Bollettino di Geofisica Teorica ed Applicata Vol. 61, n. 2, pp. 131-144; June 2020
DOI 10.4430/bgta0297

# Seismic data random noise attenuation using LLSP smoothing

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(Received: 14 January 2019; accepted: 2 September 2019)

***ABSTRACT*** Interpreting and studying reflection seismic data containing low levels of noise has become easier and more accurate. Random noise decreases the quality of seismic data considerably, and suppressing it is an important step in seismic data processing. In this study, we introduce a method called local least squares polynomial (LLSP) smoothing based on the Savitzky-Golay filter to diminish seismic random noise. This filter is based on smoothing the data fitting a curve in the least squares method which can eliminate random noise significantly. The LLSP smoothing has two main notable advantages. First, simple mathematics governing it which makes it convenient for data processing. Second, its excellent ability to preserve the signal waveform after application. The proposed method is applied on two synthetic models and real seismic data. For a more accurate investigation of the method's efficiency, the results are compared with the techniques: frequency offset deconvolution filter, wavelet transform, and singular value decomposition. The final results in both synthetic and real data show that the proposed method is as powerful as other well-known techniques for random noise attenuation, also the signal information is preserved during filtering.

**Key words:** seismic random noise, LLSP smoothing, FX deconvolution, wavelet transform, SVD.

## 1. Introduction

The reflection seismic method is one of the most common methods used in exploring hydrocarbon reservoirs. Seismic signals that are reflected from different layers of the Earth contain important information, so the high quality of these seismic signals helps extract information of Earth interior (Sheriff and Geldart, 1995). In a reflection seismic method, noise is a significant factor in affecting the signal and hiding important information associated with it. Various noises are observed in seismic data, an important category of which is random noise that is observed as random oscillation at all times and all frequencies. The attenuation of random noise is very important in seismic data processing, and proves very difficult when the signal-to-noise ratio is low. There are several ways to suppress this type of noise, including some relatively new methods described here in brief:

- wavelet and curvelet transforms. In wavelet transform, the coefficient of correlation of a signal with a base wavelet is calculated in different scales and shifts. Then, because the random noise has high frequencies, the coefficients associated with the high frequencies are removed and, then, the inverse wavelet transform is used (Anvari *et al*., 2017). Curvelet transform has a rotation property in addition to the scaling and shifting properties and enables this transform to identify the edges of the events and is suitable for processing signals having singularities (Oliveira *et al*., 2012);

- Empirical Mode Decomposition (EMD) method. In this method, a signal is decomposed into simple oscillatory functions named intrinsic mode functions (IMF). The first IMFs have high oscillations and represent high frequency random noise, so the first IMFs can be removed and the other IMFs accumulated and a random noise-free signal can be obtained (Huang *et al*., 1998);
- Singular Value Decomposition (SVD) method. This method is based on matrix algebra and it is very effective in random noise reduction. In this method, seismic data or traces are considered as a matrix, then this matrix is written as a sum of some matrices that are called eigen images. The first eigen images represent high energies or primary events and the last ones represent low energies or random noise. Therefore, by eliminating the last eigen images, random noise in seismic data can be attenuated (Bekara and Van der Baan, 2007);
- Frequency offset deconvolution filter (FX filter). The basis of this method is that if seismic events are linear in a seismic section, then these events are periodic and predictable in the frequency-offset domain, while the random noise is uncorrelated and unpredictable from one trace to another. So, by designing a filter in frequency-offset domain that can predict the behaviour of data in the next steps, one can obtain the predictable part of data or primary events and delete the random noise. This filter is named Frequency offset deconvolution filter (FX filter) and designed by applying complex Wiener filter theory (Treitel, 1974; Harris and White, 1997);
- median filter. The basis of this method is to smooth data. In this filter, the algorithm used to smooth the data is such that a finite number of data samples are selected, then arranged in ascending order and the median of this data is selected as smoothed value. Then, a sample is moved forwards and this algorithm is repeated for later examples (Bagheri and Riahi, 2016). This filter has been developed in many ways, such as a weighted median filter (Kumar *et al*., 2007) and the Decision Based Medians (DBM) filter that has a very good performance for data with low signal-to-noise ratio (Bagheri *et al*., 2017);
- Savitzky-Golay (S-G) filter. Savitzky and Golay (1964) applied this method to attenuate high frequency noise of chemical spectrum data; it was demonstrated that this filter reduces high frequency noise while maintaining the shape and the height of the signal waveform (Schafer, 2011a, 2011b). In recent years, other researchers have applied this method in various fields, such as: reconstruction of Normalized Difference Vegetation Index (NDVI) time-series data, reconstruction of Moderate-Resolution Imaging (MODIS) - Enhanced Vegetation Index (EVI) time-series data, electrocardiogram (ECG) denoising, remote sensing image merging, and seismic random noise reduction. Liu *et al*. (2016) applied an S-G filter on synthetic and real seismic data for random noise reduction. They demonstrated that the S-G filter performs better than other more renowned methods.

In this paper, the method Local Least Squares Polynomial (LLSP) approximation based on the S-G filter is proposed to diminish seismic random noise. The method is applied to two synthetic models and real seismic data. For an accurate investigation, the results of the filter are compared with the results of the FX filter, wavelet transform and SVD. We used these three methods for comparison because the FX filter is specifically designed for seismic random noise attenuation and gives good results (Liu *et al*., 2012); the wavelet method has been widely used in the geophysical field and has acceptable results (Miao and Moon, 1994) and SVD is a very effective method in the decomposition and processing of data and gives excellent results for random noise attenuation (Bekara and Van der Baan, 2007). The results in both synthetic and real data confirm the power of the LLSP smoothing method for reducing the level of random noise and increasing the signal-to-noise ratio, while the signal is not affected significantly.

## 2. S-G filter

In 1964, the chemist Abraham Savitzky and physicist Marcel Golay proposed a method of data smoothing based on local least squares polynomial approximation. Savitzky and Golay (1964) were interested in attenuating the high frequency noise of data obtained from chemical spectrum analysers, and they demonstrated that least squares smoothing reduces noise while maintaining the shape and height of the waveform (Savitzky and Golay, 1964).

Today, the filters that are designed using this method are known as the S-G filter (Orfanidis, 1996). The S-G filter, acting as a low pass filter, has a structure similar to the Finite Impulse Response (FIR) filter, and the function is controlled by two parameters: the length of the window or the length of the polynomial fitting area that is called approximation interval, and the degree of polynomial (Schafer, 2011a, 2011b). The S-G filter has simple mathematics and brief calculations. In fact, the S-G method has two important features, one of which is the simple mathematics governing it and another is preserving the signal waveform after applying it to the data, both being the strengths of the S-G method.

## 3. Local Least Squares Polynomial (LLSP) smoothing

The basis of this method is that a polynomial is fitted to the odd number of the data points in the least squares procedure; the value of this polynomial is then calculated for the midpoint of this subset, and this value is equal to the smoothed amount of the midpoint (Člupek *et al.*, 2007). Fig. 1 gives a clear explanation of this method.

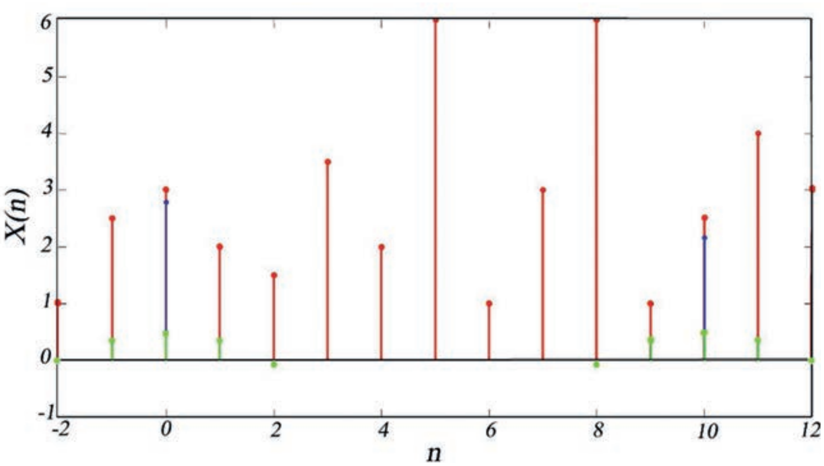


Fig. 1 - Local polynomial fitting by least squares method. Red points are input samples and blue points are the smoothed amount for the middle sample. Green points are the impulse response of the LTI system.

In Fig. 1, samples of a signal X[n] are shown with red points. $2n + 1$ samples centred at $n = 0$ are considered, we now intend to find coefficients of a polynomial $P(n)$ of degree $N$ fitted to this set:

$$P(n) = \sum_{k=0}^{N} a_k n^k \quad (1)$$

The error of this fit in the least squares is equal to:

$$\varepsilon_N = \sum_{n=-M}^{M} [P(n) - x(n)]^2 = \sum_{n=-M}^{M} [\sum_{k=0}^{N} a_k n^k - x(n)]^2 \quad (2)$$

In this equation, $M$ is the half width of the approximation interval. In Fig. 1, a special case is considered for $N = 2$ and $M = 2$. Using this polynomial, we can obtain the smoothed value for the central point at $n = 0$. The output value or the smoothed value at $n = 0$ is equal to: $y(0)$

$$y(0) = P(0) = a_0 \quad (3)$$

The output value for the next sample is obtained by shifting the approximation interval by one sample and repeating this operation, and this will be repeated until the last sample. In each step, different polynomials are obtained. Another example is shown on the right side of Fig. 1. In this case, the centre of the approximation interval is $n = 10$ and the output value is obtained using this polynomial for $n = 10$ (Schafer, 2011a, 2011b).

In general, the approximation interval does not need to be symmetric around the evaluation point. This leads to nonlinear phase filters, which can be useful for smoothing at the ends of finite-length input sequences (Schafer, 2011a, 2011b). Savitzky and Golay (1964) showed that at each position, the smoothed output value obtained by sampling the fitted polynomial is identical to a fixed linear combination of the local set of input samples; i.e. the set of $2M + 1$ input samples within the approximation interval are effectively combined by a fixed set of weighting coefficients that can be computed once for a given polynomial order $N$ and approximation interval of length $2M + 1$. That is, the output samples can be computed by a discrete convolution of the form:

$$y(n) = \sum_{n=-M}^{M} h(m) x(n-m) = \sum_{m=n-M}^{n+M} h(n-m) x(m). \quad (4)$$

In other words, the selection of the approximation interval and the fitting polynomial can be considered as a Linear Time-Invariant (LTI) system, so that the output of this system is equal to the smoothed signal.

In Fig. 1, the values $X(n)$ are the shifted impulse response of the LTI system, $h$ [0 - $m$] and $h$ [10 - $m$]. To show that we can find a single finite duration impulse response that is equivalent to least squares polynomial smoothing for all shifts of the $2M + 1$ - sample interval, we must first determine the optimal coefficients of the polynomial in Eq. 1 by differentiating $\varepsilon_N$ in Eq. 2 with respect to each of the $N + 1$ unknown coefficients and setting the corresponding derivative equal to zero. These yields, for $i = 0, 1 \ldots, N$:

$$\frac{\partial \varepsilon_N}{\partial a_i} = \sum_{n=-M}^{M} 2n^i [\sum_{k=0}^{N} a_k n^k - x(n)] \quad (5)$$

which, by interchanging the order of the summations, becomes the set of $N$+1 equations in $N$+1 unknowns:

$$\sum_{k=0}^{N}\left(\sum_{n=-M}^{M} n^{i+k}\right)a_k = \sum_{n=-M}^{M} n^i x(n) \qquad i = 0,1,\ldots, N \tag{6}$$

The Eqs. 6 are known as the normal equations for the least squares approximation problem. We now write the normal Eqs. 6 in the matrix form. To do this, it is helpful to define a (2$M$ + 1) by ($N$ + 1) matrix A = $\{\alpha_{n,i}\}$ as the matrix with elements:

$$\alpha_{n,i} = n^i, \qquad -M \le n \le M, \; i = 0,1,\ldots,N \tag{7}$$

This matrix is called the design matrix for the polynomial approximation problem (Press *et al.*, 2007). The transpose of A is $A^T$ and the product matrix $B = A^TA$ is an ($N$ + 1) × ($N$ + 1) symmetric matrix with elements:

$$\sum_{n=-M}^{M} \alpha_{i,n}\alpha_{n,k} = \sum_{n=-M}^{M} n^{i+k} = \beta_{i,k}, \qquad k = 0,1,\ldots, N \;, \quad i = 0,1,\ldots, N \tag{8}$$

Furthermore, if we define the vector of input samples as $X = \{x\,[-M], \ldots.. x\,[0], \ldots.. x\,[M]\}^T$ and define $a = [a_0, a_1, \ldots.. a_N]^T$ as the vector of polynomial coefficients, then it follows that the Eqs. 6 can be represented in matrix form as:

$$Ba = A^TAa = A^Tx. \tag{9}$$

Therefore, the solution for the polynomial coefficients can be written as:

$$a = (A^TA)^{-1} A^T x = Hx. \tag{10}$$

Now recall that the output for the group of samples centred on $n = 0$ is $y(0) = a_0$, i.e. we only need to obtain the coefficient $a_0$. Furthermore, it can be seen that we only need the 1st row of the matrix H. Then, by the definition of matrix multiplication, the output will be:

$$y(0) = a_0 = \sum_{m=-M}^{M} h_{0,m}\, x(m). \tag{11}$$

In Eq. 11, $h_{0,m}$ is the elements of the 1st row of the matrix H. Therefore, comparing this equation to the second term of Eq. 4 with $n = 0$, we observe that:

$$h(-m) = h_{0,m} \qquad -M \le m \le M. \tag{12}$$

Note that this equation gives $h\,[-m]$ since, as shown in Eq. 4, the impulse response is flipped with respect to the input in evaluating discrete convolution. Efficient matrix inversion techniques can be employed to compute only this first row rather than the entire matrix H (Press *et al.*, 2007). In Fig. 2, a simple example of the application of the LLSP smoothing filter is shown.

According to Fig. 2, the lower polynomial degree, the higher level of smoothing, or, the narrower passband. But if the polynomial degree is higher, the level of smoothing is lower, but

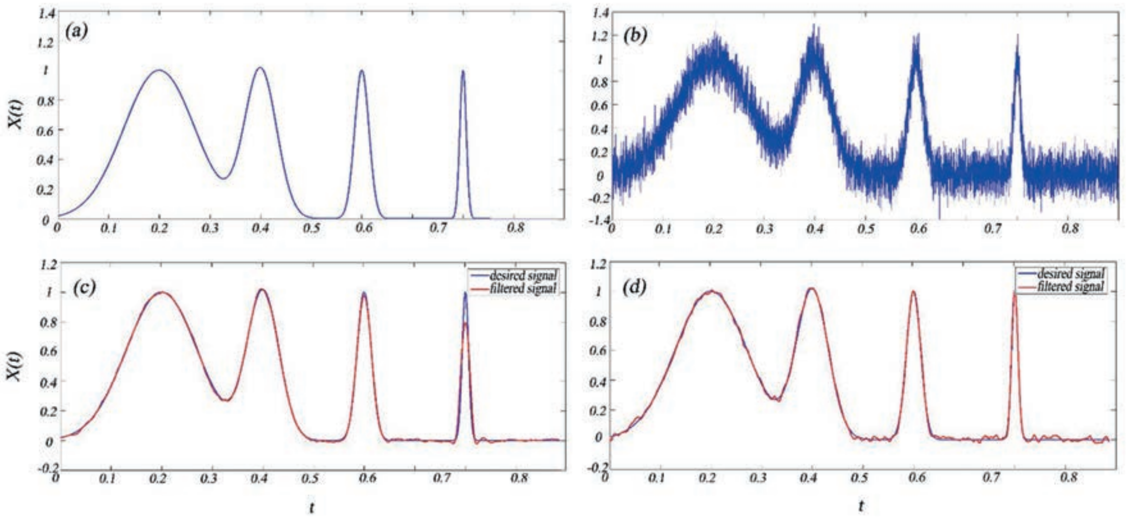


Fig. 2 - Signals: a) desired signal without noise; b) noisy signal; c) smoothed signal for $M = 75$ and $N = 2$; d) smoothed signal for $M = 75$ and $N = 6$.

the waveform is better preserved. Another point is that the level of smoothing is proportional to the value of $M$, that is, the larger value of $M$, the more smoothing.

## 4. Application

In this section, the LLSP smoothing is applied on synthetic and real seismic data. Synthetic models are linear and hyperbolic so that the operation of the method is tested to tackle linear and nonlinear events. Also, synthetic models are designed to be as close as possible to real models. Real data include a Common Mid-Point (CMP) gathered from the Viking Graben area in the North Sea.

### *4.1. Synthetic model 1*

In Fig. 3, a synthetic CMP is shown that includes three reflectors. In this model, the sampling interval is 1 ms and the number of traces is 70. Random noise (white Gaussian noise) with 2dB signal-to-noise ratio is added to this model, then the LLSP smoothing filter for $M = 11$, $N = 2$, FX filter, wavelet method with soft threshold and SVD method are applied to it.

After applying the LLSP smoothing filter on the noisy data, the signal-to-noise ratio is equal to 19 dB, after applying the FX filter, this ratio is equal to 15 dB, after the wavelet method, this ratio is 8 and after the SVD method this ratio is equal 12 dB. This indicates the better performance of the LLSP smoothing filter for random noise reduction. In Fig. 4, the difference between the noisy data and the denoised data shows useful information.

As can be seen in Fig. 4, we find that the difference between noisy data and denoised data by LLSP smoothing filter almost contains random noise, which means that this filter maintains the primary events or useful signals. But in the case of the FX filter, wavelet transform, and SVD method, the waveform is affected considerably after filtering and these methods have lost some of the main signals.

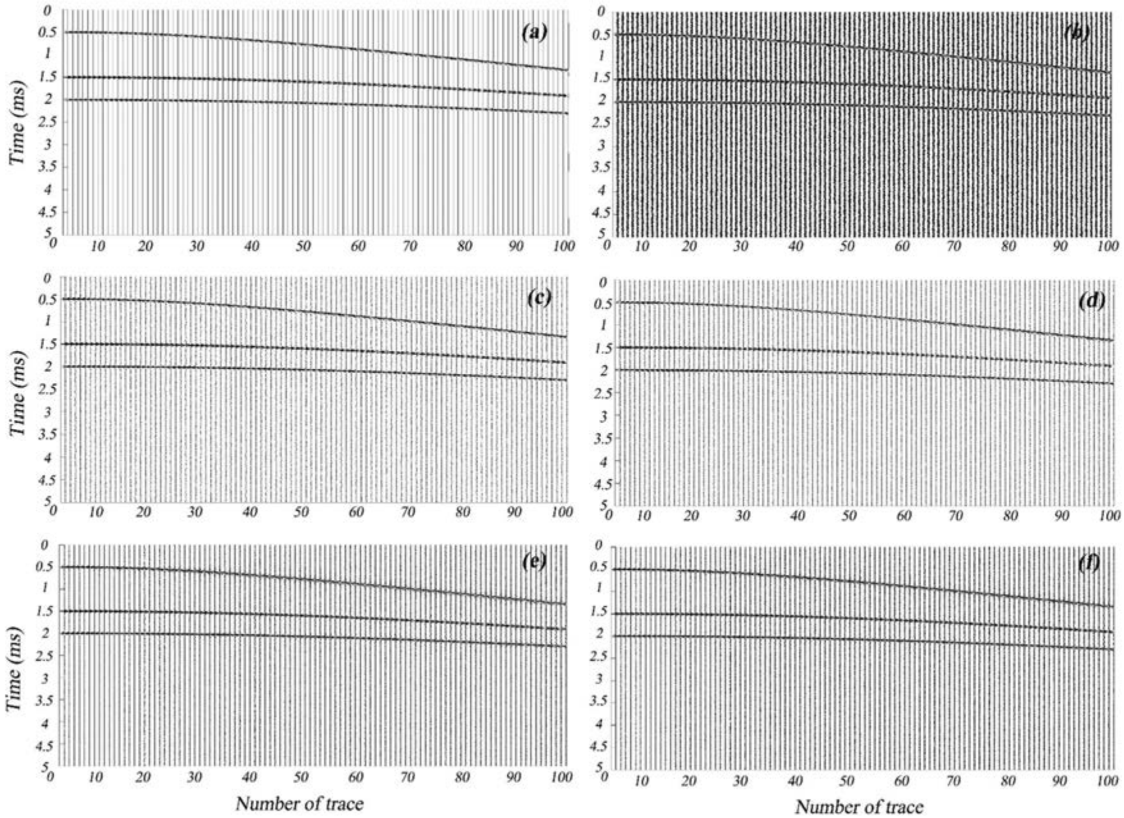


Fig. 3 - Data: a) synthetic seismic data; b) noisy data; c) denoised data with LLSP smoothing filter; d) denoised data with FX filter; e) denoised data with wavelet method; f) denoised data with SVD method.

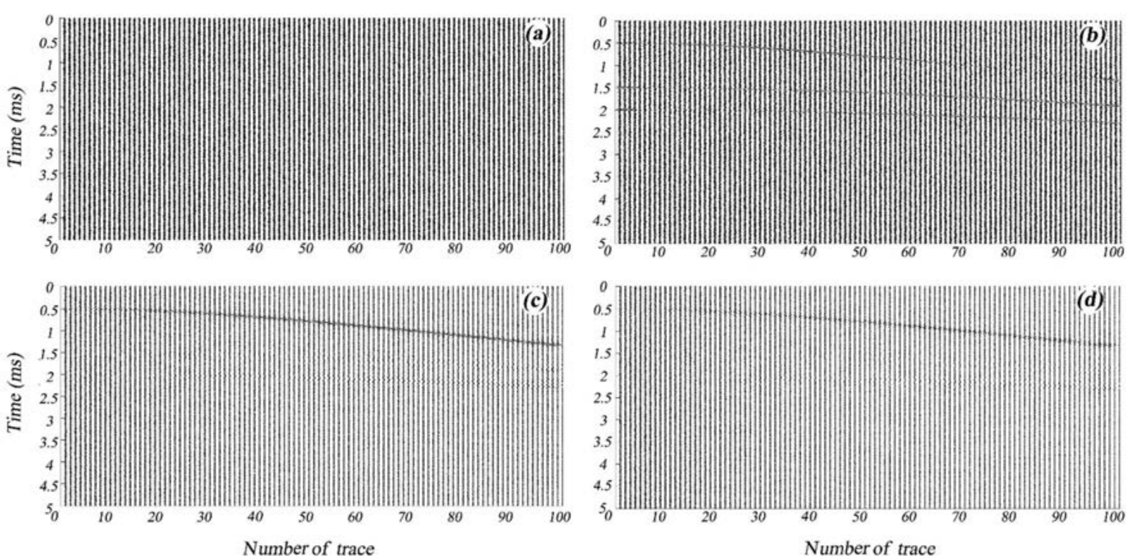


Fig. 4 - Difference between noisy data and denoised data: a) LLSP smoothing filter; b) FX filter; c) wavelet method; d) SVD method.

One of the characteristics of a suitable filter is that it does not distort the frequency contents of the main signal, namely that main signal has the same frequency spectrum before and after applying the filter. For this purpose, in Fig. 5, the frequency spectrum of the first trace of the synthetic data, noisy data and the denoised data with the LLSP smoothing filter of Fig. 3 are shown.

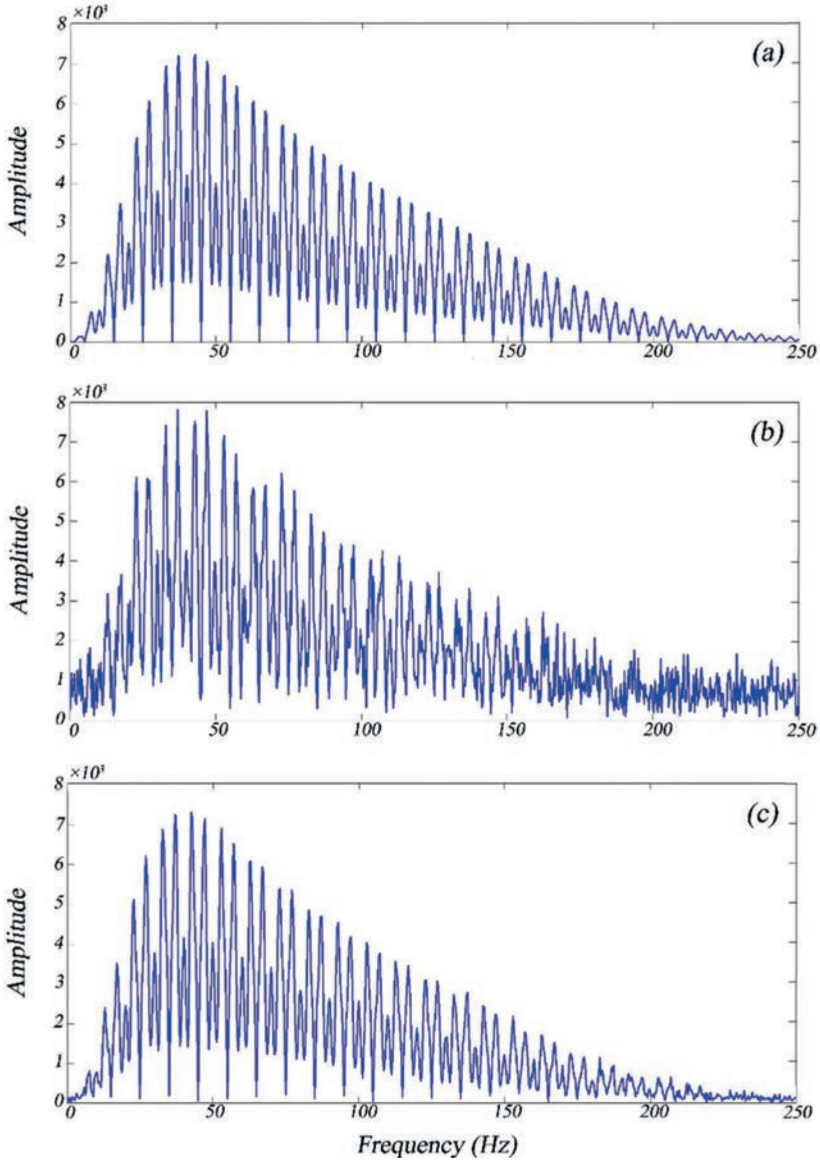


Fig. 5 - The frequency spectrum of the first trace of Fig. 3 data: a) synthetic data; b) noisy data; c) denoised data with LLSP smoothing filter.

As shown in Fig. 5, we see almost the same frequency spectrum for the synthetic trace and denoised trace. In other words, we find that the frequency contents of the trace remain almost unchanged after applying the LLSP smoothing filter and this is another advantage of this filter.

*4.2. Synthetic model 2*

In Fig. 6 a synthetic CMP stacked section is shown. This section has 4 reflectors, the sampling interval is 2 ms and the number of traces is 100. Random noise (white Gaussian noise) with signal-to-noise ratio 1 dB has been added to this section. Then this section is denoised by using the LLSP smoothing filter for $N = 4$ and $M = 45$, FX filter, soft threshold wavelet method, and SVD method.

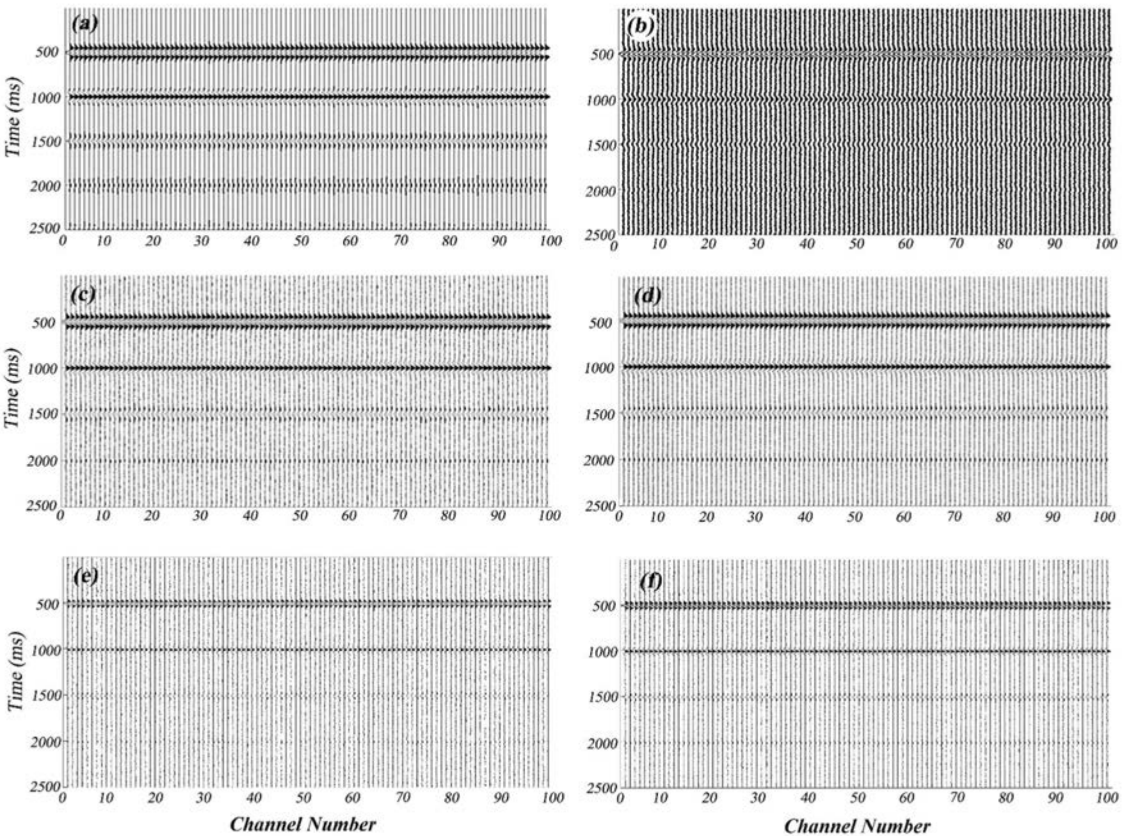


Fig. 6 - Sections: a) synthetic CMP stacked section; b) noisy section; c) denoised section with LLSP smoothing filter; d) denoised section with FX filter; e) denoised section with wavelet method, f) denoised section with SVD method.

After denoising, the signal-to-noise ratio for the LLSP smoothing filter was 15 dB, for the FX filter it was 17 dB, for the wavelet method it was 10 dB and for the SVD method it was 13 dB. The better result of the FX filter has two reasons. The first is that this filter is based on linear events and in this section we have linear events. The second is when the signal-to-noise ratio is low, after applying LLSP smoothing filter, very weak oscillations appear in parts of the signal that are zero. This is the only flaw of the S-G filter, though this filter still keeps the signal waveform very well. The LLSP smoothing filter also has better results than the wavelet and SVD methods.

In Fig. 7 the difference between the noisy and denoised section for the LLSP smoothing filter is shown. Following Fig. 7, we find that the difference between the noisy and denoised section by the LLSP filter only contains random noise, and this filter maintains the primary events very well.

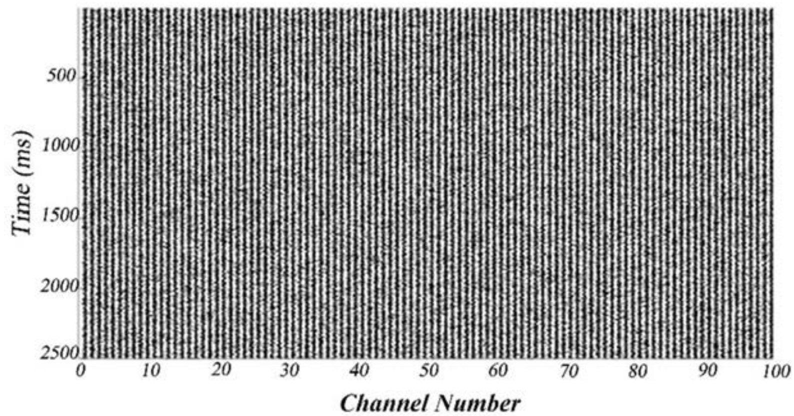


Fig. 7 - Difference between noisy and denoised model by LLSP smoothing filter.

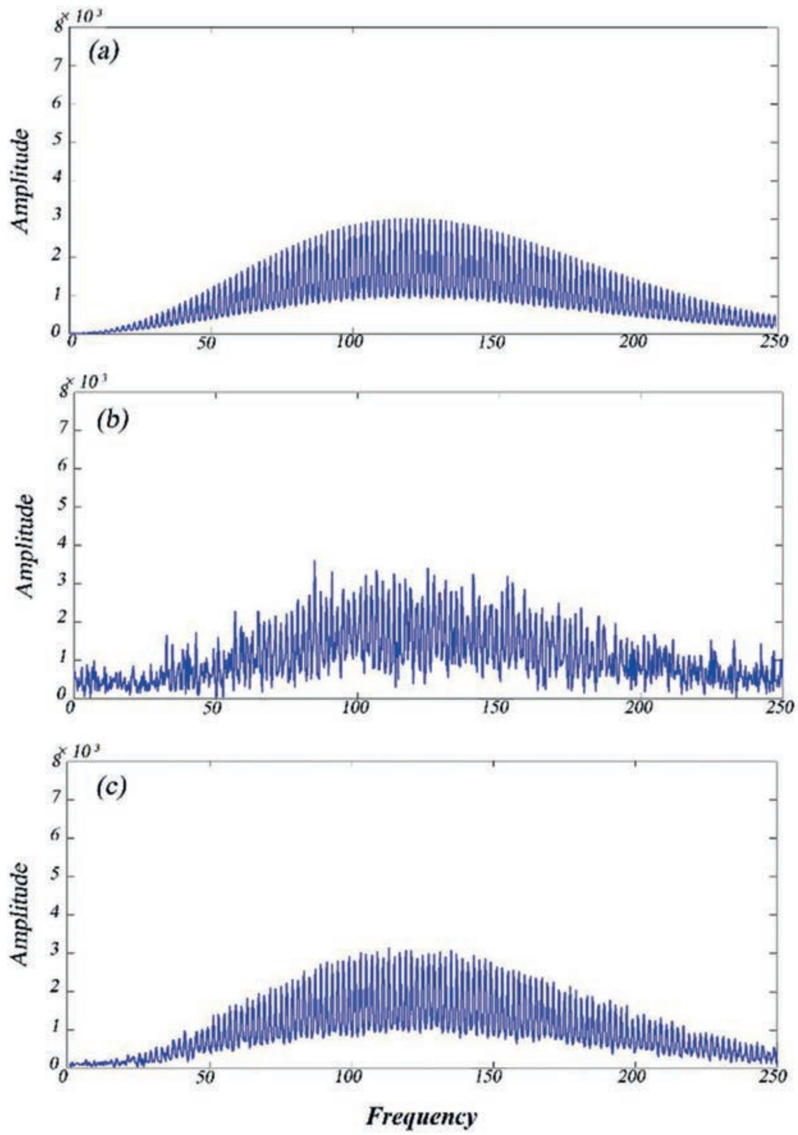


Fig. 8 - Frequency spectrum of the first trace of Fig. 6 section: a) synthetic section; b) noisy section; c) denoised section with LLSP smoothing filter.

We also investigated the performance of the LLSP smoothing filter in the frequency domain for the section of Fig. 6. In Fig. 8, the frequency spectrum of the first trace of the synthetic section, noisy section and denoised section with LLSP smoothing filter of Fig. 6 are shown.

As indicated in Fig. 8, we see almost the same frequency spectrum for the synthetic trace and denoised trace. In other words, we find that the frequency content of the first trace is almost unchanged after applying the LLSP smoothing filter.

### *4.3. Real data*

In this section we use real seismic data. This data concerns the Viking Graben area in the North Sea, which contains 1001 shot records at a distance of 25 m. Each shot is recorded by 120 receivers at a distance of 25 m in 6 seconds. The sampling interval is 4 ms, the depth of cable receiver is 10 m and the depth of the source is 6 m. First, we separate the data of one CMP from the shot records. Then we apply the LLSP filter for $N = 2$ and $M = 11$, the FX filter, wavelet method with soft threshold and SVD method on this data. The results are shown in Fig. 9.

In Fig. 10, to better understand the operation of the LLSP smoothing filter, the part of the data of Fig. 9 separated with the red frame is zoomed.

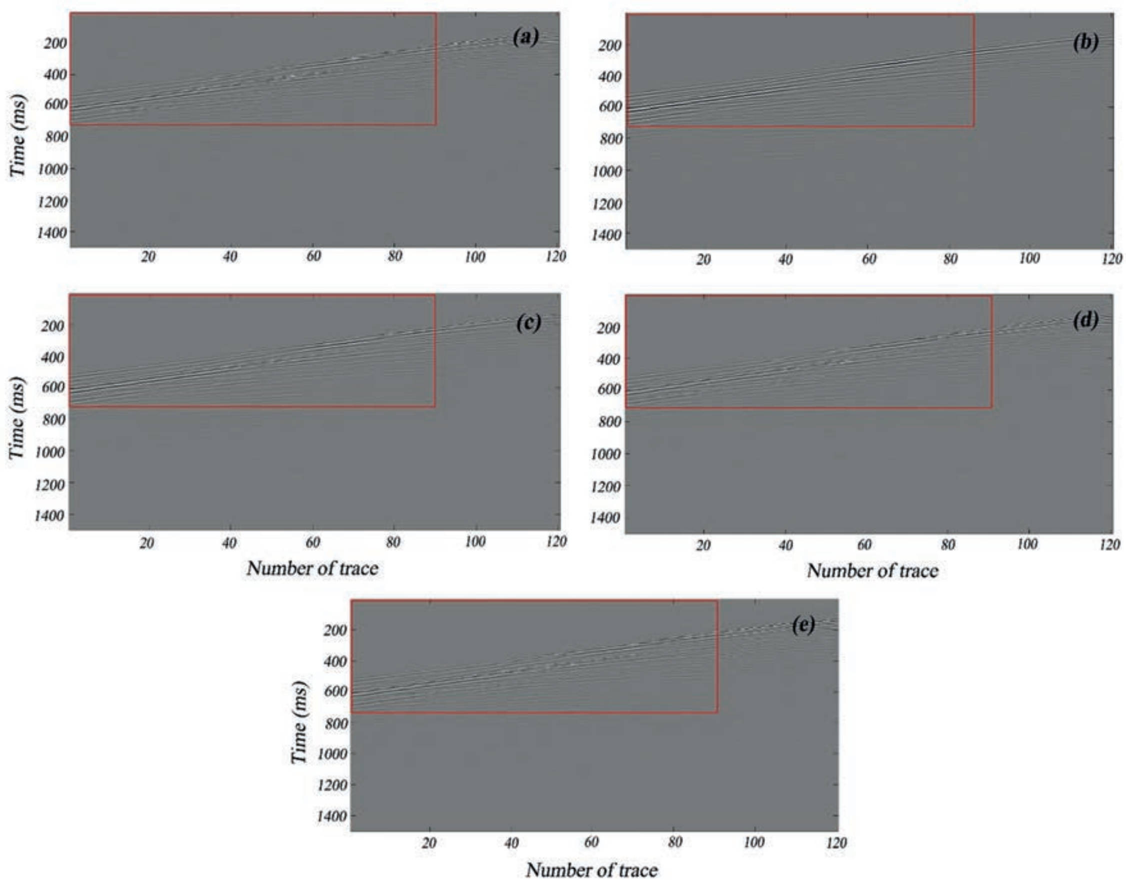


Fig. 9 - Data: a) noisy real data; b) denoised data with LLSP smoothing filter; c) denoised data with FX filter; d) denoised data with wavelet method; e) denoised data with SVD method.

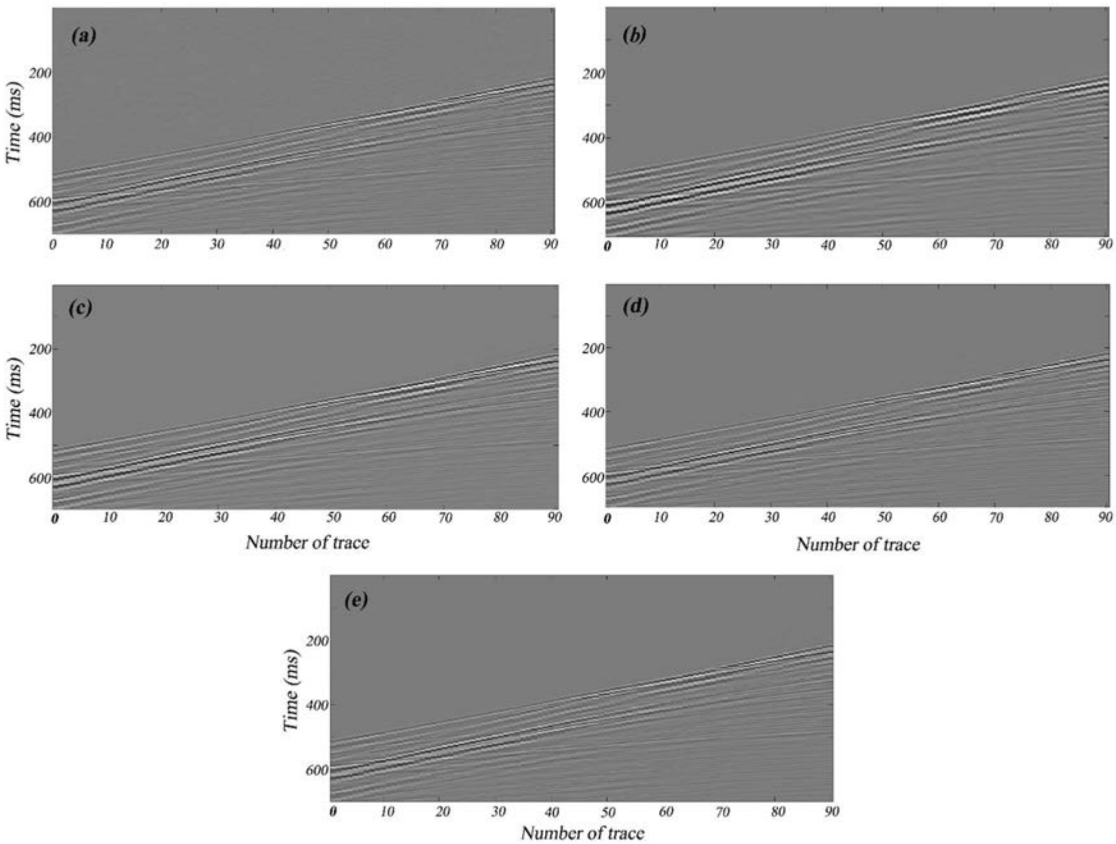


Fig. 10 - A close-up of the data in Fig. 9 in red box: a) noisy real data; b) denoised data with LLSP smoothing filter; c) denoised data with FX filter; d) denoised data with wavelet method; e) denoised data with SVD method.

As shown in Figs. 9 and 10, we find that the LLSP smoothing filter effectively attenuates the random noise and, due to the non-linearity of the events, it has a better performance than the FX filter, wavelet and SVD methods.

## 5. Discussion and conclusion

The LLSP smoothing filter is a novel tool for seismic processing and we have applied this method to eliminate seismic random noise and recover the valid seismic signals in synthetic and real seismic data. This filter is based on data smoothing by fitting the local polynomial with the least squares method. This filter has two notable advantages. First, simple mathematics governing it which make it convenient for data processing. Second, its excellent ability to preserve the signal waveform after application. This means, when we use this filter, we do not lose the main signals. In other words, this filter keeps the frequency spectrum of the signal unchanged and the main signal has the same frequency spectrum before and after applying the LLSP smoothing filter. However, the LLSP smoothing filter has one weakness: when the signal-to-noise ratio is low, after applying the filter, very weak oscillations appear in parts of the signal that are zero, but it still maintains the main signal. The function of this filter is controlled by two parameters, polynomial degree and approximation interval. There is no special procedure to select these two parameters. In fact, the smaller the polynomial degree or the larger the approximation interval, the narrower passband or the more data smoothing. In other words, selecting the optimal parameters depends on the amount of required smoothing. Another point is that in the LLSP smoothing filter, a specific cut-off frequency can be obtained by a different combination of parameters. The difference between these filters is only within the width of the transition regions. This is inversely proportional to the length of the impulse response and is a familiar property of FIR discrete-time low pass filters and the larger approximation interval, the longer impulse response length. In this paper, the filter was applied on synthetic and real seismic data to reduce the random noise. For a more accurate analysis, the results of the proposed filter were compared with the results of the FX filter, wavelet and SVD methods. We have seen that the LLSP smoothing filter yields better results. Also, the simpler mathematics of the filter is another advantage compared to the FX filter, wavelet and SVD methods which makes it much easier to apply. Therefore, it can be confidently concluded that the LLSP smoothing filter is more efficient the than FX filter, wavelet and SVD methods.

**Acknowledgements.** The Institute of Geophysics, University of Tehran is acknowledged for supporting this research.

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