(t\text{-}\Delta k)$ and $b^m(t\text{-}\Delta k)$ are the real and imaginary components of the time varying part of the tide-generating potential associated with each band and lagged by Δk days. The unknown coefficients sought are the u_k and v_k for the diurnal and semidiurnal constituents, and the U_{M4} and V_{M4} coefficients for each shallow-water constituents of frequence FM_4. Fixing the expansion of the response formalism at K=1 gives an adequate six-parameter description of the admittance curves in both the diurnal and the semidiurnal bands, respectively. It adds up to a total of 16 parameters to be estimated. Details on the response method related to altimetry, the parameter selection and the conversion from response weights to amplitudes and phases for the individual constituents can be found in (Andersen, 1994; 1995) and (Cartwright and Ray, 1991).

The use of the response formalism for the diurnal and semidiurnal band was chosen because the applied FES94.1 ocean-tide correction only removes the major 13 constituents in these tidal bands (Le Provost et al., 1994). Consequently, several small diurnal and semidiurnal constituents of astronomical origin remain, and these can conveniently be estimated using the response formalism.

4. Interpolation of shallow-water tidal estimates

From the estimation procedure above, a set of 16 tidal parameters were derived in each along-track normal point and at each crossover location. In Fig. 1 the location of the tidal estimates obtained at each along-track normal points (hereinafter called along-track estimates) are shown with a circle. The location of estimates obtained at crossover locations (hereinafter called crossover estimates) are marked with a plus sign. The along-track and crossover estimates must be treated differently. First, because the crossover estimates are based on twice as many observations, and secondly because non-tidal variability influence will have a different influence on the different type of estimates.

To interpret and map the tidal solutions, the tidal estimates must be interpolated onto a regular grid. For all grids a resolution of 0.5° latitude by 0.5° longitude was used. Below a refined method of merging the along-track estimates with the crossover estimates in the interpolation procedure is described. Collocation or krieging is normally used for the interpolation. As covariance function, a second-order isotropic Markov covariance function like

$$c(r) = C_0(1 + \frac{r}{\alpha})e^{(-r/\alpha)}, \tag{3}$$

is frequently used where r is the spatial distance, C_0 is the error variance, and α is the correlation length (where a 50% correlation is obtained). An estimate of the error variance on the estimated parameters is output from the estimating procedure (Eq. 2), when using standard least square technique (i.e. Jackson, 1979). The error variance depends on the number of observations used, as all T/P observations were assigned the same accuracy of 2 cm. Tidal parameters or estimates, obtained at crossover locations, will generally have lower error variances than estimates obtained in the along-track normal points, as twice as many observations will be available at the crossover locations.

Due to the high spatial variation of tides in shelf regions, it is important to use a combination of crossover and along-track tidal estimates. The crossover data gives the most accurate tidal estimates. However, the along-track data adds very valuable information on the spatial variation of the tides. Furthermore, including along track-data enhances the spatial coverage of the model, as empirical ocean tide models are only valid where observations are available. This is seen for example, in Fig. 1 where the inclusion of along-track observations enables tidal modelling in the southernmost North Sea between The Netherlands and Great Britain, where no crossovers could be computed.

Tidal solutions obtained on an ascending and descending track seldom match due to non-tidal ocean variability propagating into the tidal solution. A closer investigation showed that this mismatch or error has a long wavelength character related to the individual tracks. One explanation may be that ocean variability maps differently into the tidal solutions due to the limited number of altimetric observations. Therefore an additional covariance function for this error can be introduced in the interpolation like

$$c(r) = C_0(1 + \frac{r}{\alpha})e^{(-r/\alpha)} + D_0(1 + \frac{r}{\beta})e^{(-r/\beta)}, \tag{4}$$

where D_0 and β are the signal variance and correlation length for this non-tidal error. The values

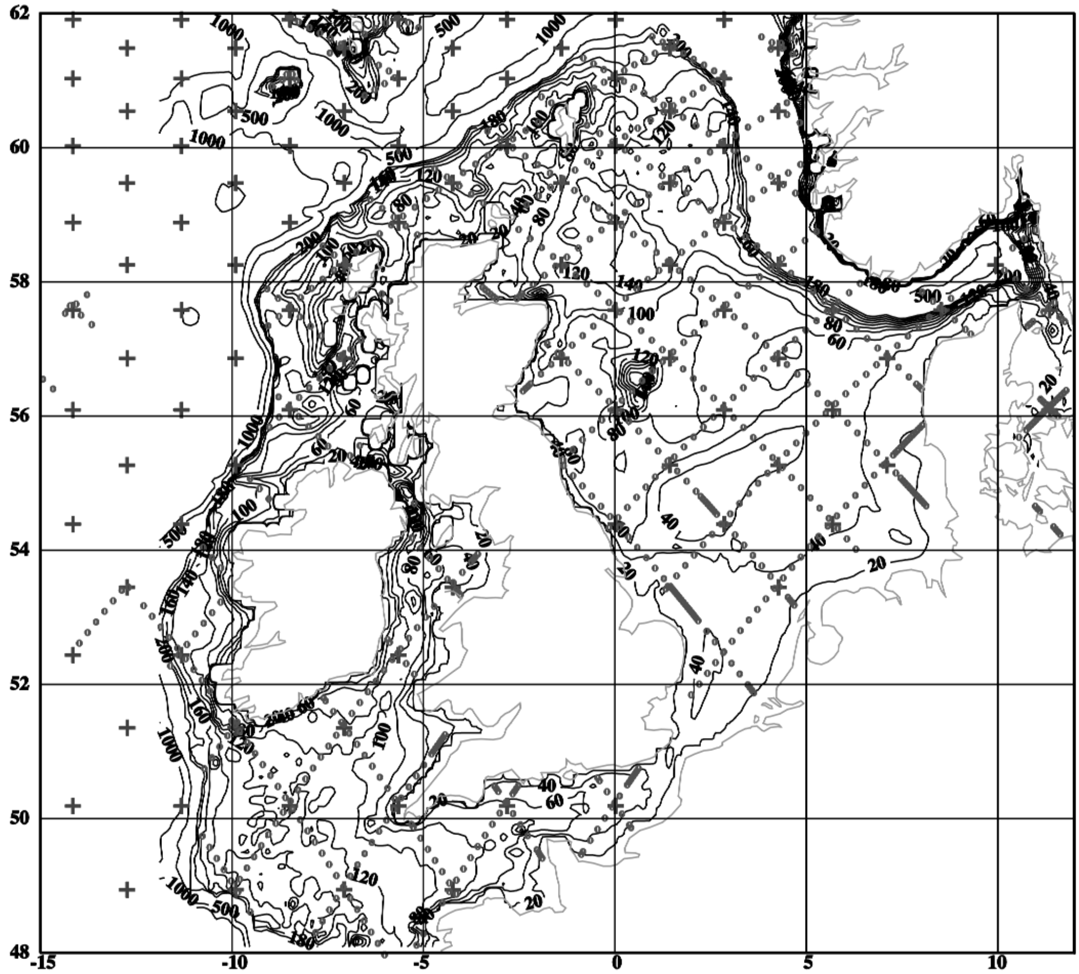

Fig. 1 - Bathymetry and location of TOPEX/POSEIDON data on the northwest European shelf. Crossover locations are marked with a cross and along track data are marked using circles. Only selected bathymetry contours are shown.

for D_0 and β were fixed at 5 cm^2, and at 2000 km, respectively. Details on this covariance function modification can be found in (Andersen and Knudsen, 1998). The additional error variance function was only applied to tidal estimates on the same track. For estimates on different tracks and at crossover locations this error is assumed to be uncorrelated and D_0 was fixed at zero.

5. Satellite altimetry

The ocean altimeter Pathfinder product for the T/P exact repeat mission (ver. 2 containing 154 repeats) was used for this study. A full description of this data set can be found through the NASA Pathfinder homepage at the following Internet address: "http://neptune.gsfc.nasa.gov/ocean.html".

The altimetric observations were processed using the set of standard corrections provided.

These were: geophysical corrections (solid earth tide, load tide, pole tide, cross track geoid correction and a 100% inverse barometer correction); media corrections (ionosphere, wet and dry troposphere); and instrumental corrections (sea state bias and instrumental corrections). The orbits were based on the JGM-3 geopotential model (Tapley et al., 1996). A special data set without the ocean tide correction applied was provided for this study (R. D. Ray, personal communication). Subsequently, ocean tides were removed using the FES94.1 pure hydrodynamic ocean tide model (Le Provost et al., 1994). The FES94.1 ocean tide model was preferred to the newer ocean tide models like FES95.1, CSR3.0, OSU-TPXO3 and AG95.1 (Shum et al., 1997; Egbert, 1997), because all these recent models have already been fitted to T/P observations once. The NASA Pathfinder products are delivered as normal point data, containing stacked along-track observations. The locations of crossover positions were then calculated and the observations were interpolated onto the positions of the crossover locations using an along-track spline interpolation. Besides this crossover data set, another data set consisting of every third along-track normal point (roughly 16 km apart) was used in shallow water for a depth of less than 300 metres. These two data sets, will in the following, be called the crossover data set and the along-track data set. In Fig. 1, the bathymetry of the northwestern European Shelf region is shown along with the location of all T/P crossover and along-track data used in this investigation.

6. The M_4 shallow-water constituent

In Fig. 2(a) the empirical M_4 constituent from T/P data is presented and in Figure 2(b) the M_4 constituent from the Flather hydrodynamic shelf model is shown. Both have similar amplitudes to the southwest of Britain, in the Irish Sea, and along the British Channel. The empirical T/P model captures the same rapid change in amplitude and phase in the English Channel, as found in the high resolution hydrodynamic shelf model, as well as in the models by (Le Provost and Fornerino, 1985) and (Pingree and Maddock, 1978). On the west coast of Britain, both models agree on the phase propagation towards the coast perpendicular to the strike direction of the shelf, as they have amplitudes increasing towards the coast, a characteristic of shallow water constituents (i.e. Pugh, 1987).

In the English Channel both models agree on the location of two tidal amphidromes, showing how the phase of M_4 propagates from the centre of the Channel to the west and east, as also described by (Le Provost and Fornerino, 1985). The models disagree in amplitude close to the south coast of Great Britain and especially in the southern part of the North Sea. Here, the T/P model has amplitudes of around 4 cm (54EN, 4EE), whereas the Flather shelf model has very

Table 2 - RMS comparison with 168 coastal and pelagic tide gauges in the northwesten European Shelf region.

Model	M_4 (cm)
T/P model (0.5° by 0.5°)	3.92
Flather shelf model (0.11° by 0.16°)	4.29

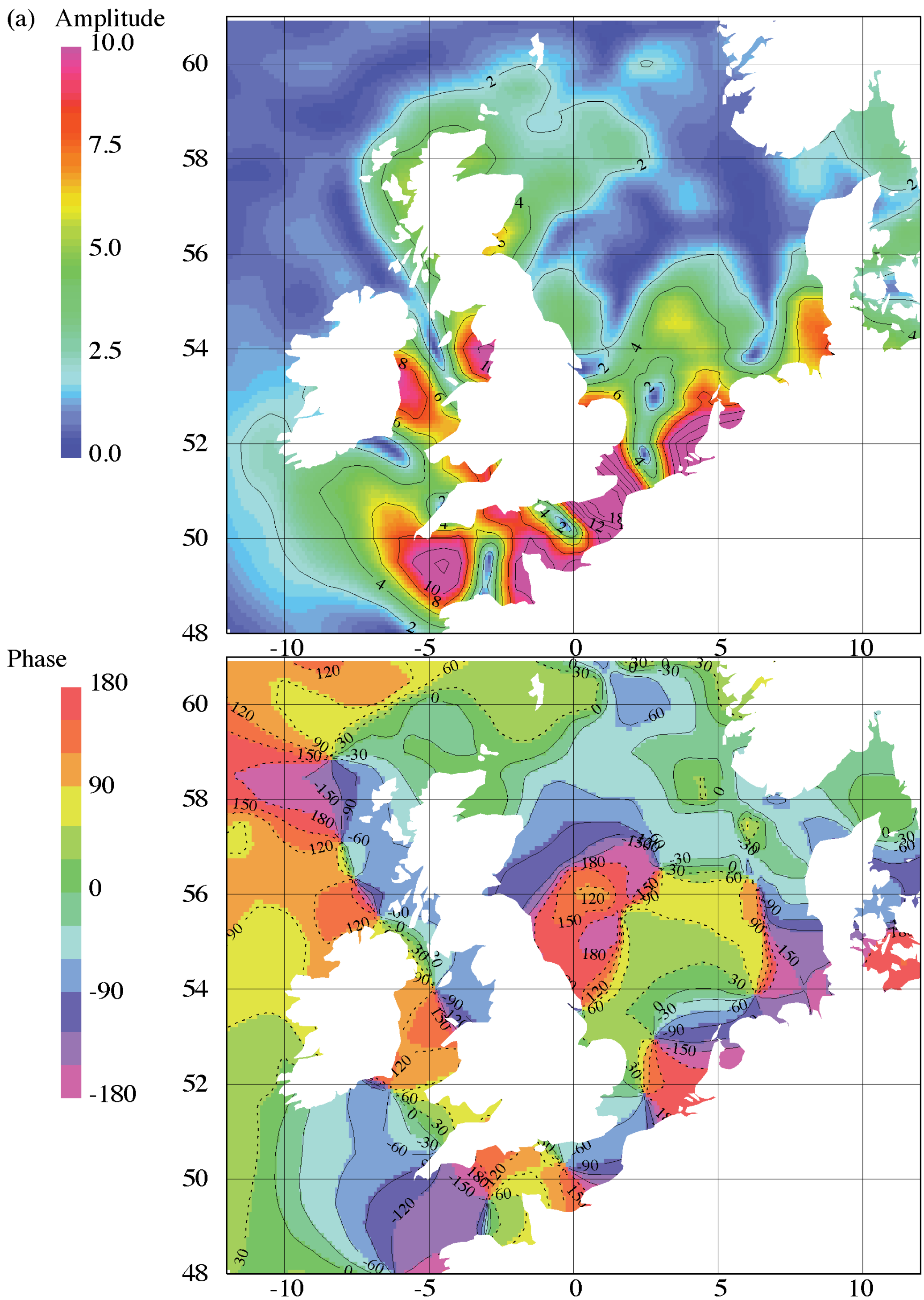

Fig. 2 - The M_4 constituent in the northwest European shelf region from T/P data (a) and from the Flather hydrodynamic shelf model (b). Amplitudes are in centimetres, and phases are in degrees with respect to Greenwich. The contour intervals are 1 centimetre and 30°, respectively.

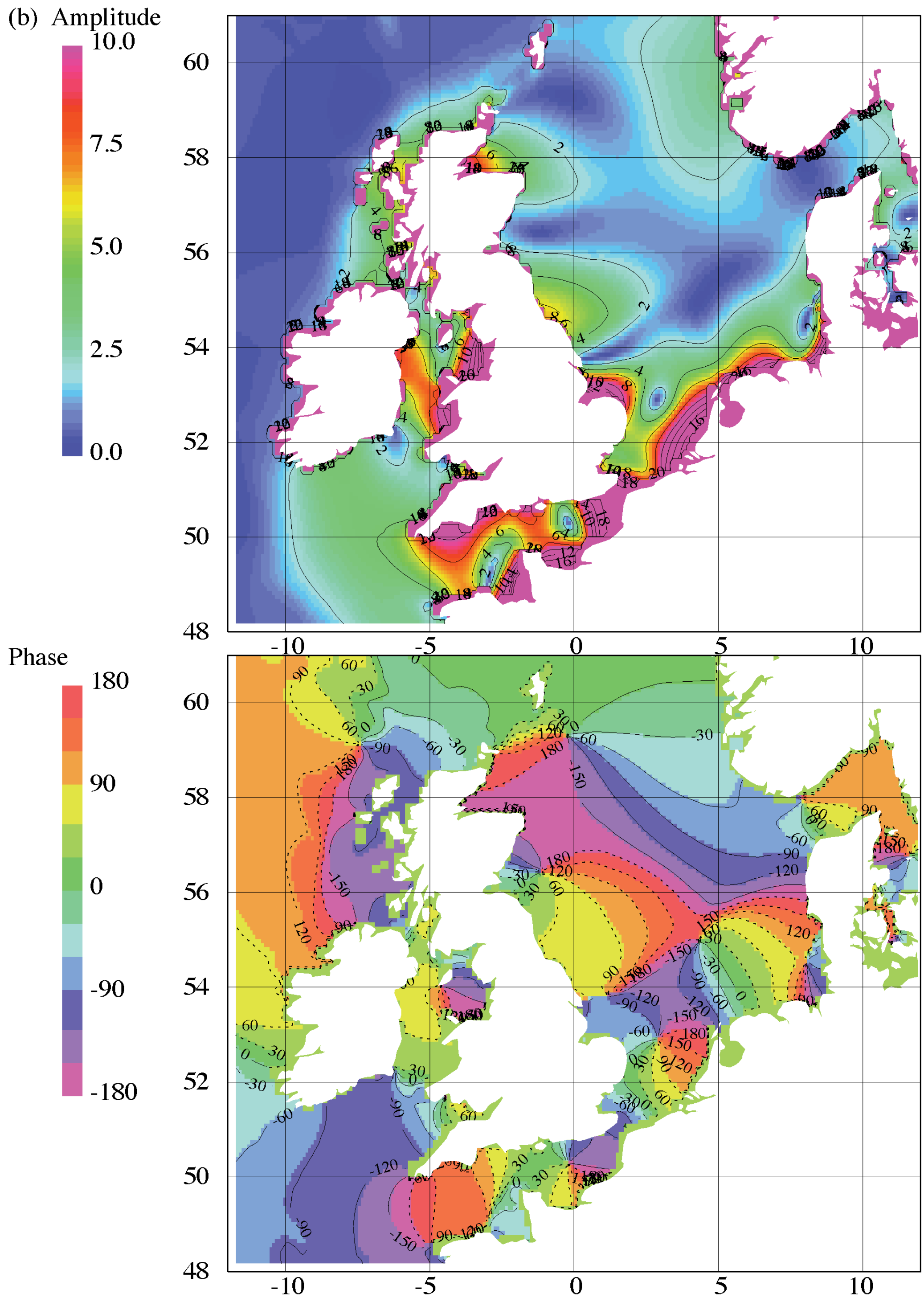

Fig. 2 - continued.

small amplitudes of around 1 cm.

The Root Mean Square (RMS) comparison with 168 tide gauges on the northwestern European Shelf is presented in Table 2 as RMS vector differences taking into account both amplitude and phase discrepancies (Andersen et al., 1995). The empirically T/P derived model for M_4 compares significantly better with tide gauges than the Flather hydrodynamic shelf model. However, this difference can largely be subscribed to the much better fit to pelagic tide gauges in the southern North Sea for the T/P derived model. The comparison with a pelagic tide gauge subset in the southern North Sea reveals, that the empirical T/P model has the same amplitude as the tide gauges. Apparently, the hydrodynamic model misses this large region of 4 cm amplitudes.

7. Conclusions

The largest shallow-water constituent on the northwestern European Shelf, the fourth diurnal M_4, was derived accurately from altimetry. A comparison with 168 tide gauges showed that it compares better with pelagic tide gauges than the high resolution hydrodynamic shelf model by Flather (3.92 versus 4.29 cm RMS comparison). This study was confined to the northwestern European Shelf, which has very profound shallow-water tides, comparable with the astronomical diurnal tides, and many tide gauge observations of shallow water constituents. Shallow-water tides depend on the local tidal regime combined with the local bathymetry, and no shelves have similar shallow-water tides. Generalisation by referring to other shelves is therefore not straightforward, and each shelf must be investigated separately. In the future, further development within shallow-water tide modelling from altimetry can be expected with longer time series and the forthcoming launch of the T/P follow-on satellite called Jason-1.

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