Bollettino di Geofisica Teorica ed Applicata Vol. 58, n. 1, pp. 55-73; March 2017
DOI 10.4430/bgta0189

Integration of aeromagnetic geophysical data with other exploration data layers based on fuzzy AHP and C-A fractal model for Cu-porphyry potential mapping: a case study in the Fordo area, central Iran

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(Received: January 21, 2017; accepted: April 22, 2017)

ABSTRACT Today, using only one geophysical layer creates a large percentage of errors in the mineral exploration field. For this reason, the integration of different exploration layers with the geophysical layer for mineral exploration is suggested. The fuzzy analytical hierarchy processes (AHP) method is a manner of mineral prospectivity mapping (MPM) that is generally used for mineral exploration. This method has been applied for multi-criteria decision-making problems. In this paper, fuzzy AHP and geospatial information system (GIS) were used to generate a prospectivity model for Cu-porphyry mineralization on the basis of a conceptual model and geo-evidence layers derived via geological, geochemical, and geophysical data in the Fordo area, central Iran. Fuzzy AHP was utilized to determine the weights belonging to each criterion. Three geoscientists' knowledge of exploration of Cu-porphyry mineralization has been applied to assign weights to evidence layers in fuzzy the AHP MPM method. After assigning normalized weights to all evidential layers, a fuzzy operator was applied to integrate weighted evidence layers. The results demonstrate the acceptable outcomes for Cu-porphyry exploration. For further evaluation of the prospectivity model, C-A fractal model was used for the MPM. Finally, the C-A prospectivity map was confirmed by checking three target areas in the field.

Key words: fuzzy analytical hierarchy process, mineral prospectivity mapping, C-A fractal, Cu-porphyry, central Iran.

1. Introduction

Discovering new mineral deposits and diagnosing prospective zones within the region of interest is the ultimate purpose of mineral exploration. To achieve this goal, multiple data sets, or layers should be collected, analyzed, and integrated for mineral prospectivity mapping (MPM) in the region of interest (Bonham-Carter, 1994; Carranza, 2008). As a result of the elaboration of geological domains, there is ambiguous knowledge on evaluating the relative importance of geo-evidential features as indicators for prospecting a deposit type. The fuzzy analytical hierarchy processes (AHP) MPM method, introduced in this paper, can assign more realistic weights to individual evidence layers in comparison with other MPM methods.

MPM generates a predictive model for tracing prospective regions as a multiple criteria decision-making (MCDM) function. Using this method, producing evidential maps, combining evidential maps, and finally ranking promising target areas for further exploration have been performed (Bonham-Carter, 1994; Carranza, 2008; Yousefi *et al.*, 2012; Ghodratabadi and Feizi, 2015; Feizi *et al.*, 2017). There are two types of methods to assign evidential weights and combine evidential maps for MPM: knowledge- and data-driven. Both are used for MPM (Bonham-Carter, 1994; Pan and Harris, 2000; Carranza, 2008). Fuzzy AHP (FAHP) is a knowledge-driven method.

Knowledge-driven MPM methods are proper in less-explored areas with no or only a few known mineral deposits of the type sought (Bonham-Carter, 1994; Carranza, 2008). In these MPM approaches, a geoscientist's expert judgement is applied to weight evidential features. Boolean logic, index overlay, evidential belief functions, and fuzzy logic (Chung and Moon, 1990; Moon, 1990; An *et al.*, 1991; Bonham-Carter, 1994; Carranza, 2008) are examples of knowledge-driven methods.

Because of AHP limitations, the fuzzy modification of AHP (FAHP) was proposed and is the subject of this study. This paper shows implementation of the FAHP method for discovering new mineral deposits (specifically Cu-porphyry) and analysing prospective zones within the region of interest. With the idea of reducing the costs of exploration, the knowledge of three decision makers (DMs) who have major experience in Cu-porphyry exploration was called upon to allocate weights to evidential layers in the FAHP method. Then a fuzzy operator analysed the weighted evidential layers. In this paper, we used different evidence layers consisting of geological, geochemical, and geophysical data to generate prospectivity of Cu-porphyry mineralization in the Fordo prospecting area, Qom Province, Iran. Finally, we evaluated the generated prospectivity model using the location of the known mineral occurrences as testing points. For further evaluation of the prospectivity, we used a combination of concentration-area (C-A) fractal model with MPM for generating C-A mineral prospectivity. Finally we evaluated the generated prospectivity model using the location of the known mineral occurrences as testing points and three target areas.

2. The study area and geological setting

The Fordo area is part of the Orumieh-Dokhtar magmatic arc in central Iran. The location of the studied area in the physiographic-tectonic zoning map of Iran is illustrated in Fig. 1. This magmatic arc is the most important metallogenic area within the region and hosts the majority of the larger metal deposits such as copper and iron (Hassan-Nezhad and Moore, 2006). The studied area is located in the south of the Qom province. The most impressive lithological features in the studied area are the volcanic rocks with basic combinations (Ghalamghash and Babakhani, 1996). These features are included: andesite, andesibasalt, and megaporphytite andesite. There are tuffaceous sandstone and limestones in the east and SE of the investigated area. Hematitation, limonitation, silisification and argillic alterations are visible in central parts that have been formed in in a N-S trend in the east and NE of the studied area (Mansouri and Feizi, 2016). Based on a 1:100,000 geological map of Kahak, three NE-SW faults have been seen. According to study results conducted around the faults, alterations and ferrous fluid have been observed (Fig. 1). The existence of Cu-porphyry mineralization is predictable due to the eroded ring alterations (argillic, phyllic, propilitik), which can be detected by satellite images. Also, stockworks are

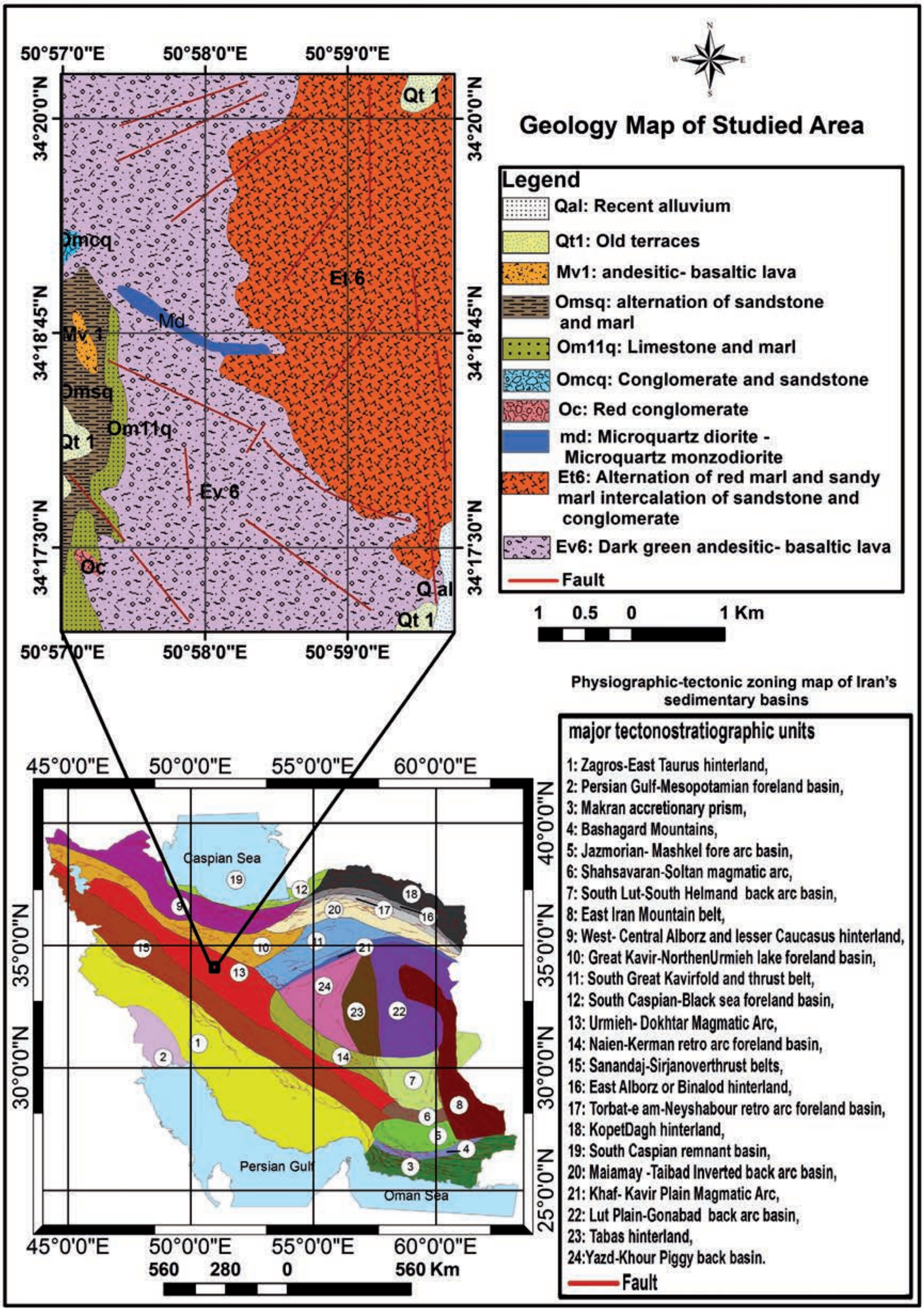

Fig. 1 - Physiographic-tectonic zoning map of Iran's sedimentary basins (modified from Arian, 2013) and location of the study area.

visible around the microquartz diorite - microquartz monzodiorite intrusive rocks in the Fordo area (Ghalamghash and Babakhani, 1996). Cu-porphyry deposits can be distinguished from other granite-related deposits by specific features such as stockworks (Sillitoe, 2010).

3. Methods and results

3.1 Analytical hierarchy process (AHP) method

AHP is a multi-criteria decision method that uses hierarchical structures to represent a problem and then develops priorities for alternatives based on the judgement of the user (Saaty, 1980). AHP involves the three basic steps comprising construction of a hierarchy, priority setting, and logical consistency (Macharis *et al.*, 2004). These steps are described in the following.

3.1.1. Construction of a hierarchy

In this step, the complex problem is decomposed into a hierarchical structure with decision elements including objective and attributes (i.e., criterion map layer and alternatives). The hierarchical structure which is used to generate MPM in this study is illustrated in Fig. 2.

3.1.2. Priority setting

The method of deriving evidential weights via AHP involves pairwise comparisons of criteria according to their relative importance with respect to a proposition (Carranza, 2008). The DM uses a standardized comparison scale of nine levels that is shown in Table 1 (Saaty, 2005; Dăgdeviren, 2008).

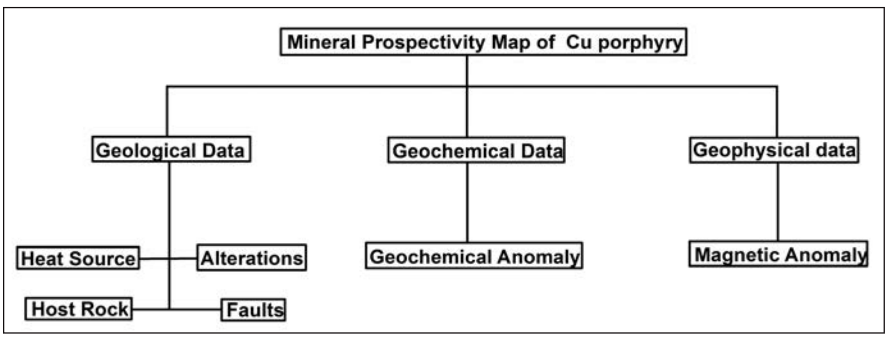

Fig. 2 - Hierarchy used for prospectivity mapping.

Table 1 - Scales for pairwise comparison (Saaty, 1980).

Preferences expressed in numeric variables	Preferences expressed in linguistic variables
1	Equal importance
3	Moderate importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2, 4, 6, 8	Intermediate values between adjacent scale values

Let $C = \{C_j | j = 1, 2, ….., n\}$ be the set of criteria. The results of the pairwise comparison on n criteria can be summarized in an $(n \times n)$ evaluation matrix A, in which every element a_{ij} $(i, j = 1, 2, ….., n)$ is the quotient of weights of the criteria as shown in Eq. 1 (Dăgdeviren, 2008):

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \qquad a_{ii} = 1, \qquad a_{ji} = \frac{1}{a_{ij}}, \qquad a_{ij} \neq 0 \tag{1}$$

The mathematical process commences to normalize and find the relative weights for each matrix. The relative weights are given by the right eigenvector (w) corresponding to the largest eigenvalue (λ_{max}) as:

$$Aw = \lambda_{max} W \tag{2}$$

If the pairwise comparisons are completely consistent, the matrix A has rank 1 and $\lambda_{max} = n$. In that case, weights can be obtained by normalizing any of the rows or columns of A matrix (Dăgdeviren, 2008).

3.1.3. Logical consistency

The quality of the output of the AHP is strictly related to the consistency of the pairwise comparison judgments. The consistency is defined by the relation between the entries of A as follows:

$$a_{ij} \times a_{jk} = a_{ik} \tag{3}$$

A $n \times n$ matrix (n = number of factors or criteria), such as a pairwise comparison matrix, is consistent if it has one eigenvalue with a value equal to n; otherwise it has at most n eigenvalues with values varying around n (Saaty, 1977). When the pairwise comparison matrices are completely consistent, the priority (or weight) vector corresponds to the right eigenvector (w). Therefore, the highest eigenvalue (λ_{max}) is equal to n. In case the inconsistency of the pairwise comparison matrices is limited, λ_{max} deviates slightly from n. This deviation ($\lambda_{max} - n$) is used as a measure for inconsistency. This measure that is divided by ($n - 1$) yields the average of the other eigenvectors (Macharis *et al.*, 2004). The consistency index (CI) is:

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{4}$$

The final consistency ratio (CR), on the basis of which one can conclude whether the evaluations are sufficiently consistent, is calculated as the ratio of the CI and the random index (RI is given in Table 2) and it corresponds to the degree of consistency that automatically arises when completing at random reciprocal matrices with the values on the 1-9 scale (Macharis *et al.*, 2004):

$$CR = \frac{CI}{RI} \tag{5}$$

Table 2 - Some random inconsistency indices (RI) generated by Saaty (1977).

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The number 0.1 is the accepted upper limit for *CR*. If the final *CR* exceeds this value, the evaluation procedure has to be repeated to improve consistency. The measurement of consistency can be used to evaluate the consistency of DMs as well as the consistency of all the hierarchy (Dăgdeviren, 2008).

3.2. FAHP method

In spite of the popularity of AHP, this method is often criticized for its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision maker's perception to exact numbers (Deng, 1999). Since fuzziness and vagueness are common characteristics in many decision-making problems, a FAHP method should be able to tolerate vagueness or ambiguity (Mikhailov and Tsvetinov, 2004). In other words, the conventional AHP approach may not fully reflect a style of human thinking because the decision makers usually feel more confident to give interval judgements rather than expressing their judgements in the form of single numeric values. FAHP, however, is capable of capturing a human's appraisal of ambiguity when complex multi-attribute decision-making problems are considered (Erensal *et al.*, 2006). This ability comes about when the crisp judgements are transformed into fuzzy judgements. Zadeh (1965) published his work on fuzzy sets, which described the mathematics of fuzzy set theory. This theory, which was a generalization of classic set theory, allowed the membership functions to operate over the range of real numbers (0, 1). The main characteristic of fuzziness is the grouping of individuals into classes that do not have sharply defined boundaries (Hansen, 2005). The uncertain comparison judgement can be represented by the fuzzy number. In the fuzzy extension of AHP, the weights of the nine-level fundamental scales of judgments are expressed via the triangular fuzzy numbers (TFN) in order to represent the relative importance among the hierarchy criteria (Karimi *et al.*, 2011) (Fig. 3). A fuzzy number M on $R(M\in M(R))$ is a TFN if its membership function $\mu Mx{:}R\rightarrow(0, 1)$ is equal to

$$\mu_M(X) = \begin{cases} (x-l)/(m-l), & l \le x \le m \\ (u-x)/(u-m), & m \le x \le u \\ 0 & otherwise \end{cases} \tag{6}$$

where $l\le m\le u$, l and u stand for the lower and upper value of the support of M respectively, and m gives the modal value of the membership function $\mu M(x)$. Here $M(R)$ represents all fuzzy sets, and R is the set of real numbers. The triangular fuzzy number can be denoted by (l, m, u). The support of M is the set of elements $X\in R1<x<u$.

The steps of applying FAHP that are suggested by Chang (1996) are used in this study. The paper published by Karimi *et al.* (2011) is used to describe these steps as follows.

Step 1: a group of t DMs is used to prepare pairwise comparisons. Each DM individually will construct a pairwise comparison matrix (PCM), as shown in Eq. 7 for each criterion:

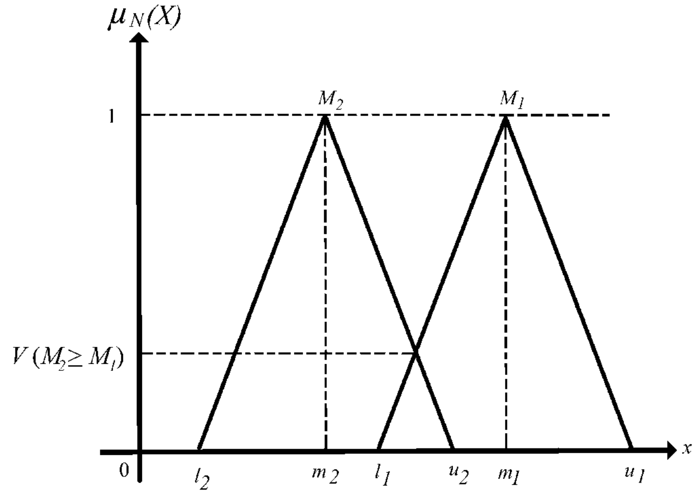

Fig. 3 - Evaluation of two TFNs M1 and M2.

$$DM_p = \begin{bmatrix} a_{11p} & a_{12p} & \cdots & a_{1mp} \\ a_{21p} & a_{22p} & \cdots & a_{2mp} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1p} & a_{n2p} & \cdots & a_{nmp} \end{bmatrix} \quad p = 1,2,\dots,t \tag{7}$$

where m is the number of alternatives for each criterion and t is the number of DMs. For instance in Fig. 2, the number of alternatives m for geological data criterion is 3. Then, a comprehensive PCM is constructed by integrating the grades of all DMs via Eq. 8. In this way, the PCM values of DMs are transformed into TFN to make the fuzzy evaluation matrix:

$$l_{ij} = \min(a_{ijp}), m_{ij} = \sum_{p=1}^{t} \frac{a_{ijp}}{t}, u_{ij} = \max(a_{ijp}), M_{ij} = (l_{ij}, m_{ij}, u_{ij}), \ p = 1,\dots,t, \ i = 1,\dots,m, \ j = 1,\dots,m \tag{8}$$

where min (a_{ijp}) and max (a_{ijp}) indicate minimum and maximum values of PCMs prepared by DMs for each i, j respectively. This step is used to construct fuzzy evaluation matrices with respect to criteria and alternatives of designed hierarchy.

Step 2: the value of the fuzzy synthetic extent with respect to the ith object of m alternatives for each criterion is defined by the following:

$$S_i = \sum_{j=1}^{m} M_{ij} \otimes \left[\sum_{k=1}^{m}\left(\sum_{j=1}^{m} M_{KJ}\right)\right]^{-1}, \ i,j,k = 1,\dots,m \tag{9}$$

where all the Mij are TFNs after construction of the fuzzy evaluation matrix. The symbol $\otimes$ indicates the fuzzy multiplication operator. Considering two TFNs $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$, their operational laws are as follows:

$$(l_1, m_1, u_1) \otimes (l_2, m_2, u_2) \approx (l_1 l_2, m_1 m_2, u_1 u_2) \tag{10}$$

$$(l_1, m_1, u_1)^{-1} = (1/l_1, 1/m_1, 1/u_1) \tag{11}$$

Step 3: as $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two TFNs, the degree of possibility (V) of $M_2 \geq M_1$ is defined by:

$$V(M_2 \geq M_1) = \begin{cases} 1 & if\ m_2 \geq m_1 \\ 0 & if\ l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise \end{cases} \tag{12}$$

Fig. 3 represents the evaluation of two TFNs M_1 and M_2.

Step 4: to compare M_1 and M_2, it is necessary to consider both values of $V(M_2 \geq M_1)$ and $V(M_1 \geq M_2)$. The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $Mi(i = 1,\ldots, k)$ can be defined by:

$$V(M \geq M_1, \ldots, M_k) = V[(M \geq M_1)\ and\ (M \geq M_2)\ and\ \ldots\ (M \geq M_k)] = \min V(M \geq M_i),\ i = 1, \ldots, k \tag{13}$$

Assume that: $d(B_i) = \min V(S_i \geq S_k)$, $k = 1,\ldots, m$; $k \neq i$. Then the weight vector is given by:

$$W' = (d'(B_1), \ldots, d'(B_m))^T \tag{14}$$

where B_i (i=1,. . ., m) are m elements.

Step 5: via normalization, the normalized weight vectors are:

$$W = (d(B_1), \ldots, d(B_m))^T \tag{15}$$

where W is a non-fuzzy number.

The point should be noted that the extent analysis method on FAHP is a reliable method except in cases where irrational zero weight is assigned to some useful decision criteria and alternatives. The weights determined by the extent analysis method in such cases do not represent the relative importance of decision criteria or alternatives and cannot be used as their priority. For integrating the generated continuous fuzzy evidential maps, we used a fuzzy operator for comparison purposes. A variety of operators can be applied, such as “And”, “Or”, “Algebraic Product”, “Algebraic sum”, and “Gamma” operators (Bonham-Carter, 1994; Carranza, 2008). The idea of using fuzzy logic in mineral potential mapping is to consider the spatial objects on a map as members of a set. In classical set theory, the membership of a set is defined as true or false (1 or 0), whereas membership of a fuzzy set (μ) is expressed on a continuous scale from 0 to 1 (Bonham-Carter, 1994). The values of fuzzy membership can be chosen based on the subjective judgement of an analyst (Nykänen *et al.*, 2008).

4. Results and discussion

4.1. Cu-porphyry mineralization model

Cu-porphyry deposits are formed from postmagmatic hydrothermal fluids related to granitoid porphyritic intrusive rocks commonly having hydrothermal alterations. In Cu-porphyry deposits, primary ore minerals are dominantly structurally controlled and are spatially and genetically associated with felsic to intermediate porphyritic intrusions (Sillitoe, 2010). Therefore, proximity to intrusive contacts (endo- and exo-contacts) represents favourability for Cu-porphyry mineralization (Yousefi and Carranza, 2015). Cu-porphyry deposits can be distinguished from other granite-related deposits such as skarns and mantos by their large size and structural control, mainly stockworks, porphyry stock, veins, vein sets, fractures, and breccias (Sillitoe, 2010). In the formation of Cu-porphyry deposits, once the magma solidifies, high-temperature fluids are released into the solidified porphyry and its surrounding host rocks. The fluids, which are often mineral-rich, take the path of least resistance and travel through cracks and fractures, which facilitate the passage of magmas and the circulation of hydrothermal fluids (Sillitoe, 1997). It is generally accepted that fault zones act as major channel ways for deeply sourced melts as well as hydrothermal fluids (Pirajno, 2010). Fault architecture has been used to investigate porphyry systems as described in many studies worldwide (Pirajno, 2010), as well as in the Urumieh-Dokhtar belt of Iran (Ghasemi and Talbot, 2006). Cu-porphyry deposits in the Urumieh-Dokhtar belt of Iran show geochemical halos of indicator elements, mainly Cu, Mo, Au, Ag, Zn, Pb, As, and Sb (Yousefi and Carranza, 2015).

4.2. Geo-exploration evidence layers

In this paper, geological, geochemical, and geophysical evidential data are selected considering the experiences of previous experiments of Cu-porphyry deposit exploration in the study area. For this, based on the experiences, the foregoing explanation of geological setting and deposit model of Cu-porphyry mineralization, and the available data in the Fordo area, we used the following geo-exploration evidential layers as the most significant regional scale criteria for prospecting the deposit type sought in the study area, lithology of intrusive rocks as heat sources, host rock lithology, alterations, faults, lithogeochemical anomalies interpreted from analyses of rock samples taken from outcropping rocks that extended in the study area, and airborne magnetic anomaly.

For obtaining the evidential layers of intrusive rocks as heat source and host rock lithology, these layers were generated from the geological map of the study area. In this paper, microquartz diorite - microquartz monzodiorite were presented as heat source lithology. In addition, dark green andesitic - basaltic lava were introduced as host rock lithology. Also, the density map of faults in the studied area was generated as a layer.

Alterations that are significant in Cu-porphyry prospecting were extracted by remote sensing. To separate alterations, a spectral analysis was carried out on the ASTER satellite imagery data of the investigated area to map spectral signatures associated with the hydrothermal alterations. Phyllic, Argillic, propylitic, and iron oxide were manifested as important alterations in Cu-porphyry. To separate alteration zones, False Color Composite (FCC), Principal Component Analysis (PCA), and Least Squares Fitting (LS-Fit) techniques have been applied on ASTER data and iron oxide, argillic, phyllic, and propylitic zones have been separated (Feizi and Mansouri 2012, 2013a, 2013b).

Lithogeochemical anomalies, which came from the results of analyzing 131 rock samples taken from outcropping rocks in the Fordo area, have been mapped using C-A fractal model. Cheng *et al*. (1994) proposed the C-A fractal model for separating geochemical and geophysical anomalies (Eq. 16). In this area, the element content of Cu, Pb, Zn, Ag, and Au in the samples were used as indicators for Cu mineralization. For this purpose, statistical societies of the mentioned elements were specified by using the C-A fractal method. Then, the last geochemical population (the highest priority) of Cu, Pb, Zn, Ag, and Au, were separated for utilize in integration.

Airborne geophysical magnetic data was used to define magnetic anomaly respectively with reduction-to-the-pole (RTP) technique. The RTP technique transforms total-magnetic-intensity (TMI) anomalies to anomalies that would be measured if the field were vertical (assuming there is only an inducing field). This RTP transformation makes the shape of magnetic anomalies more closely related to the spatial location of the source structure (Mansouri *et al.*, 2015; Golshadi *et al.*, 2016). This evidential map was categorized into two classes. In the fuzzy logic MPM method, based on the expert judgements, two, three, or another number of classes can be used for categorizing evidential features in evidential maps (Luo and Dimitrakopoulos, 2003; Porwal *et al.*, 2004, 2006).

After classifying the evidential features into classes in evidential maps, the classes must be assigned weights to appraise their relative importance for prospecting the demanded deposit type (Bonham-Carter, 1994; Carranza, 2008). To do this, classes of evidential maps were specified with scores within (1, 10) range [Table 3: Porwal *et al.* (2004, 2006)]. The determined weighted evidence layers used for MPM of Cu-porphyry deposit in the study area are shown in Fig. 4.

Table 3 - Summary of evidence maps, classes, and their corresponding weights for Cu-porphyry prospectivity mapping.

Data	Evidential layer	Class	Class score
Geological data	Heat source	microquartz diorite - microquartz monzodiorite	10
	Host rock	dark green andesitic - basaltic lava	10
	Alterations	Phyllic	10
		Argillic	8
		propylitic	6
		iron oxide	4
	Faults	10 m buffer	10
		10 - 30 m buffer	9
		30 - 50 m buffer	8
		50 - 70 m buffer	7
		70 - 90 m buffer	6
		90 - 110 m buffer	5
		110 - 130 m buffer	4
		130 - 150 m buffer	3
		150 - 170 m buffer	2
		170 - 190 m buffer	1
Geochemical data	Lithogeochemical sample	Cu	10
		Pb	8
		Zn	6
		Ag	4
		Au	2
Geophysical data	Airborne magnetic	Magnetic anomaly	10

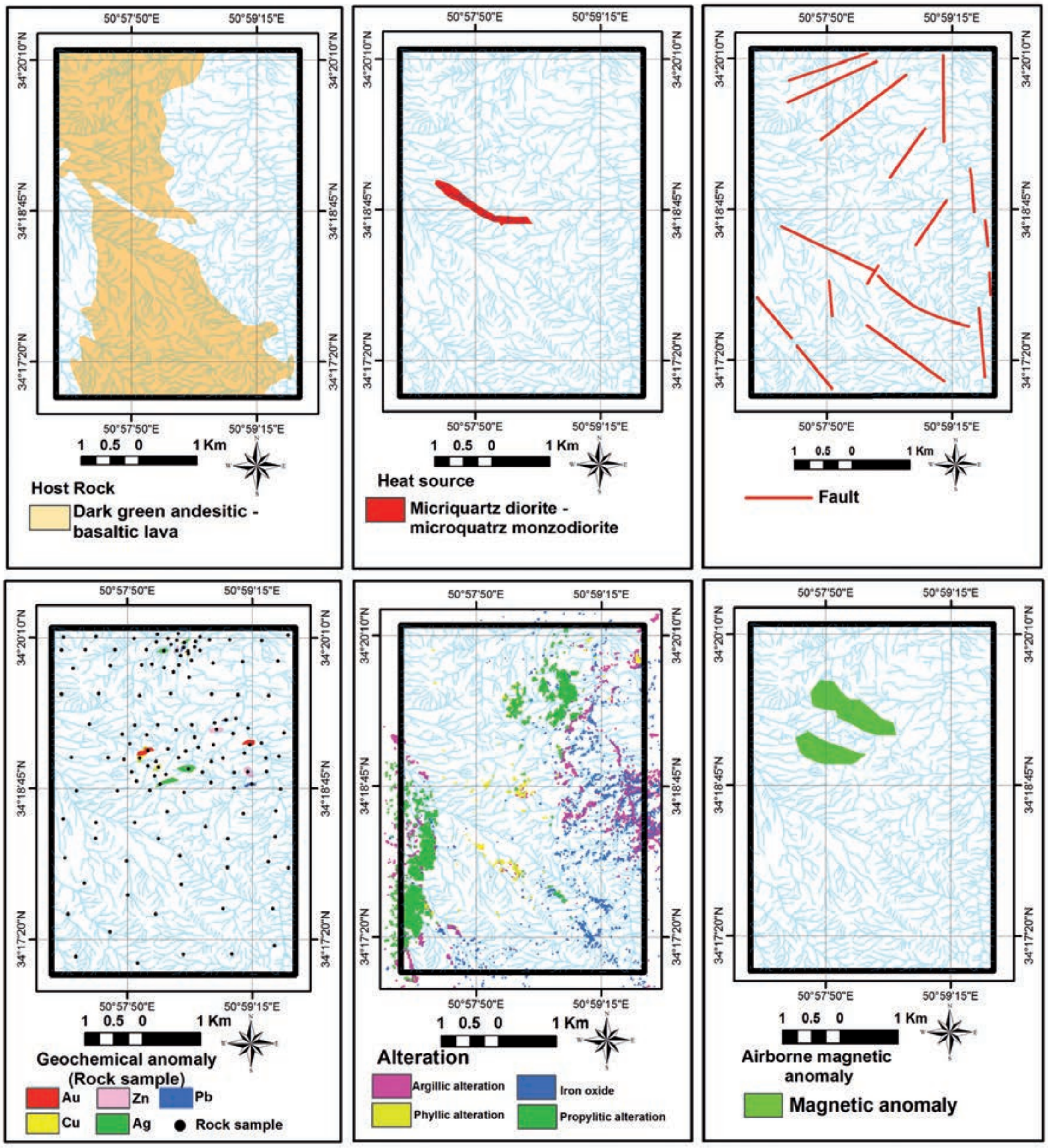

Fig. 4 - Derived geo-evidential layers used in FAHP MPM method.

In the fuzzy logic MPM method, an appropriate cell or pixel size should be used in GIS. This paper recommended a minimum pixel size with respect to requirements and accuracy of evidential layers. Hence, an appropriate cell size can be based on ASTER data pixel size, which is 15 m. Therefore, we used a pixel size of in all of the maps in this study. Furthermore, all pixel values of all evidential layers were normalized between ranges of 0 to 255. This range was selected because ASTER data Digital Numbers (DNs) are in this range. *Ergo*, pixel sizes and values of all evidential layers in this paper convert to ASTER data pixel sizes and values.

4.3. FAHP prospectivity model

4.3.1. Weighting of individual evidential layers

In this study, three DMs were used to create pairwise comparison matrices. After constructing the decision hierarchy for MPM, the criteria to be used in the appraisement process are the weights determined by the AHP method. Three DMs are given the task of constructing individual PCM by use of the scale in Table 1.

All consistency ratios obtained from the PCM for administering AHP are presented in Table 4. All of them are less than 0.1. So, the results are used to make fuzzy evaluation matrices (Table 5). Fuzzy evaluation or pairwise matrices with respect to the criteria of Cu-porphyry deposit, geology alternatives, constructed by DMs which are transformed into TFN is shown in Table 6. Because there is one alternative for both geochemistry and geophysics, pairwise matrices were not generated.

Table 4 - CR of the pairwise comparison matrix.

CI	Criteria Geology	Geochemistry	Geophysics
DM1		0.0469	
DM2		0.0171	
DM3		0.0723	
CI	Geological data Heat source	Alterations Faults	Host rock
DM1		0.0406	
DM2		0.0466	
DM3		0.0207	

Table 5 - Fuzzy evaluation matrix with respect to criteria of Cu-porphyry prospectivity model.

Criterion	Geological data	Geochemical data	Geophysical data
Geological data	(1, 1, 1)	(2.5, 2.6667, 3)	(3, 6, 8)
Geochemical data	(0.3333, 0.3750, 0.4)	(1, 1, 1)	(0.5, 3.1667, 5)
Geophysical data	(0.1250, 0.1667, 0.3333)	(0.2, 0.3158, 2)	(1, 1, 1)

Table 6 - Fuzzy evaluation matrix for geological alternatives.

Criterion	Heat source	Alterations	Faults	Host rock
Heat source	(1, 1, 1)	(2.5, 3.1667, 4)	(2.5, 3.6667, 4.5)	(1.5, 5.1667, 7)
Alterations	(0.25, 0.3158, 0.4)	(1, 1, 1)	(1.5, 2.3333, 3)	(0.2, 3.4, 6)
Faults	(0.2222, 0.2727, 0.4)	(0.3333, 0.4286, 0.6667)	(1, 1, 1)	(0.2, 2.5667, 4.5)
Host rock	(0.1429, 0.1935, 1)	(0.1667, 0.2941, 5)	(0.2222, 0.3896, 5)	(1, 1, 1)

Weights of eight evidential layers of information can be determined by FAHP. For example, the method of obtaining normalized weights from the fuzzy evaluation matrix with respect to geological data alternatives is proposed. From Table 6, the value of the fuzzy synthetic extent was calculated from Eq. 9 as follows:

$S_{Heat\ Source} = (7.5,13,16.5) \otimes (1/45.4667,1/26.1944,1/13.7373) = (0.1650,0.4963,1.2011)$

$S_{Alterations} = (2.95,7.0491,10.4) \otimes (1/45.4667,1/26.1944,1/13.7373 = (0.0649,0.2691,0.7571)$

$S_{Faults} = (1.7556,4.2650,6.5667) \otimes (1/45.4667, 1/26.1944, 1/13.7373) = (0.0386,0.1629,0.4780)$

$S_{Host\ rock} = (1.5317,1.8773,12) \otimes (1/45.4667,1/26.1944,1/13.7373) = (0.0337,0.0717,0.8735)$

Eq. 12 was used to compare these fuzzy values as follow:

$d_{(S\ Heat\ Source > S\ Alterations)} = 1$, $d_{(S\ Heat\ Source > S\ Faults)} = 1$, $d_{(S\ Heat\ Source > S\ Host\ rock)} = 1$

$d_{(S\ Alterations > S\ Heat\ Source)} = 0.7227$, $d_{(S\ Alterations > S\ Faults)} = 1$, $d_{(S\ Alterations > S\ Host\ rock)} = 1$

$d_{(S\ Faults > S\ Heat\ Source)} = 0.4843$, $d_{(S\ Faults > S\ Alterations)} = 0.7955$, $d_{(S\ Faults > S\ Host\ rock)} = 1$

$d_{(S\ Host\ rock > S\ Heat\ Source)} = 0.6253$, $d_{(S\ Host\ rock > S\ Alterations)} = 0.8038$, $d_{(S\ Host\ rock > S\ Faults)} = 0.9015$

Then, Eq. 13 was used to assign priority weights:

$d'_{(S\ Heat\ Source)} = (1,1,1) = 1$

$d'_{(Alterations} = (0.7271,1,1) = 0.7271$

$d'_{(Faults)} = (0.4843,0.7955,1) = 0.4843$

$d'_{(Faults)} = (0.6253,0.8038,0.9015) = 0.6253$

The weights vector is $W = (1, 0.7271, 0.4843, 0.6253)$. Finally, the normalized weights vector was calculated as $W = (0.3531, 0.2552, 0.1710, 0.2208)$. This process was applied on all evaluation matrices to obtain the final normalized weights vector. Table 7 shows the normalized weights for all Cu-porphyry prospecting alternatives. These weights show that rock sample anomalies, heat source, and alterations, respectively, have higher values, so they have a significant effect on Cu-porphyry prospecting.

Table 7 - Weight of each criterion and alternative to evaluate Cu-porphyry prospectivity map.

Criterion	Weight	Alternative	Weight	Final Weight
Geological data	0.6219	Heat source	0.3531	0.2195
		Alterations	0.2552	0.1587
		Faults	0.1710	0.1063
		Host rock	0.2208	0.1373
Geochemical data	0.3276	Lithogeochemical anomaly	1	0.3276
Geophysical data	0.0505	Magnetic anomaly	1	0.0505

4.3.2. Integration of evidential layers

For producing the prospectivity pattern, the weights of each individual evidence layer (Table 7), calculated by using the FAHP approach, was multiplied in the corresponding evidence layers. Then the final prospectivity map (Fig. 5) was generated by integrating the weighted evidential maps using fuzzy "Gamma" = 0.9.

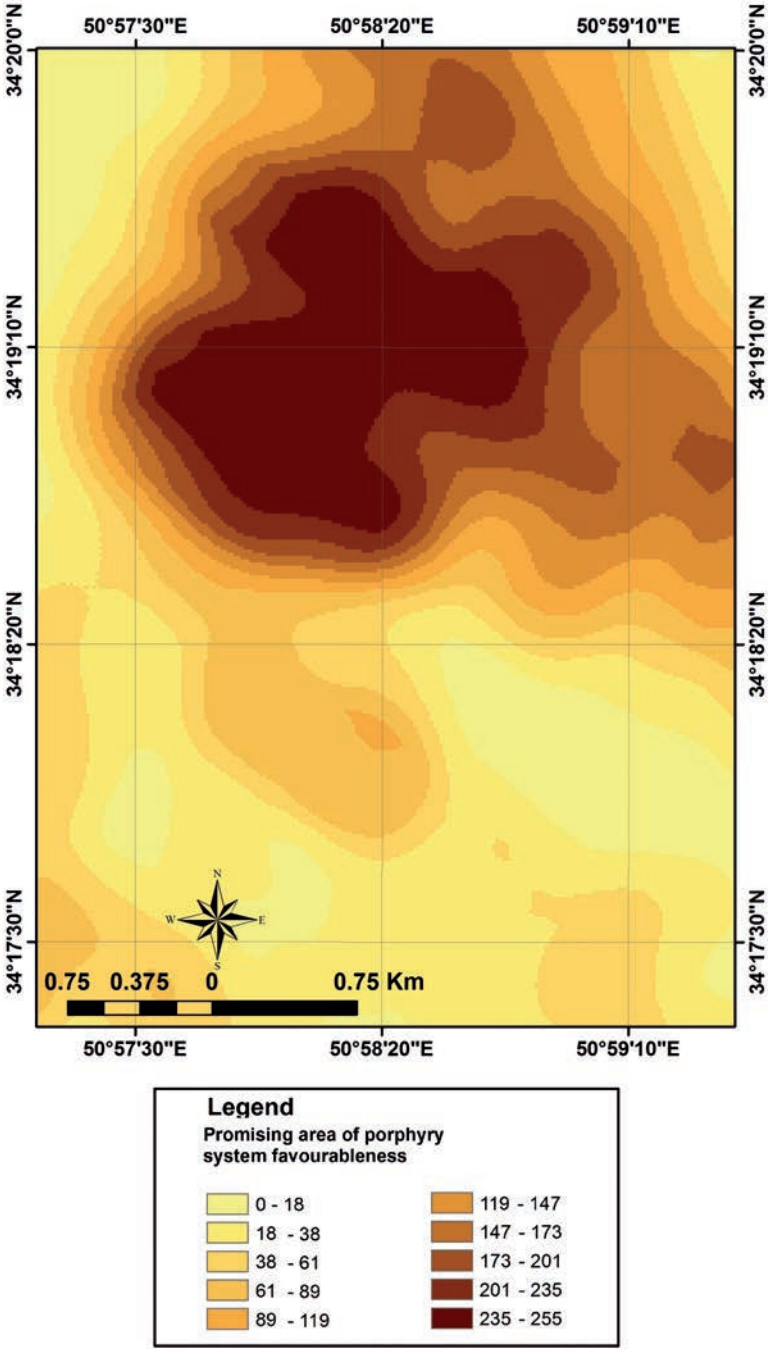

Fig. 5 - Prospectivity map for Cu mineralization in Fordo area generated using FAHP method.

4.3.3. Evaluation of the prospectivity model

The result of integrating the data is a map depicting the favourable area for exploring Cu mineralization (Fig. 5). The legend shows the favourability values for the areas identified by the corresponding patterns. The breakpoints were determined by dividing the prospectivity values into 10 natural break (Jenks) classes. For proper evaluation of the prospectivity model and due to the fact that geological features have fractal dimensions, we used a combination of the C-A fractal model with MPM as C-A mineral prospectivity mapping (C-A MPM) method.

Feizi *et al.* (2017) used the application of the C-A fractal model for discretization of continuous-value evidence maps and classification of the prospectivity model. For implementing C-A MPM, we extract the value of each pixel in the prospectivity map. Afterward, the C-A method was applied to the pixel value data of the prospectivity map. The C-A fractal model demonstrates further correlation in the relationship between the obtained results with the geological, geochemical, geophysical, and remote sensing information. As a result, its most useful features are the simple implementation and the ability to compute quantitative anomalous thresholds (Cheng *et al.*, 1994; Goncalves *et al.*, 2001; Cheng and Li, 2002). Cheng *et al.* (1994) proposed the C-A fractal model for geochemical and geophysical anomalies' separation from backgrounds in order to characterise the distribution of:

$$A\,(\rho \le \upsilon) \propto \rho^{-a1};\; A\,(\rho \le \upsilon) \propto \rho^{-a2} \tag{16}$$

where $A\,(\rho)$ denotes the area with concentration values greater than the contour value ϱ (pixel values in this paper); υ represents the threshold; and a1 and a2 are characteristic exponents. The area $A\,(\rho)$ for a given ϱ is equal to the number of cells multiplied by cell area with pixel values greater than ϱ. The breaks between straight-line segments on the log-log plot and the corresponding values of ϱ have been used as cut-offs to separate pixel values into different components, representing different causal factors, such as geological differences, geochemical processes and mineralizing events (Cheng *et al.*, 1994; Goncalves *et al.*, 2001; Yasrebi *et al.*, 2013; Mansouri *et al.*, 2015; Feizi *et al.*, 2016). Unlike the most conventional methods, the C-A model generates different classes of pixel values on this basis and also puts into account the spatial and geometrical properties of the real-world features on the ground. Using the C-A model, we obtained three breakpoints (Fig. 6a) for defuzzification of the prospectivity model as a C-A prospectivity map (Fig. 6b). The C-A prospectivity map shows that all of the mineral occurrences of Cu-porphyry mineralization have been predicted by the uppermost class as very high favourability for Cu-porphyry mineralization. Results indicate that areas with high and very high favourability are in the northern half of the map. These areas are good candidates for further exploration of Cu mineralization. In addition, the adaption of the prospectivity map with mineral occurrences of Cu mineralization, three target areas with very high favourability, were checked, and the C-A prospectivity map was confirmed by geological observations (Fig. 7). Fig. 7 shows a 3D map of the Fordo area based on the C-A method, with pictures from copper zones. It is necessary to mention that the TERRA satellite has a back-looking telescope with a resolution of 15 m in the VNIR that matches with the wavelength of the band 3 that is used to extract 3D information provided in Fig. 7. Based on field observation, copper mineral indexes were evident. As Fig. 7 shows, the copper hydrocarbonates like malachite and azurite covered the surface of the rocks. These minerals were recognized in the central parts of propylitic and argillic rings. The

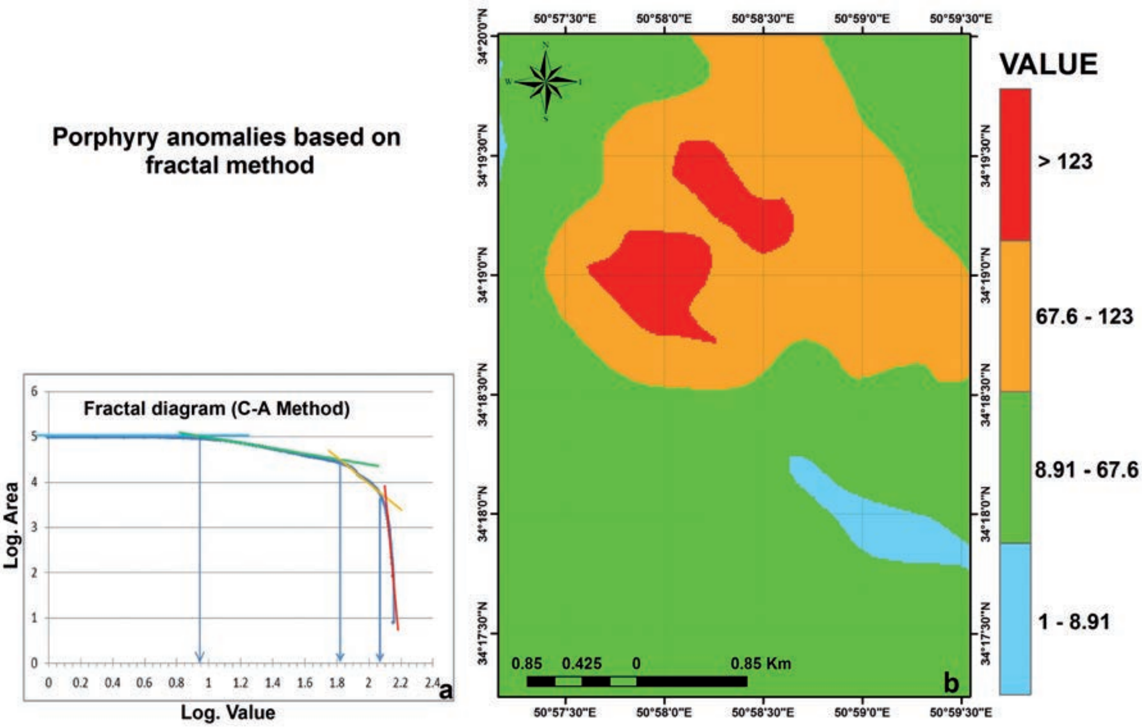

Fig. 6 - a) Cu mineralization: log-log graph of area versus pixel value of prospects; b) C-A prospectivity map.

presence of three dominant strikes: NE-SW, N-S, NW-SE were recognized in the three studied areas. The result of integration between alteration and lineaments indicate a circle band that goes from the NW corner to the east, SE and to the west. There is probably a porphyry system, caused by the propylitic alteration zone around the argillic alteration zone, especially in the central part of the area. The overlapping between argillic alteration and iron oxide zones indicates the presence of sulfide deposits. The phyllic alteration zone exists in the middle of the band, especially where the intrusive bodies are. Therefore, Cu-porphyry mineralization was observed and confirmed in the Fordo area.

5. Conclusions

Today, using only one geophysical data layer creates a large percentage of errors in the mineral exploration field. For this reason, integration of different exploration layers with the geophysical layer is suggested. As a result of the elaboration of geological domains, there is ambiguous knowledge for evaluating the relative importance of geo-evidential features as indicators for prospecting a favorable deposit type. So in MPM, target areas in final prospectivity models are influenced by weighting methods and weighted input evidence layers. The FAHP MPM method, introduced in this paper, can assign realistic weights to individual evidence layers. Inspection of the final prospectivity model in this paper shows that the applied method has delineated some new targets as well. For further evaluation of the prospectivity model and due to the fact that

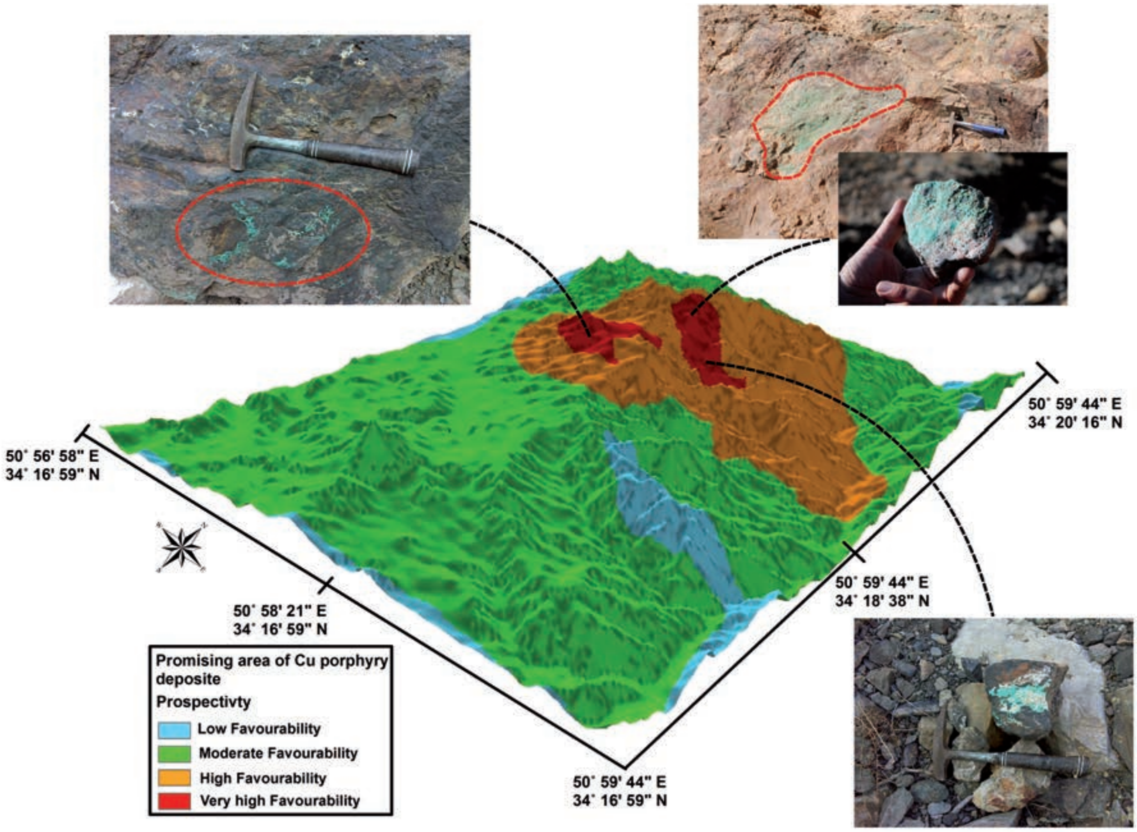

Fig. 7 - C-A prospectivity map of Fordo area which confirmed by check field of three target areas.

geological features have fractal dimensions, we used the C-A MPM method. The possibility of the data leading to finding the deposit appears higher when the fractal analysis is added to the MPM. The C-A prospectivity map which was generated with the provided method, was confirmed by a field check of three target areas. Therefore the target areas generated in the final prospectivity model can be used to follow up exploration of the Cu mineralization.

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