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Multivariate simulation of a multi-element deposit, based on the different transformations. Case study: Mehdiabad deposit, Iran

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ABSTRACT Modelling of multivariate complex deposits with the presence of several correlated attributes is a very challenging issue in the mining industry which can be addressed using existing multivariate analysis method. In this study, some of these multivariate methods, such as Step-wise Conditional Transformation (SCT), Minimum/maximum Autocorrelation Factors (MAF) and Projection Pursuit Multivariate Transform (PPMT), were applied to a data set of Mehdiabad deposit. The data set is containing core samples to be analysed for Pb, Zn, Cu, and Ag. At the first stage, the variables were transformed by mentioned methods and a set of validations were performed to the transformation results. Next, the transformed variables were simulated using sequential Gaussian simulation and the results were analysed as well. Based on the validation reviews, it was concluded that the PPMT could present more reliable outcomes. Furthermore, for every transformation, the grade-tonnage curves for each transformed variable were calculated based on the E-type values of the simulations and the discrepancies between them were also investigated. The results of this study can be also used in mine planning and risk measurement during mining.

Key words: multivariate modelling, Step-wise Conditional Transformation, Minimum/maximum Autocorrelation Factors, Projection Pursuit Multivariate Transform, grade-tonnage curve.

1. Introduction

Evaluation of the mineral resources can be done by estimation of the tonnage and grade that are expected to be recovered during mining. This may affect the further mining planning and also the financial benefits of the mining operation at the early stage of production (David, 2012; Peattie and Dimitrakopoulos, 2013; Hosseini *et al.*, 2017). Geostatistical estimation methods are the most common approaches in the case of mineral resource estimation which provide conditionally unbiased value as close as possible to the actual value. However, these methods have an unavoidable drawback which is the smoothing effect that will generally lead to overestimating the tonnage above the economic cut-off grade (Assibey-Bonsu *et al.*, 2015). To overcome this issue, stochastic simulation methods can be used in order to omit the smoothing effect (Chilès and Delfiner, 1999). These methods present a more accurate evaluation of the grade uncertainty through the multiple realisations, and also try to minimise the spatial variability of the true grades. It also can be useful in mine design, mine planning studies (Dimitrakopoulos, 2010; Rossi and Deutsch, 2013). Simulation of individual variables is more prevalent using easily accessible methods (Deutsch and Journel, 1992; Goovaerts, 1997; Dimitrakopoulos and Luo, 2004; Chilès and Delfiner, 2012). However, in the case of multivariable deposits, the matter is to jointly simulate the interested attributes (especially for the deposits with more than two interested variables) in the way that the original features can be reproduced.

There are several approaches proposed for joint simulation of attributes (David, 1988; Myers, 1989; Verly, 1993; Goovaerts, 1997; Soares, 2001; Chilès and Delfiner, 2012). Some of them, especially those that rely on the Linear Model of Coregionalisation (LMC) are difficult to apply for more than three variables. Furthermore, the multivariate geostatistical simulation should be applied to variables with multivariate Gaussian distribution which rarely exists in the case of coregionalised data (Hosseini and Asghari, 2019). Besides, integration of the univariate transformed Gaussian variables would not satisfy the assumption of multi-Gaussian (Barnett, 2015; Battalgazy and Madani, 2019). As an alternative, it is possible to implement multivariate analysis methods which transform the variables to an orthogonal space (uncorrelated), so that the variables can independently be simulated and back-transformed to an original space (Boucher and Dimitrakopoulos, 2012).

The most common method for multivariate analysis of data set is Principal Component Analysis (PCA) which is widely used in hundreds of research. PCA is a linear combination of variables which aims to reduce the dimensionality of the data set providing uncorrelated factors. A modified version of the PCA called Minimum/maximum Autocorrelation Factor (MAF) (Desbarats and Dimitrakopoulos, 2000) is able to produce spatially uncorrelated factors rather than just for lag zero based on the LMC with two structures (Bandarian *et al*., 2008). Also, MAF can be applied with no need to model the coregionalisation (Rondon, 2012). Silva and Costa (2014) used MAF for grade estimations of the variables instead of using cokriging. In another research, MAF was applied to a geochemical data set in order to extract and model the mineralisation factor (Ghane and Asghari, 2017).

In some deposits, relations between the variables do not follow a linear relationship. Hence, the application of non-linear based methods would yield better results for such cases. Leuangthong and Deutsch (2003) proposed a multivariate non-linear Gaussian transform named Step-wise Conditional Transformation (SCT) to produce uncorrelated variables for lag zero. This method, as a prerequisite of all Gaussian-based simulation algorithms, orthogonalises and normalises simultaneously the variables into Gaussian distribution. However, this method has a restriction when dealing with several attributes or small data set. To explore this issue, Barnett *et al*. (2014) introduced the Projection Pursuit Multivariate Transformation (PPMT) which is able to model and transform highly multivariate and complex data. The uncorrelated transformed variables will be modelled independently and back-transformed subsequently producing original correlations and complexity such as heteroscedasticity, nonlinearity, and constraints among the variables.

The main aim of this study is to evaluate the performance and efficiency of transformations and their affects on grade-tonnage curves and resource estimation. Therefore, three different multivariate transformation methods, namely MAF, SCT, and PPMT, were applied for the joint simulation of the interested variables. Then, the efficiency and the performance of the mentioned methods were assessed based on several factors (reproduction of original mean, variance, variograms, and correlations between the variables and the cross-variograms) and the best one was chosen. Finally, the grade-tonnage curves for each transformed variable (for every method) were calculated and compared to find which method can produce the grade and tonnage of variables for optimum resource modelling.

2. Geology and data set

2.1. Geology and deposit description

The Mehdiabad deposit is located 116 km SE of the city of Yazd in the central Iranian tectonic block. It is a world-class Cretaceous deposit and one of the most important metallogenic province for zinc-lead mineralisation (Ghazanfari, 1999; Ghorbani, 2013). The geologic map of the Mehdiabad area is shown in Fig. 1. According to Maghfouri (2017), Mehdiabad deposit is a syn-sedimentary mineralisation related to SedEx (sedimentary exhalative) type based on the host rocks, minerals and rock forming, petrographical evidence, and other geological/geochemical

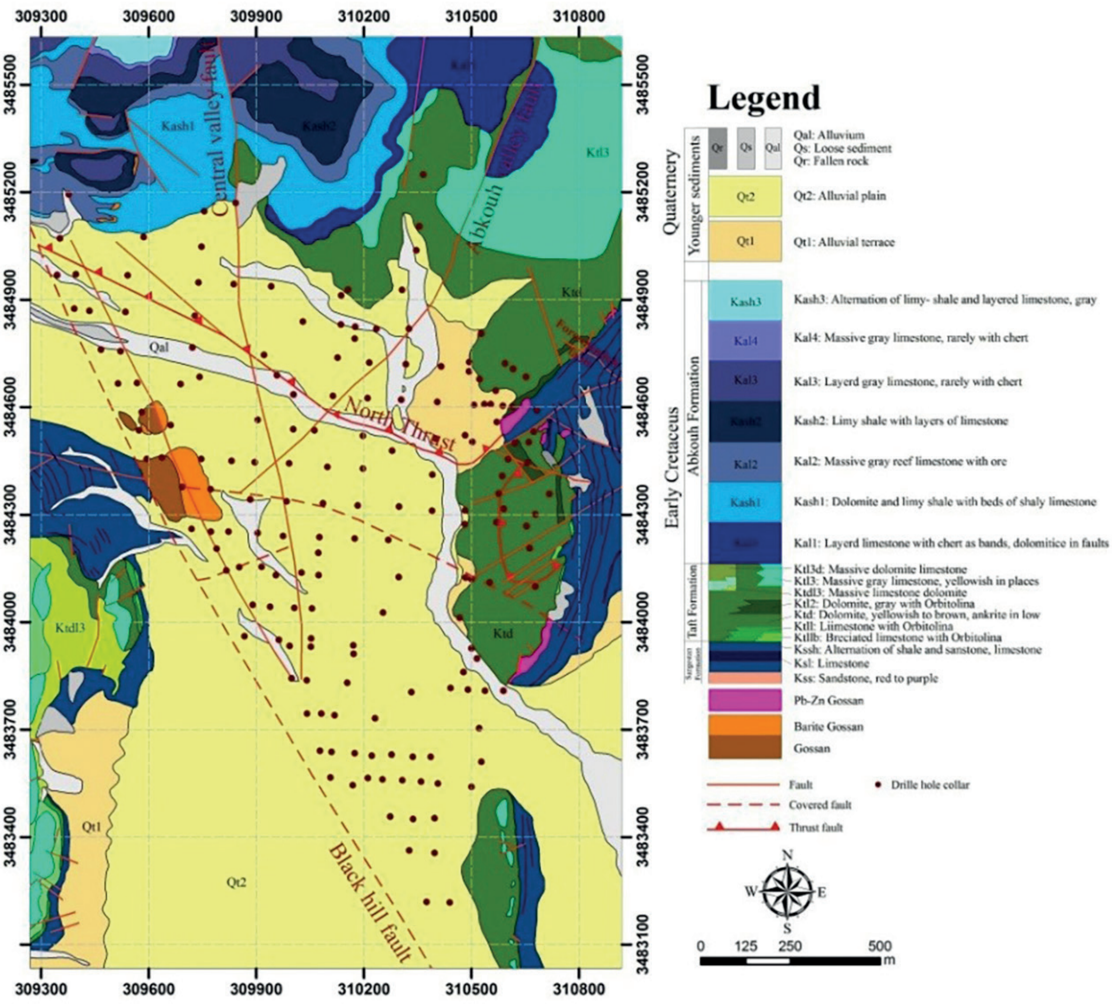

Fig. 1 - The geological map of the Mehdiabad area (Hosseini and Asghari, 2019).

features. The main geological formations of the deposit are Sangestan, Taft, and Abkouh which are formed by different geological units. The mineralisation is mainly controlled by the faults which are the most important structural characteristics of the studied area. The Mehdiabad deposit is divided into two parts: the sulfide and the oxide zones. The sulfide part, which is covered by alluvial sediments (up to 250 m thickness), occurs mainly in the Taft formation and consists of dolomite and ankerite limestone, where the main sulfide minerals are Galena, Sphalerite, Barite, Pyrite, and Chalcopyrite. The oxide part is located in the north-western part of the deposit and it is separated from the sulfide zone by faults. The mineralisation in the oxide zone occurred within the Abkouh formation and in karst limestones and it is divided into two mineral associations: a) the red zinc minerals consist of Iron Hydroxide, Goethite, Hematite, Hemimorphite, and Smithsonite; b) the white zinc minerals consist of Hydrozincite, Hemimorphite, and Smithsonite (Reichert, 2007).

2.2. Data set

In this study, it was used a data set containing 4493 core samples, collected from 85 drillholes. The length of samples was between 0.5-1.0 m and the length of composites was considered with 1.0 m along a NW-SE direction (Fig. 2) within the west sulfide part of the Mehdiabad deposit, called zone 220. The mean distance between the drillholes is about 100 m. Also, based on the geological knowledge, four elements namely copper, lead, zinc, and silver which are analysed by ICP-OES method, were selected for further statistical and geostatistical analysis. The descriptive statistics of the mentioned variables are shown in Table 1 and Fig. 3. Furthermore, the correlation coefficients between the variables are demonstrated in Table 2 and Fig. 4. The highest correlation between the variables belongs to Pb and Ag while the lowest correlation is related to Pb and Cu which is near zero. It should be mentioned that the outlier values were replaced using Q-Q plot (quantile-quantile plot). The Q-Q plot, is a graphical technique which compares the distribution of two data sets. The Q-Q plot is a scatter plot of two sets of quantiles against one another. If same

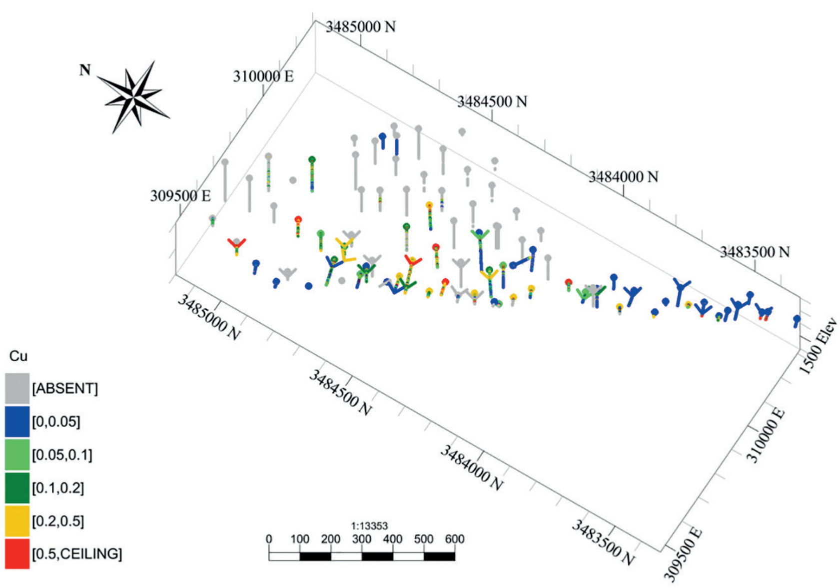

Fig. 2 - The distribution of the drillholes located in zone 220.

Table 1 - Descriptive statistics of the variables.

	Zn	Pb	Ag	Cu
Number	4493	4493	4493	4493
Mean	3.28(%)	1.440(%)	33.83(ppm)	0.140(%)
SD	3.52	1.510	36.86	0.183
Variance	12.37	2.280	1358.93	0.033
Skewness	18.20	2.340	2.85	2.110
Kurtosis	5.99	7.660	10.68	5.380
Min	0.00	0.001	0.00	0.000
Max	20.00	10.110	243.60	1.010
Quartiles: Q1	0.99	0.470	14.98	0.006
Quartiles: Q2	2.21	1.050	26.00	0.076
Quartiles: Q3	4.31	2.000	48.60	0.200

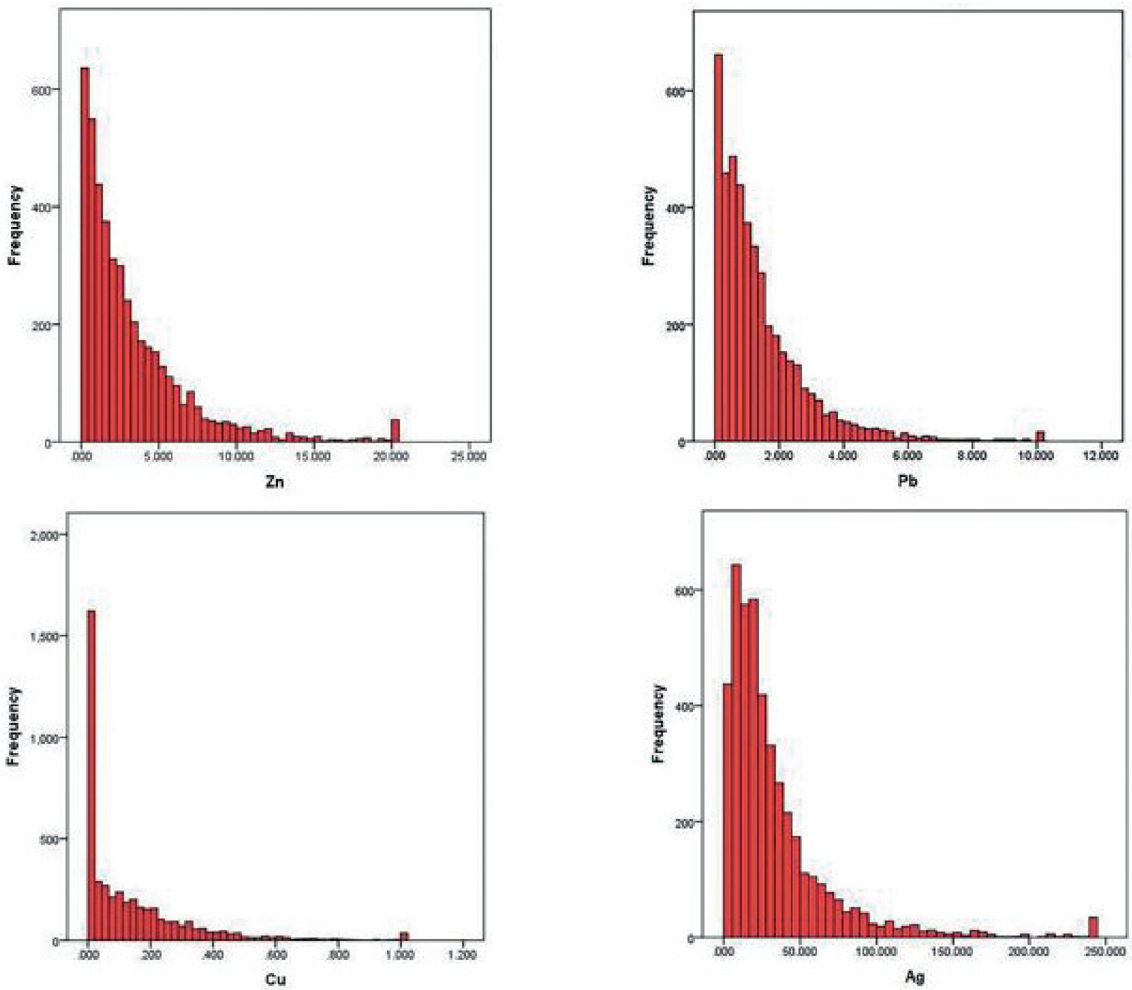

Fig. 3 - Histograms of the variables.

distributions plot against each other, the scatter plot should follow a straight linear structure along the 45 degree line. The discontinuity in tails of the Q-Q plot reveals us the outlier values. The thresholds obtained using Q-Q plot for zinc, lead, silver, and copper were 20 (%), 10.116 (%), 243.6 (ppm) and 1.01 (%), respectively.

Table 2 - The Spearman correlation coefficients between the variables.

	Zn	Pb	Ag	Cu
Zn	1.00			
Pb	0.57	1.00		
Ag	0.48	0.74	1.00	
Cu	-0.25	0.02	0.21	1.00

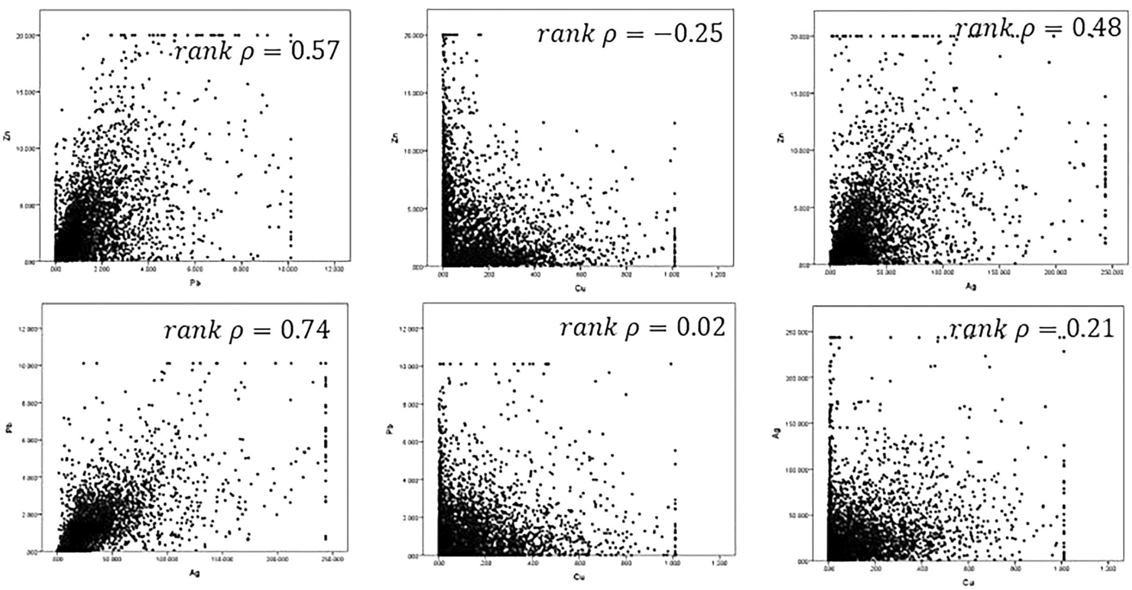

Fig. 4 - The scatter plots of the variables along with the Spearman correlations between them.

3. Methodology

3.1. Step-wise Conditional Transformation (SCT)

SCT, firstly introduced by Rosenblatt (1952), is a multivariate extension of the normal score transformation, as it attempts to remove complexity such as heteroscedasticity, nonlinearity, and constraints among the variables (Leuangthong and Deutsch, 2003). Furthermore, the technique attempts to decorrelate all of the variables at a zero lag distance. The method is stepwise application of the normal score transformation. For applying this method the first variable simply was transformed to normal score. In next step, the second variable was divided according to the conditional probability class of the first variable, before having the normal score transformation independently executed on each discretised bin. The third variable was transformed conditional to the probability class of the first and second, and so on. This stepwise process is given by Eq. 1:

$$
\begin{aligned}
y_1' &= G^{-1}[F_1(z_1)] \\
y_2' &= G^{-1}\left[F_{2|1}(z_2|z_1)\right] \\
y_3' &= G^{-1}\left[F_{3|1,2}\left(z_3\middle|z_{1,2}\right)\right] \\
y_n' &= G^{-1}\left[F_{n|1,\ldots,n-1}\left(z_n\middle|z_{1,\ldots,n-1}\right)\right]
\end{aligned}
\tag{1}
$$

where Z_i, i=1, … , n are the original variables, $F_i(z_i)$ are cumulative distribution function (CDF) of each original variable, $G(.)$ is the standard Gaussian distribution and Y_i', i=1, … , n are the SCT variables.

3.2. Min/max autocorrelation factors (MAF)

MAF is modified version of PCA method that provides uncorrelated factors for all lag distances (spatially uncorrelated factors). This method can be divided into two different approaches: the model-based and the data-driven methods. The model-based method is based on the application of two structure LMC. LMC is a geostatistical tool which investigates the linear combination between two regional variables by calculating the direct variograms of the variables and the cross-variogram between the variables. Calculating LMC is somehow tough to apply especially with the presence of several variables (Vargas-Guzmán and Dimitrakopoulos, 2003; Boucher and Dimitrakopoulos, 2009). In the data-driven method the factors are derived directly from the original variables (Switzer and Green, 1984). In this study, the data-driven method was used to produce spatially uncorrelated factors. The steps are discussed below:

1) calculating the correlation coefficient matrix B;
2) decomposing matrix B as $B = Q^T \Lambda Q$ where Q is eigenvector and Λ is eigenvalue;
3) second rotation of the variable using matrix A as $A = \Lambda^{-1/2}Q$
4) calculating PCA scores through matrix $Y_{PCA}(u) = AZ(u)$ where $Z(u)$ is normal transformed data;
5) calculating matrix $\Gamma_{Y_{PCA}}(h)$ containing PCA factors experimental variogram values for determined lag distances. The lag distance would be determined based on the horizontal separation of the drillholes (Debsbarats, 2001) or it can be achieved according to better decorrelation of the factors (Dimitrakopoulos, and Fonseca, 2003);
6) further rotation of PCA factors for producing MAF factors can be done using matrix Q_1 which is extracted from spectral decomposition of the matrix $\Gamma_{Y_{PCA}}(h)$;
7) the MAF factors are calculated as $F_{MAF}(u) = Q_1AZ(u) = MZ(u) = Q_1Y_{PCA}(u)$ (Rondon, 2012).

The produced factors should have normal distributions and also the cross-variograms between the factors should obtain values near zero for all lag distances.

3.3. Projection Pursuit Multivariate Transform (PPMT)

PPMT is an exciting new technique that facilitates independent Gaussian geostatistical modelling in highly complex multivariate settings. The PPMT is a modified version of Projection Pursuit Density Estimation (PPDE) method in which high dimensional complex data are transformed iteratively to multivariate Gaussian distribution from a geostatistical point of view.

PPMT will iteratively transform the data to be multivariate Gaussian. When multi-Gaussian are implemented, the final transform locations of the data are recorded for mapping purposes. The algorithm runs in three steps as shown below (Barnett *et al*., 2014; Barnett, 2017):

a) normal score to transform variables to Gaussian units through $y = G^{-1}[F(z)]$;
b) the second step is data sphering, which transforms the data to be uncorrelated and with unit variance as below:

$$x = S^{-1/2}y$$
$$S^{-1/2} = UD^{-1/2}U^T \qquad (2)$$

where $S^{-1/2}$ is the sphering matrix which is calculated by spectral decomposition of the covariance matrix of y (D is diagonal eigenvalue matrix and U is corresponding eigenvectors);
c) in final step, the projection pursuit algorithm would be run. Assume α as vector with unit length and K dimention. $p = \alpha^T X$. If x is multi-Gaussian, any α should produce a p that is univariate Gaussian. Therefore, a test statistic (named projection index), $I(\alpha)$, that evaluates univariate non-Gaussianity is defined. For any α that associated p is perfectly Gaussian, $I(\alpha)$ will be zero. Using an optimised search, projection pursuit finds the α that produces the maximum $I(\alpha)$. Then the multivariate data, X, are transformed to have a univariate Gaussian projection along α using the Gaussianisation transform. Treating the Gaussianised multivariate data as Y, the projection $p = \alpha^T Y$ is univariate Gaussian. Iterating this search and Gaussianise procedure, Y is eventually transformed to be multivariate Gaussian (Barnett *et al*., 2014; Barnett, 2017).

4. Results

In this study, three different multivariate methods namely SCT, MAF, and PPMT were applied to the west sulfide zone of Mehdiabad deposit. The analysis was done using four elements which are Pb, Zn, Ag, and Cu. The transformed variables were simulated using the conditional Sequential Gaussian Simulation (SGS) and, then, back-transformed to original distribution. SGS needs variables to be transformed to a Gaussian distribution with zero mean and unit variance. In this method, the points are first estimated using simple kriging system and, then, the variable is simulated using the normal CCDF obtained for each point (Soltani *et al*., 2104). The application of simulation provides us an opportunity to obtain any number of realisations so that we could better assess the efficiency of our methods. The simulations were carried out within a block model of 25×25×10 m^3 and number of nudes in X, Y, and Z dimension is 167,282. The performance of the mentioned methods was evaluated by several tools such as histograms of the simulation realisations, correlation coefficients of the simulated transformed variables, cross-variograms of the transformed variables, and variograms of the transformed simulated variables. Finally, considering the mentioned factors, the best multivariate method was identified and the grade-tonnage curves of the variables were calculated using the results of the best-selected approach.

4.1. Application of SCT

Due to the fact that the SCT has provided more reliable results in the case of three variables rather than four variables, two series of the data set containing three variables were considered in order to apply SCT. The first series consists of Zn, Pb, and Ag and the second series consists of Zn, Pb, and Cu. The priorities of the variables were determined based on their spatial continuity, and Zn was considered as Z_1, Pb was considered as Z_2 and also Ag-Cu were considered as Z_3. A schematic graph of the SCT steps is shown in the Fig. 5.

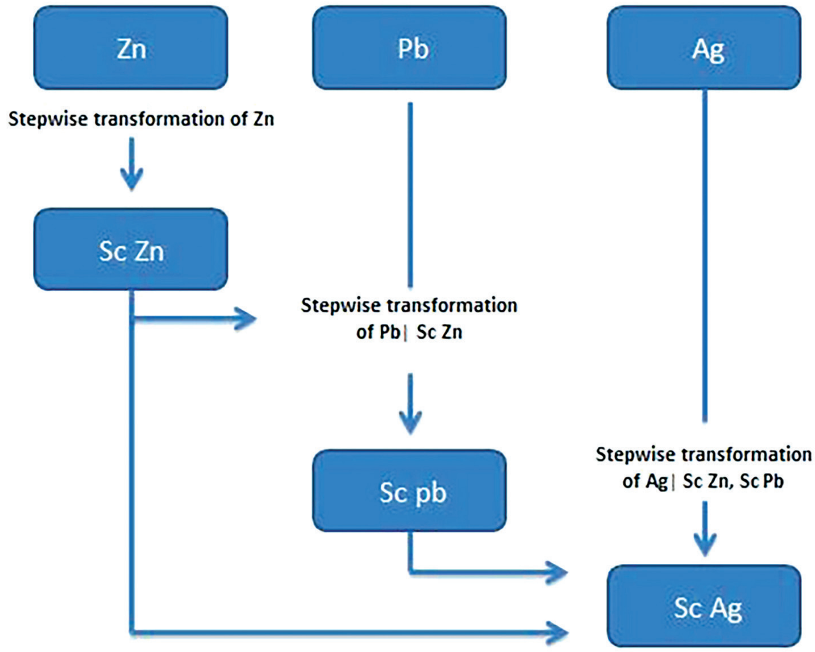

Fig. 5 - Example of schematic steps of SCT algorithm.

Application of the SCT yielded normal transformed variables with approximately mean zero and unit variance. In order to evaluate the functionality of the method, the scatter plots of the transformed variables were calculated to explore the decorrelation between them (Fig. 6). According to Fig. 4, most of the variables are well decorrelated except Ag and Cu which can be due to the separation of the variables to two series.

Besides, cross-variograms of the transformed variables were computed to investigate the decorrelations in space (Fig. 7). The experimental cross-variograms have shown values around zero for different lag distances except for Ag-Cu. Based on the experimental cross-variograms, it can be concluded that the transformed variables have not decorrelated well in space.

Furthermore, the transformed variables were simulated individually within the block model. To reach this purpose, variography was done for all variables and the anisotropies of each one were determined. Next, the simulation was performed for 100 realisations using sequential Gaussian simulation. Then, all the realisations were back-transformed to the original distributions of the variables. Moreover, the mean values of all realisations were calculated for every variable. Also, the correlation coefficients of the variables were calculated per each realisation. The validation of

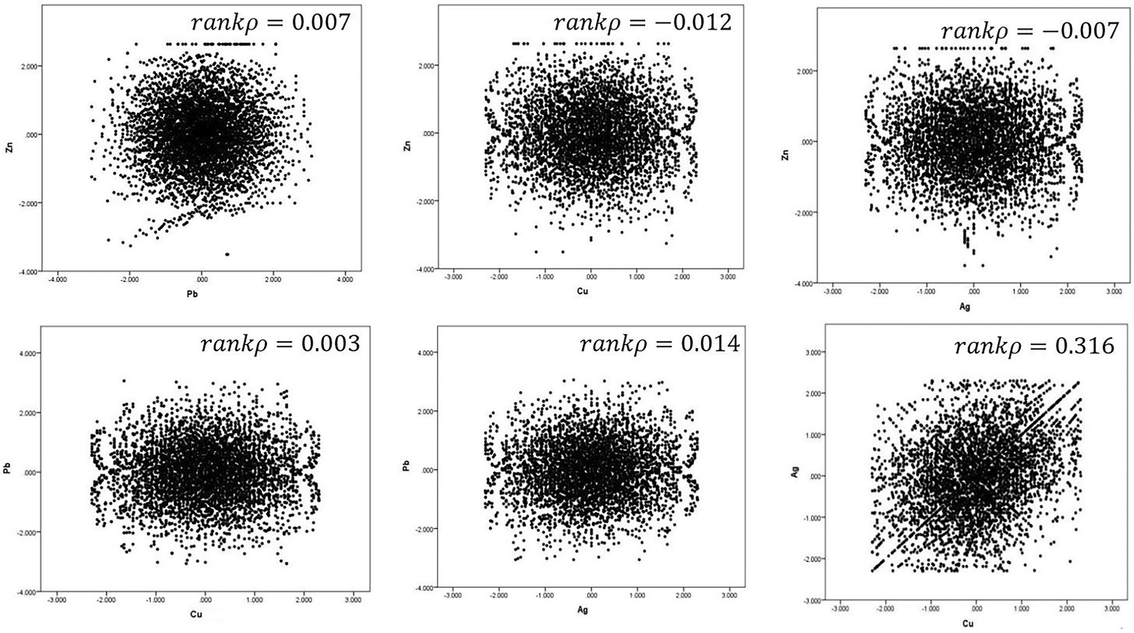

Fig. 6 - The scatter plots of the SCT transformed variables along with the Spearman correlations between them.

the results was done by comparison of the mean values and correlations of original data with the mean and correlation histograms of the simulated data (Figs. 8 and 9). The results showed that the simulations could reasonably reproduce the original mean values and also the correlations between the variables. As another validation, the experimental variograms of all 100 realisations were computed for all variables to check whether they reproduce the original variograms (from the aspects of range and sill value) or not (Fig. 10). It can be inferred that the realisation variograms could acceptably reproduce the original variograms.

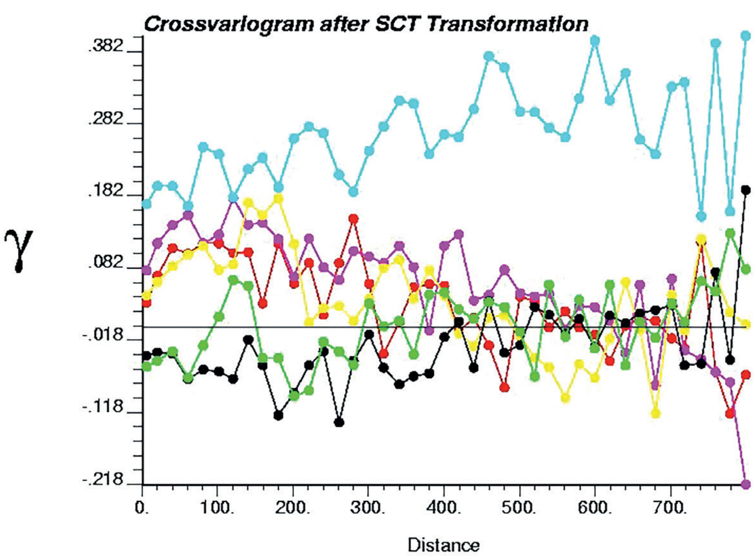

Fig. 7 - The cross-variograms of the SCT transformed variables. The red lines correspond to Pb-Zn, the purple lines correspond to Zn-Cu, the yellow lines correspond to Zn-Ag, the black lines correspond to Pb-Cu, the green lines correspond to Pb-Ag and the blue lines correspond to Cu-Ag.

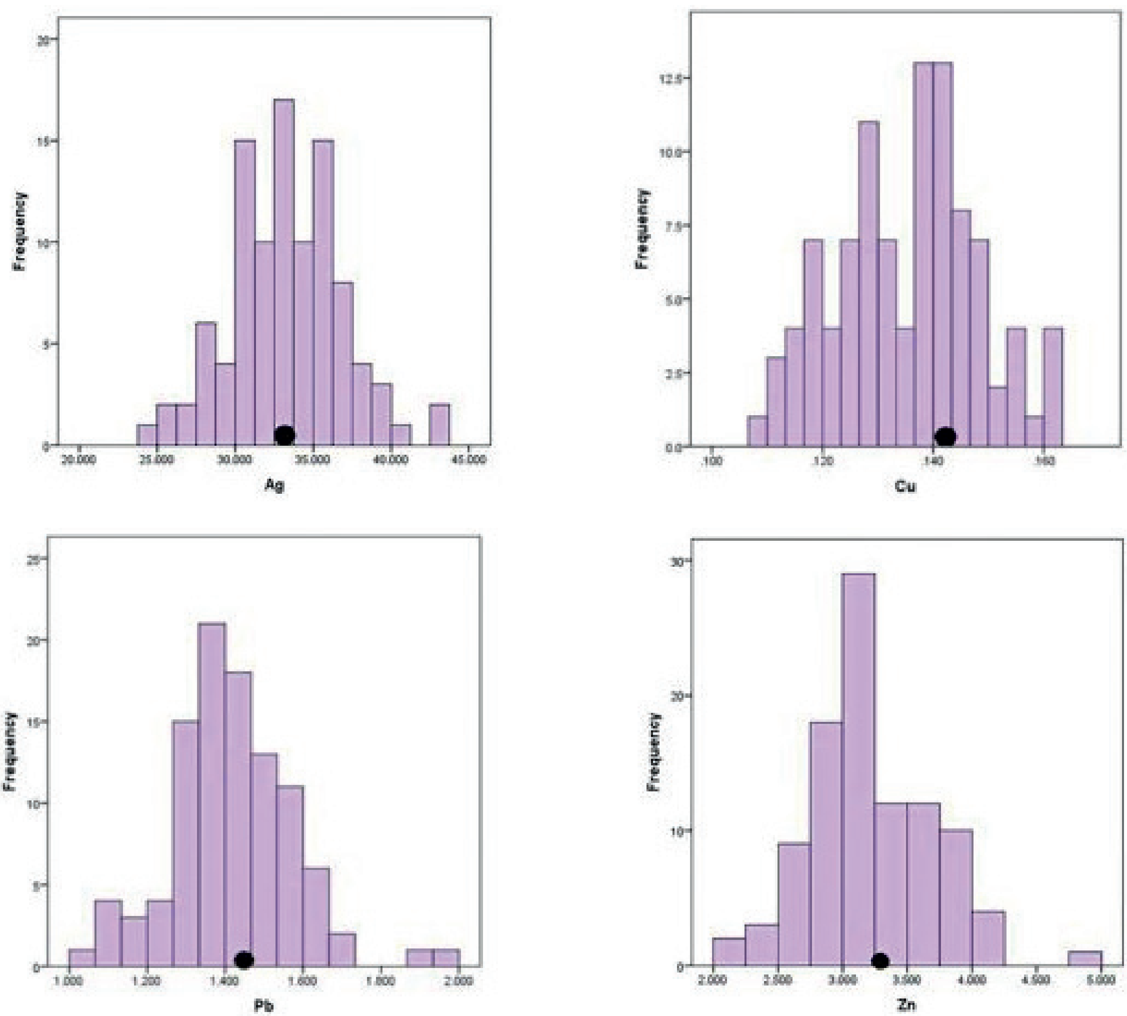

Fig. 8 - The mean histogram of SCT simulated back-transformed variables. The black dots are the original averages of the variables.

4.2. Application of MAF

As mentioned before, a data-driven MAF method is applied in this study which is able to produce decorrelated factors with no need to calculate LMC. Firstly, the variables were transformed to normal Gaussian distribution with zero mean and unit variance [$Z_{(u)}$]. Second, the correlation coefficient matrix B was computed as:

$$B = \begin{pmatrix} 1 & 0.58 & -0.19 & 0.48 \\ 0.58 & 1 & 0.02 & 0.75 \\ -0.19 & 0.02 & 1 & 0.23 \\ 0.48 & 0.75 & 0.23 & 1 \end{pmatrix}$$

where the columns are corresponding to Zn, Pb, Cu, and Ag, respectively. Then, the PCA scores were achieved using matrix A by spectral decomposition of matrix B. For the next step, the

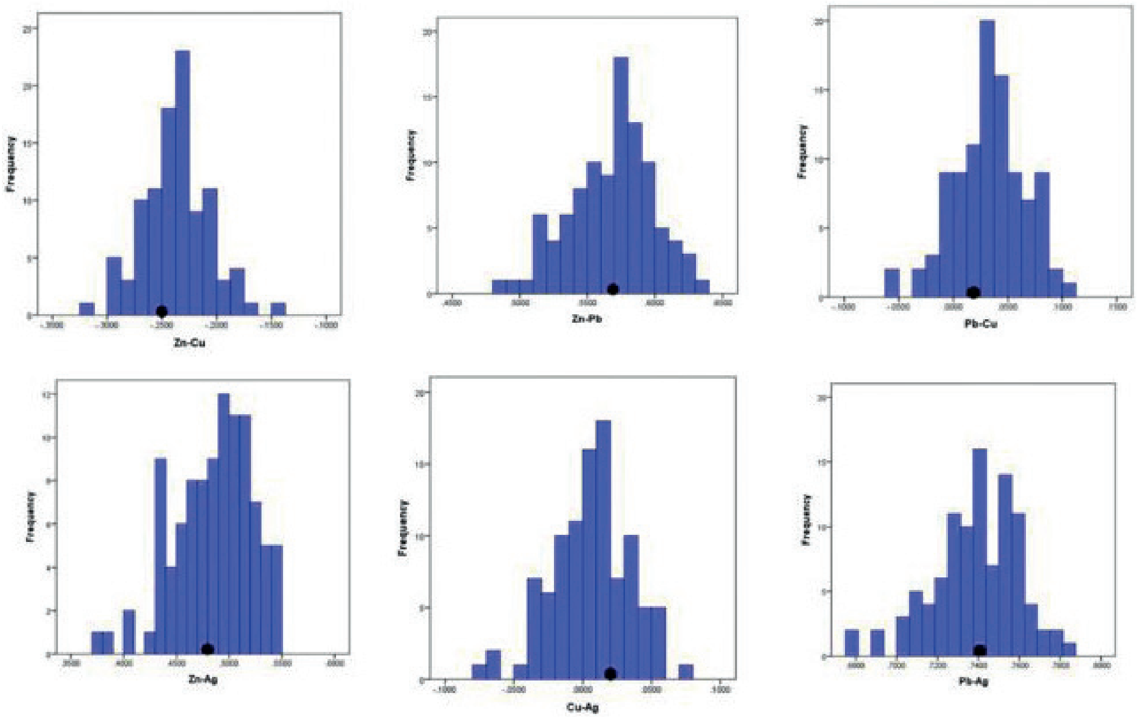

Fig. 9 - Histograms of the correlation coefficients calculated through 100 simulation realisations of the SCT transformed variables. The black dots indicated the original correlation between the variables.

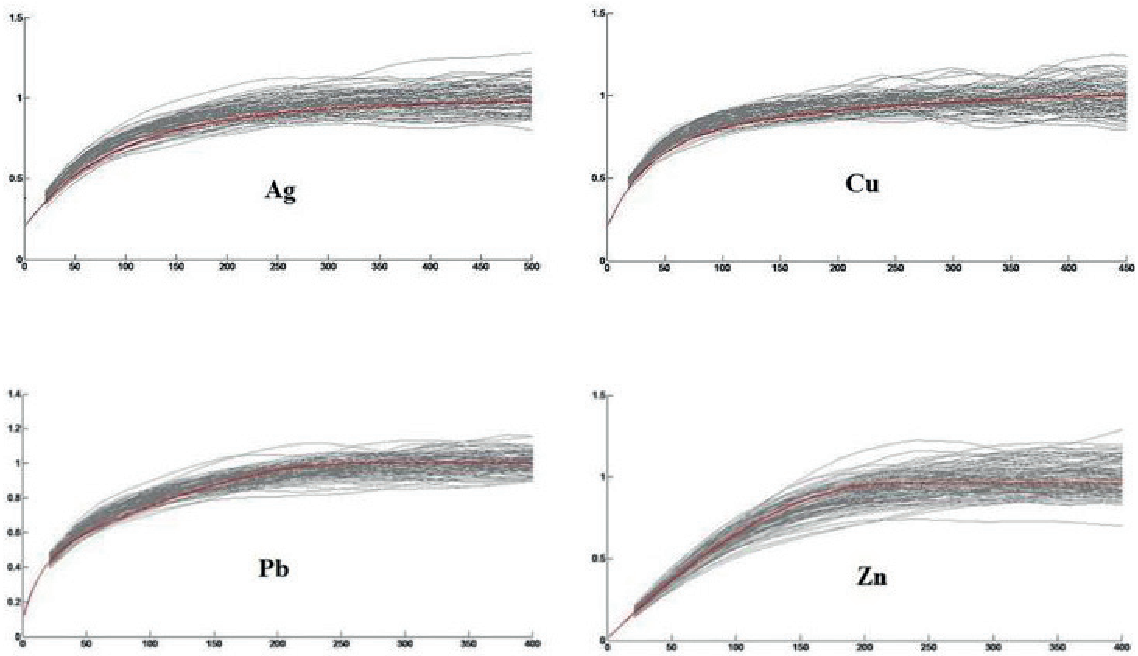

Fig. 10 - The experimental variograms of the SCT simulated back-transformed variables.

experimental variograms of the PCA factors were calculated and variogram values for lag 40 m were used to construct matrix V:

$$V = \begin{pmatrix} 0.83 & 0.004 & 0.002 & 0.03 \\ 0.004 & 0.64 & 0.003 & 0.05 \\ 0.002 & 0.003 & 0.43 & 0.09 \\ 0.03 & 0.05 & 0.09 & 0.86 \end{pmatrix} .$$

Matrix V was decomposed to matrixes Q_1 and Λ_1 and MAF factors were produced using matrix $M = Q_1A$ so that $F_{MAF} = MZ_{(u)}$:

$$M = \begin{pmatrix} -0.35 & -0.13 & 0.83 & 0.14 \\ 1.12 & -0.65 & 0.52 & -0.57 \\ -0.1 & -1.47 & -0.48 & 1.24 \\ 0.51 & -0.31 & 0.16 & 0.86 \end{pmatrix}.$$

All the MAF factors follow a normal Gaussian distribution. All the validations discussed for SCT were done for MAF results. The scatter plots of MAF factors demonstrated that the factors are well decorrelated since the correlations are almost equal to zero (Fig. 11). But, MAF factors scatter plots could not present multi-Gaussian distribution. It may be due to the non-linear nature of the data which is in contrast with the linearity assumption of the MAF. Also, in order to verify the spatial decorrelation of the factors, cross-variograms between the factors were calculated and they illustrated acceptable decorrelations for several lags (Fig. 12).

As well as the SCT, the transformed factors were simulated for further validation reviews. First, the variograms of the factors were computed considering the anisotropy and the factors were simulated for 100 realisations subsequently. Next, the simulated factors were back-transformed to the original variables for all the realisations. In addition, another back-transformation was applied

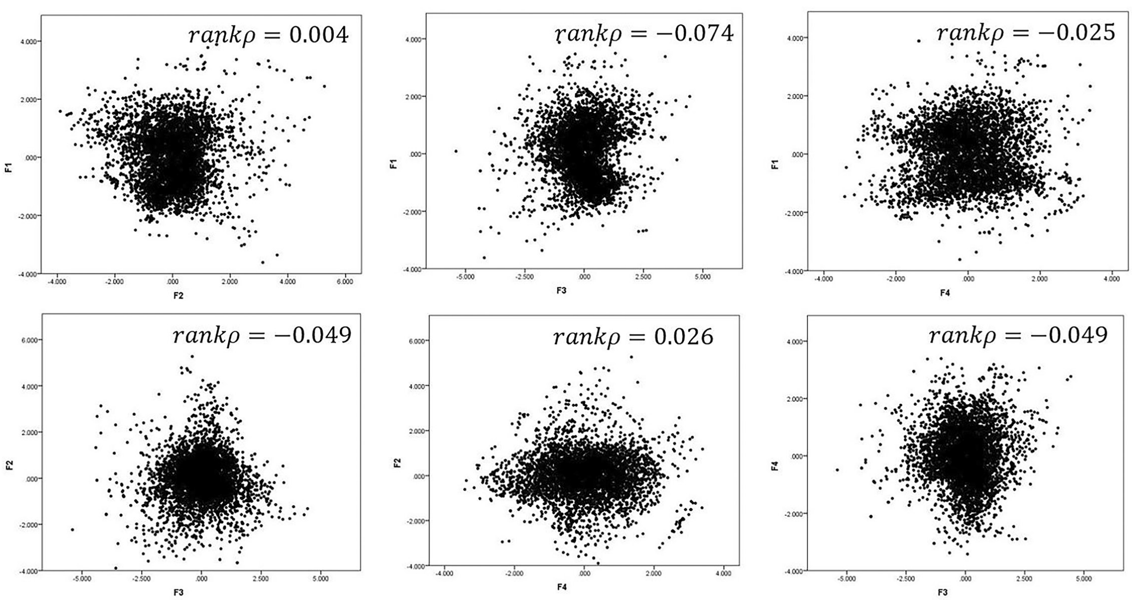

Fig. 11 - The scatter plots of the MAF transformed factors along with the Spearman correlations between them.

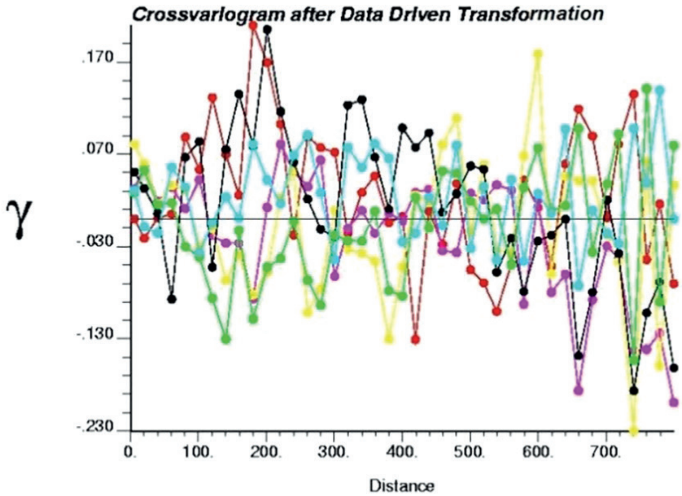

Fig. 12 - The cross-variograms of the MAF factors. The red lines correspond to F_1-F_2, the purple lines correspond to F_1-F_3, the yellow lines correspond to F_1-F_4, the black lines correspond to F_2-F_3, the green lines correspond to F_2-F_4 and the blue lines correspond to F_3-F_4.

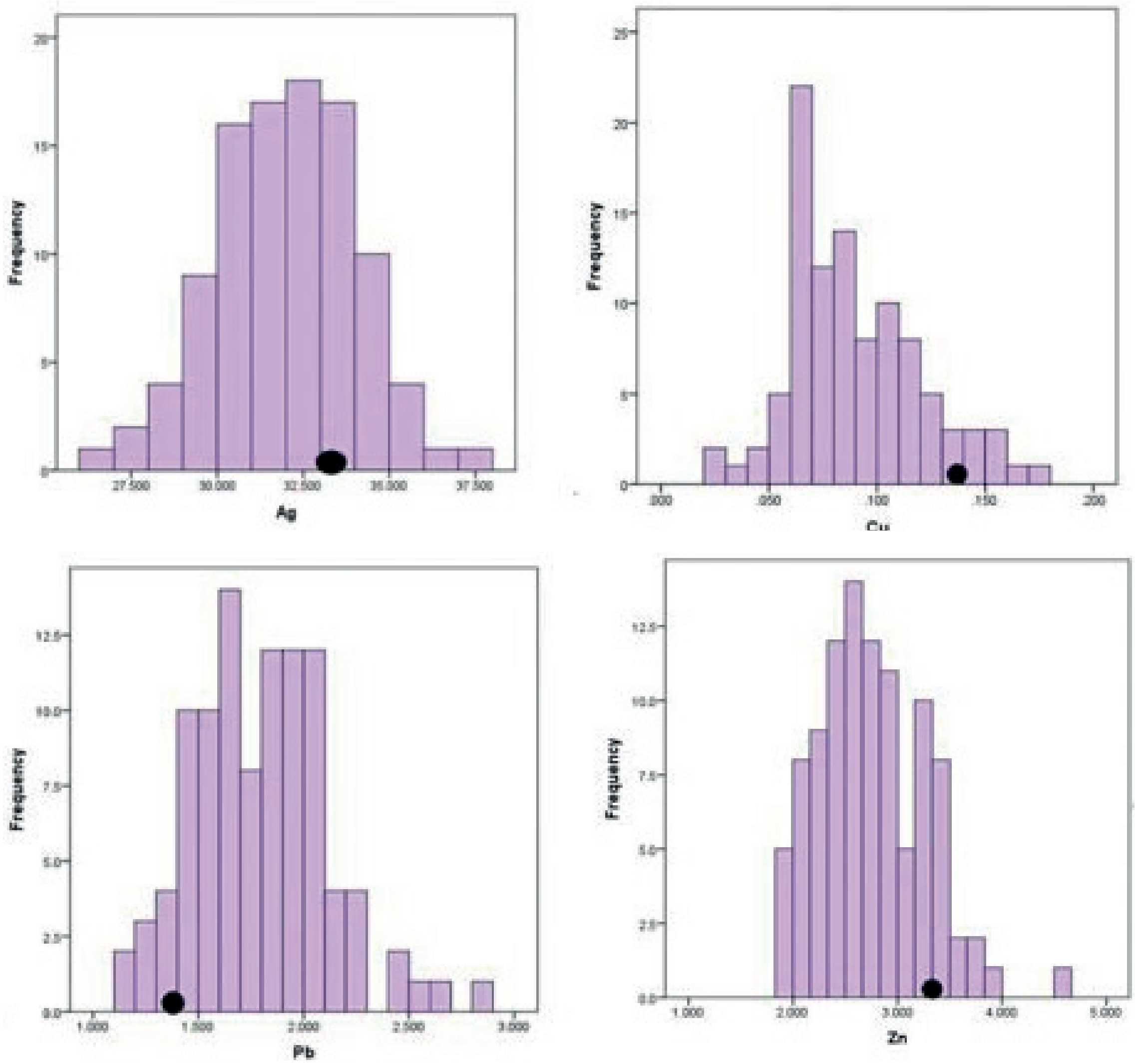

Fig. 13 - The mean histogram of simulated back-transformed variables. The black dots are the original averages of the variables.

to transform the variables into the original distribution. The validation of the results was done by comparison of the mean values and correlations of original data with the mean and correlation histograms of the simulated data (Figs. 13 and 14). The results represent poor relations between the actual mean and correlations with the simulated ones. However, the experimental variograms of the back-transformed simulated variables present good coincidence with the original variograms (Fig. 15).

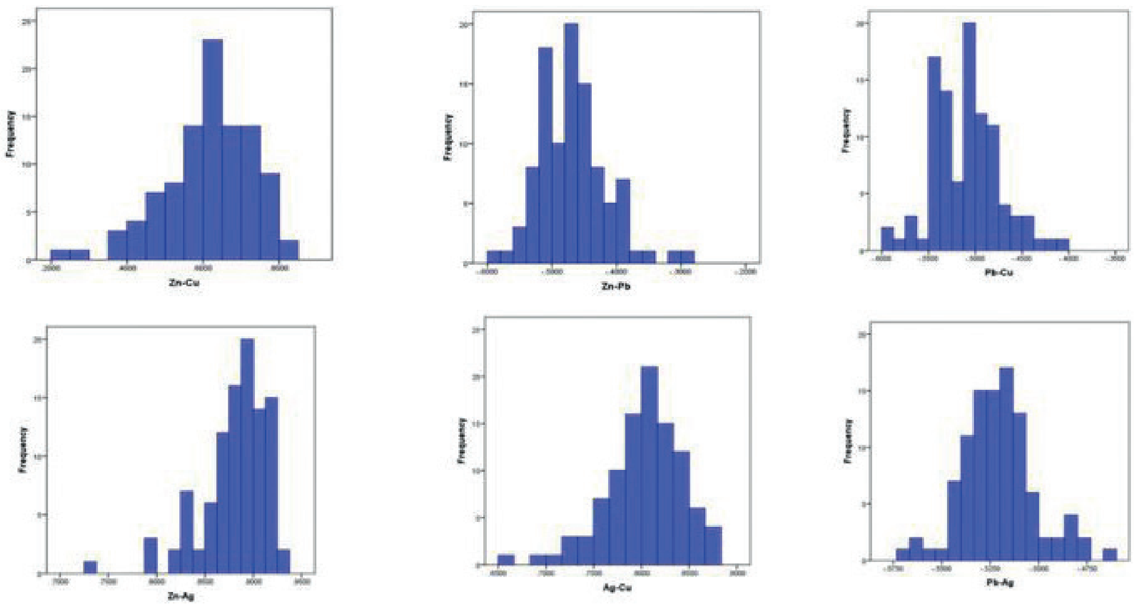

Fig. 14 - Histograms of the correlation coefficients calculated through 100 simulation realisations of the MAF transformed variables. The original correlations do not match the simulated ones at all. The original correlations of Zn-Cu, Zn-Pb, Pb-Cu, Zn-Ag, Ag-Cu, and Pb-Ag are -0.25, 0.57, 0.02, 0.48, 0.21 and 0.74, respectively.

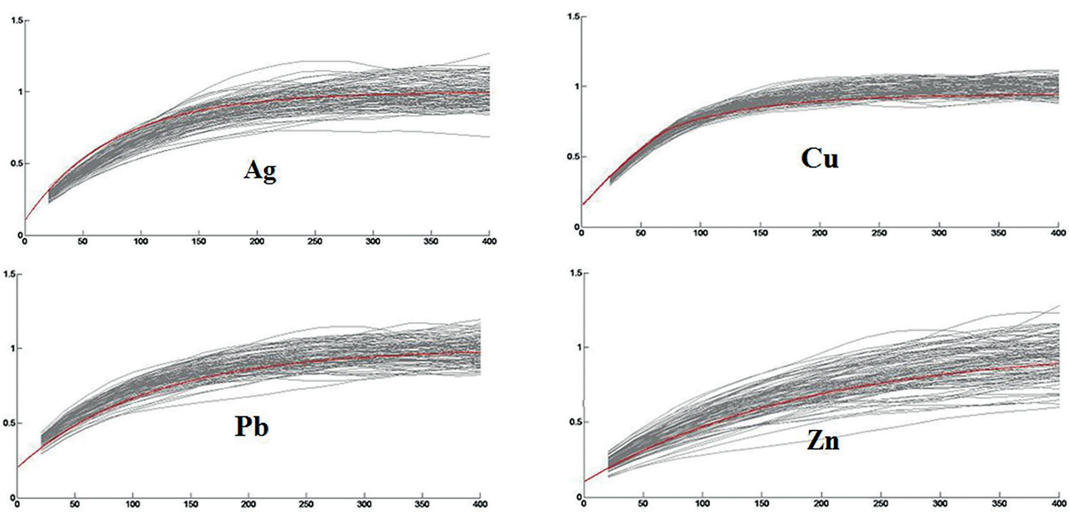

Fig. 15. The experimental variograms of the MAF simulated back-transformed variables for 100 realisations.

4.3. Application of PPMT

The PPMT, as the last method used in this study, has the capability of normalising and uncorrelating both variables together. This method was applied to the four variables and transformed them into the Gaussian distribution in an excellent way after 25 iterations. Also, the scatter plots of the transformed variables illustrate very well the decorrelation of all variables and Spearman correlation up to 2 decimal place is zero (Fig. 16). The cross-variograms of the transformed variables also present fine decorrelations of the variables for different lag distances (Fig. 17).

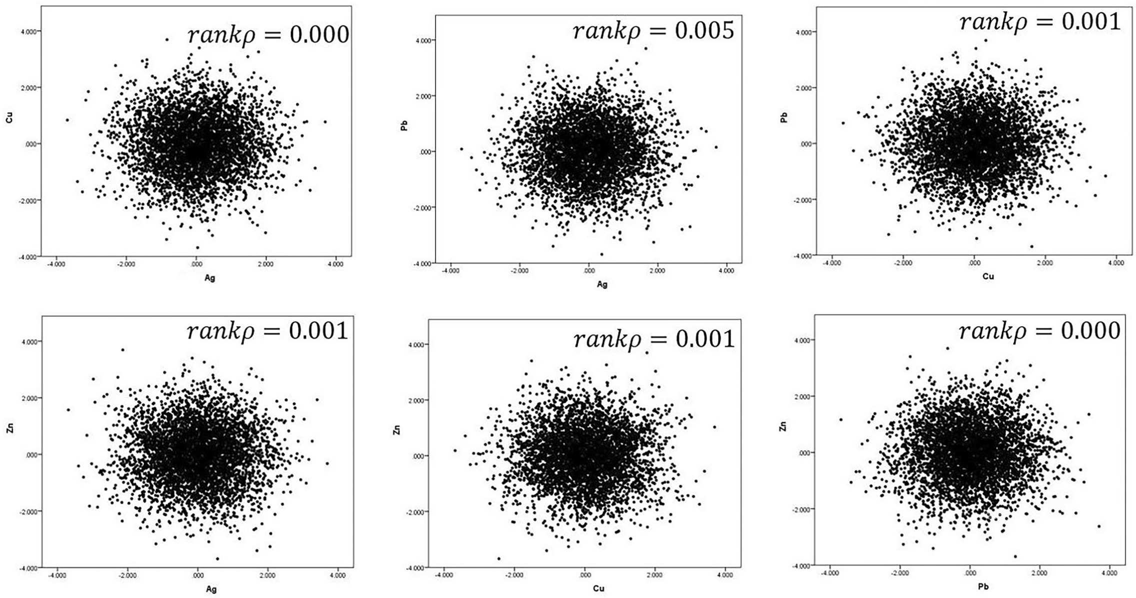

Fig. 16 - The scatter plots of the PPMT transformed variables along with the Spearman correlations between them.

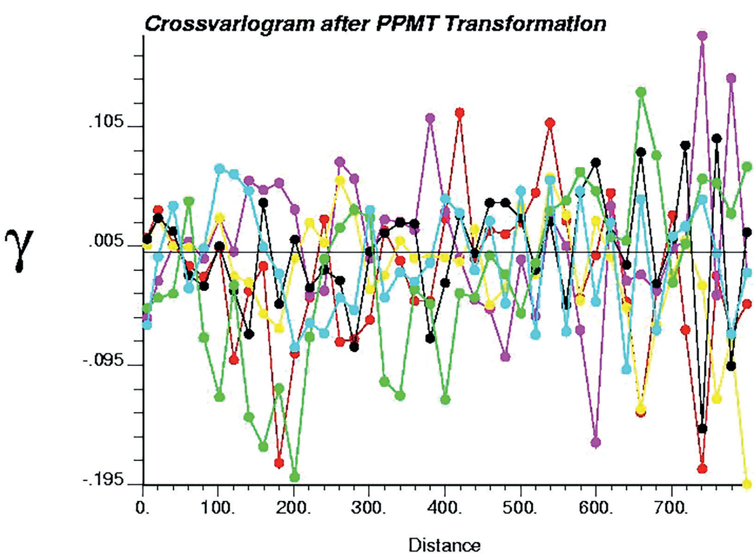

Fig. 17 - The cross-variograms of the PPMT transformed variables. The red line correspond to Pb-Zn, the purple lines correspond to Zn-Cu, the yellow lines correspond to Zn-Ag, the black lines correspond to Pb-Cu, the green lines correspond to Pb-Ag and the blue lines correspond to Cu-Ag.

Similar to the two previous methods, the transformed variables were simulated for 100 realisations using SGS. The validation of the results was done by comparison of the mean values and correlations of original data with the mean and correlation histograms of the simulated data (Figs. 18 and 19). The results represent very well the coincidence between the actual mean and correlations with the simulated ones. Also, the experimental variograms of the simulation realisations have an appropriate match with the original data variograms (Fig. 20).

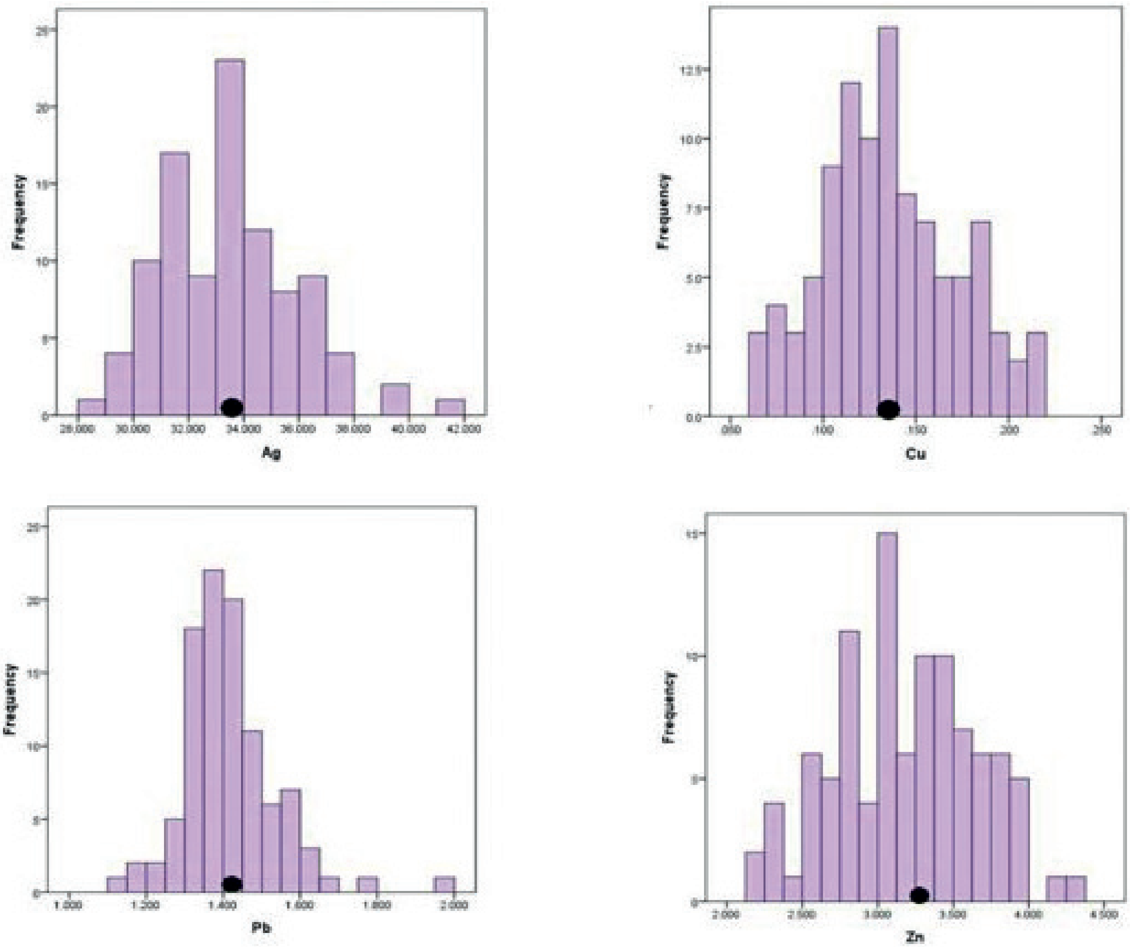

Fig. 18 - The mean histogram of the PPMT simulated back-transformed variables. The black dots are the original averages of the variables.

5. Discussion

In the previous section, three multivariate transformation methods were applied to a data set containing Pb, Zn, Cu, and Ag with the purpose of stimulating the interested variables in an uncorrelated space. Also, the performance of each method was evaluated by several factors which are the decorrelation of the variables in lag zero, the decorrelation of the variables in space, the reproduction of the original mean and correlations of the variables and, also, the reproduction of the original variograms parameters.

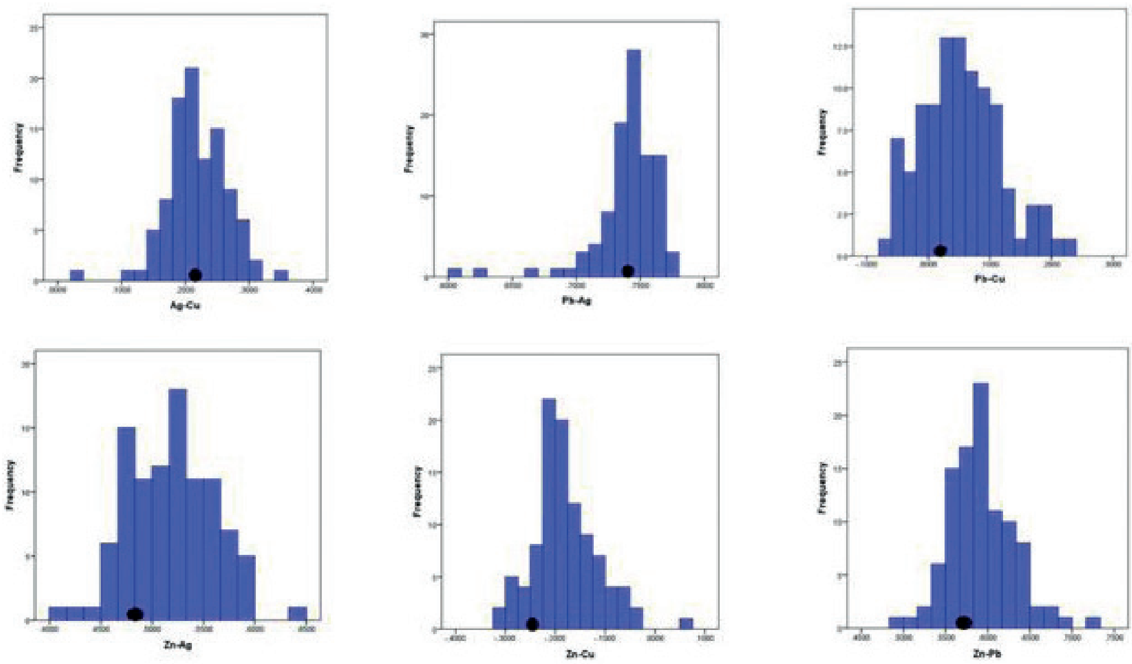

Fig. 19 - Histograms of the correlation coefficients calculated through 100 simulation realisations of the PPMT transformed variables. The black dots are the original correlations.

The statistical decorrelation of the variables (lag zero) was investigated via the scatter plots of the transformed variables. The results showed that the three methods were almost successful except a specific case for Ag-Cu which was related to the SCT. Note that these variables were transformed in different series due to the weakness of SCT in transforming four-variable data set. Besides, the scatter plots of the MAF transformed factors did not follow a multi-Gaussian

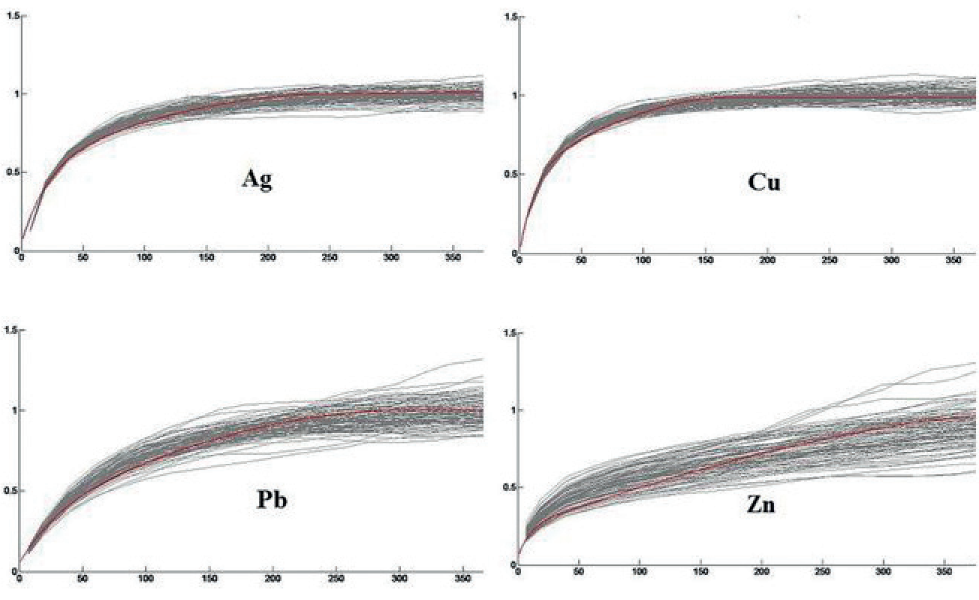

Fig. 20 - The experimental variograms of the PPMT simulated back-transformed variables for 100 realisations.

distribution. The linear nature of the MAF may cause the problem to emerge. Generally, PPMT provides better results for all variables in comparison with other methods (Figs. 6, 11, and 16).

The geospatial decorrelation of the variables (for all lag distances) was explored using the experimental cross-variograms of the transformed variables. Reaching the variogram values to about 0.3 for several increments shows that the SCT did not spatially decorrelate the variables well, especially for Ag-Cu. However, the MAF could decorrelate the factors better than the SCT. Among the methods, PPMT gained more acceptable decorrelations due to the lower values of the cross-variograms for all lags (Figs. 7, 12, and 17).

The capability of the original mean and correlation reproduction were other factors which were assessed through 100 simulation realisations in this study. The results showed that the SCT reproduced the original mean and correlations reasonably while MAF was not able to do the same due to the linear nature of the MAF. But, in contrast with the MAF, the PPMT reproduced the mentioned parameters more accurately (Figs. 8, 9, 13, 14, 18, and 19). Furthermore, the variogram parameters reproduction was the last validation to be utilised in this study. The experimental variograms of the variables for all 100 realisations were calculated and compared to the original one. The results demonstrated that all the methods could appropriately reproduce the original data variograms (Figs. 10, 15, and 20). Finally, considering all the validation results, it was concluded that the PPMT is the optimum method for modelling multivariate deposits.

Besides, for elaborating the effects of the type of the transformations on mineral resources modelling, the grades and tonnages of the interested variables were calculated using the simulated transformed variables, and the grade-tonnage curves were achieved based on several cut-off grades subsequently (Fig. 21). To reach this purpose, the 100 simulation realisations were averaged

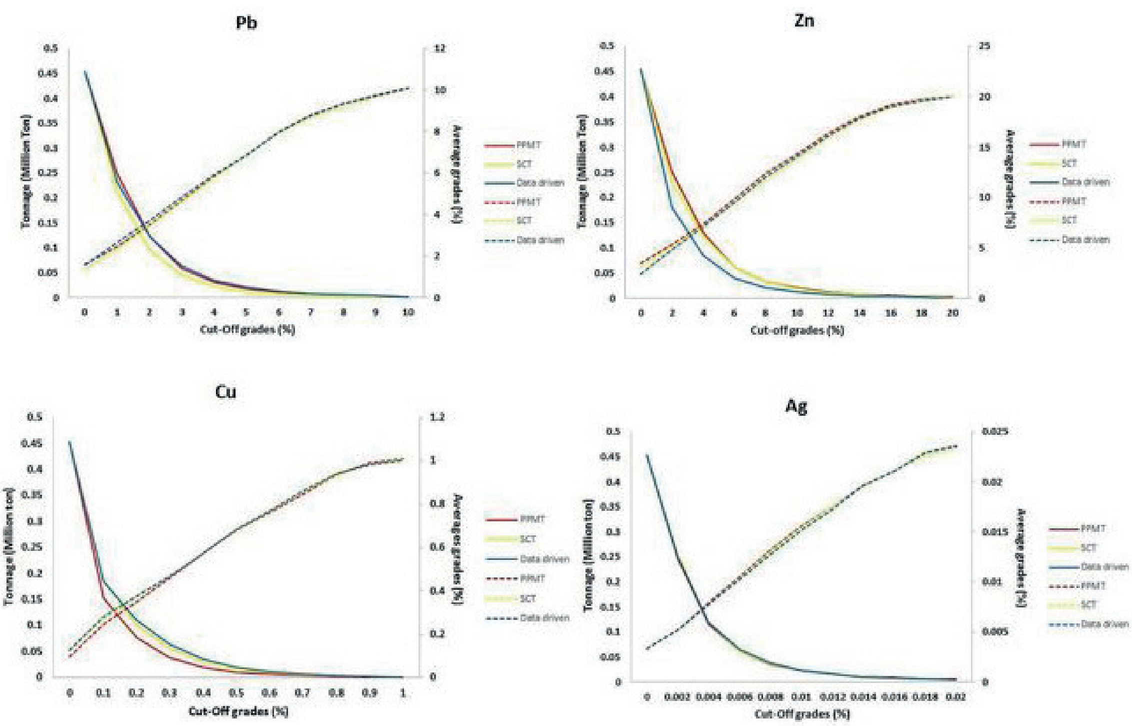

Fig. 21 - The grade-tonnage curves of the transformed variables based on the averages of the simulations.

(E-type map) for each variable in order to present a representative value for each block. Besides, the total tonnage of the zone 220 of the Mehdiabad deposit was calculated as 0.45 MT by simply multiplying the volumes of the blocks by the density of the rock type. According to Fig. 19, there are differences between the tonnages of the variables for different applied transformations. It can be inferred that the SCT and MAF transformations may underestimate the tonnages for Pb and Zn as the major elements while they overestimate the tonnages for Cu comparing to the PPMT results. All the transformations provided similar results for simulation of the Ag tonnages. To describe such discrepancies in detail, the tonnages and average grades of the elements for specific cut-off grades are demonstrated in Table 3. Also, the differences between the SCT and MAF results with the PPMT outcomes are calculated. These discrepancies may strongly affect the future mine planning.

Table 3 - The differences of the SCT and MAF tonnages and grades with respect of the PPMT results.

	Elements	Cut-off (%)	Tonnage difference (MT)	Ave Grade difference	Tonnage difference (%)	Grade difference (%)
PPMT VS SCT	Pb	2.00	0.03600	0.132000	14.500	5.36
	Zn	2.00	0.01900	0.112000	7.600	2.11
	Cu	0.20	-0.02400	-0.018000	30.900	5.40
	Ag	0.01	0.00022	0.000167	0.980	1.07
PPMT VS MAF	Pb	2.00	0.01700	-0.168000	6.976	6.83
	Zn	2.00	0.07000	0.455000	28.030	8.57
	Cu	0.20	-0.03000	-0.026000	41.540	7.60
	Ag	0.01	0.00063	0.000423	2.780	2.71

Moreover, the statistical parameters of the simulated back-transformed variables are present in Table 4. The results of the PPMT simulations indicated perfect proximities between the original mean and variance of the variables and the ones regarding the averaged simulation (Tables 1 and 4). However, there are differences between the statistics of the SCT and MAF simulations against the original ones in which SCT yielded more acceptable results comparing to the MAF.

Table 4 - The statistical parameters of the simulated back-transformed variables (average of the simulations).

Transformation	Elements	Number of blocks	Mean	Variance	Min	Max	Upper quartile	Lower quartile
PPMT	Pb (%)	167282	1.40	2.18	0	10.12	1.94	0.4500
	Zn (%)	167282	3.28	12.37	0	20.00	4.50	0.8600
	Ag (ppm)	167282	31.74	1229.23	0	243.6	40.69	11.7000
	Cu (%)	167282	0.143	0.02	0	1.01	0.15	0.0040
SCT	Pb (%)	167282	1.33	2.00	0	10.12	1.80	0.4200
	Zn (%)	167282	3.07	10.47	0	20.00	4.28	0.8000
	Ag (ppm)	167282	33.35	1261.90	0	243.6	41.19	11.9000
	Cu (%)	167282	0.13	0.03	0	1.01	0.18	0.0015
MAF	Pb (%)	167282	2.21	4.22	0	10.12	2.90	0.7800
	Zn (%)	167282	2.82	10.27	0	20.00	3.83	0.7000
	Ag (%)	167282	28.40	1013.80	0	243.6	34.91	9.7000
	Cu (%)	167282	0.055	0.01	0	1.01	0.07	0.0015

6. Conclusions

Joint simulation of the interested variables in multivariate mineral deposits can be addressed using multivariate analysis methods that aim to remove multivariate complexity and transform the attributes to an orthogonal space preparing them for further modelling and analysis. In this study, three different multivariate methods, namely SCT, MAF, and PPMT, were used for joint simulation of a data set containing Pb, Zn, Cu, and Ag related to the zone 220 of Mehdiabad deposit. The efficiency of the mentioned methods was assessed by several tools. Generally, the results showed that the PPMT yielded more reliable outcomes comparing to other two approaches. It decorrelated the variables very well both in lag zero and in other lag distances. Also, the simulated PPMT transformed variables and reproduced the original mean and correlations of the variables acceptably. Moreover, the experimental variograms of the simulation realisations had appropriate coincidence with the variograms of raw data. Moreover, the grade-tonnage curves per variables were calculated based on the averaged of the simulations in which SCT and MAF transformations may underestimate the tonnages for Pb and Zn as the major elements, while the tonnage of copper is overestimated comparing to the PPMT results. Also, the statistics of the back-transformed simulation results proved that the PPMT simulations could greatly reproduce the original statistics.

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