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Change of support based on modified localized uniform conditioning and direct block multivariate simulation on the Sungun porphyry deposit, Iran

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ABSTRACT Mining operations are planned and designed based on block models. A fundamental parameter of this type of model is the choice of block dimensions or selective mining unit (SMU) size, which affects the operation and mining costs. In the early stages of mining operations, the available information is mainly drill holes samples, which may bring negative outcomes such as over-smoothed models, especially for complicated ore deposits. Therefore, a change of support is necessary to determine the distribution of grades at an equivalent volume to the SMU size. The nonlinear localized uniform conditioning (LUC) and direct block simulation (DBSIM) methods are appropriate for optimal grade estimation of small blocks, when the input data spacing is excessively coarse. This paper presents an extension of the LUC method, incorporating the DBSIM in order to change the support on the Sungun porphyry deposit in Iran. Multiple realizations of Cu, CuO, and Mo grades are generated for the dimensions of 30×30×15 m^3, 25×25×12.5 m^3, and 20×20×10 m^3, respectively. Comparing the standard methods, the modified method proposed in this paper presents a more reliable spatial distribution of attributes and an acceptable conformity with the true data of the Sungun deposit.

Key words: localized uniform conditioning, direct block simulation, selective mining unit, resource estimation, geostatistical method, porphyry copper deposit.

1. Introduction

Mining operations are planned and designed using block models. The statistical distribution of the grades depends on the block volume to which the grade is assigned. The dimensions of blocks have a great economic impact on the mining operation. Selective mining unit (SMU) size presents the smallest block size that can be selected as the proportion of ore and waste (Leuangthong *et al.*, 2003; Jara, 2006). The grade of large blocks is estimated according to the drill spacing. Therefore, it is necessary to apply a change-of-support method to determine the distribution of grades at a volume equivalent to the SMU size. This subject has received much consideration from geostatisticians since the 1970s and a number of change-of-support methods have been proposed. The problem of change of support is addressed by utilizing nonlinear and conditional simulation methods.

In ore reserve evaluation, available information generally includes coarse exploration samples, and linear regression-based techniques are unsuitable for modelling grades of small blocks in the cases in question. One solution is to apply nonlinear-based and simulation methods at the early stages of exploration and mining project evaluations, when sparsely distributed data are often the only available information (Armstrong and Champigny, 1989; Ravenscrof and Armstrong, 1990; Pan, 1998; Emery, 2002). Direct block simulation (DBSIM) method as an alternative for sequential Gaussian simulation (SGS) and localized uniform conditioning (LUC) as a non-linear method have been developed recently (Marcotte, 1994; Emery, 2002; Godoy, 2003; Abzalov, 2006; Boucher and Dimitrakopoulos, 2009).

LUC was proposed as a solution for the mentioned problem (Abzalov, 2006). This method partitions the panels into small blocks and ranks them in an increasing order of grades, then localizes the resulted curve to the ranked blocks (Chiles and Delfiner, 1999; Wackernagel, 2002; Rivoirard *et al.*, 2014). The accuracy of the local estimation depends on the block ranking techniques. In some deposits with complex geometry, the block rankings by kriging can differ significantly from their true grade distribution. Therefore, a reliable and robust method is necessary for block ranking (Abzalov, 2014). DBSIM may serve as an appropriate solution for the LUC block ranking problem (Godoy, 2003; Boucher and Dimitrakopoulos, 2009).

The basic idea discussed in this paper is the application of modified localized multivariate uniform conditioning (LMUC) for total and oxide copper grade estimation of the complicated Sungun copper deposit. In the modified method, a ranking of small blocks is done using the DBSIM procedure (Albert, 1987; Davis, 1987; Deutsch and Journel, 1992; Glacken, 1996; Boucher and Dimitrakopoulos, 2009). To accomplish this, the realizations of every simulated block are ranked and the average of their ranks is considered for further analyses. In the next sections, the proposed method, geology of the deposit, and applied data are described. In addition, the prepared models are evaluated using cross-validation, and the performance of the proposed method is tested with satisfying results.

2. Methodology

The LUC method was first introduced by Abzalov (2006). In the following, we present a short description of the original LUC estimation algorithm and the modified algorithm proposed in this study.

2.1. Localized multivariate uniform conditioning (LMUC)

This method evaluates block grades based on a panel grade (comprising blocks), calculated using the linear regression-based techniques such as kriging (Abzalov, 2006). It represents a post-processing analysis on uniform conditioning technique to estimate the recoverable reserves using the change of support [i.e., discrete Gaussian model (DGM)]. It is assumed that if the panel grade is known, then the distribution of the blocks within that panel is also known. Six steps of the workflow for uniform conditioning are as follows:

1. estimate the panel grades using Ordinary Kriging;
2. fit the DGM to the data for normal transformation;
3. determine the change of support coefficients for the blocks and panel-sized blocks;
4. transform the panel-estimated grades to normal space;
5. transform the cut-off grades to normal space;
6. calculate the proportion and quantity of the metals (for each panel) above the cutoffs.

The standard uniform conditioning (UC) method estimates the mineralization tonnage and the grade that are recovered using the block size at the considered cut-off value (Ravenscrof and Armstrong, 1990; Chiles and Delfiner, 1999; Wackernagel, 2002). In order to incorporate LUC post processing, the 3D panels need to be split into sub-cells with cell size equal to that of the chosen small block size. All small blocks distributed in a panel are ranked in increasing order based on their kriged grade. The next step is to define the grade classes using the relationships between the tonnage of recoverable mineralization (T_v) and the cut-off grade (Z_c) estimated by UC technique for each panel. The grade class (G_{Ci}) represents the proportion of a panel whose grade is above the given cut-off (Z_{Ci}) and less than the next cut-off value (Z_{Ci+1}). Then, each ranked small block is matched to the mean grade (M_i) of its related grade class. Fig. 1 illustrates the derivation of block grades by LUC assuming continuous panel-specific tonnage (T) and mean grade curves (M), and ranking the blocks inside the panel. For further information on UC and LUC methods, interested readers are referred to the available references (Abzalov, 2006, 2014). The described algorithm on a multivariate case is summarized in Eqs. 1 to 5:

$$Q_v(Z_c) = E[Z(v) I_{Z(v) \geq Z_c} \,|\, Z^*(V)] \tag{1}$$

$$T_v(Z_c) = E[I_{Z(v) \geq Z_c} \,|\, Z^*(V)] \tag{2}$$

$$M_v(Z_c) = \frac{Q_v(Z_c)}{T_v(Z_c)} \tag{3}$$

$$Q_{2v}^* = E[Z_2(v) I_{Z_1(v) \geq Z_c} \,|\, Z_1^*(V), Z_2^*(V)] \tag{4}$$

$$T_{2v}^* = E[I_{Z_1(v) \geq Z_c} \,|\, Z_1^*(V), Z_2^*(V)] \tag{5}$$

where Q_v, T_v, and M_v stand for metal quantity, tonnage, and mean grade recovered by small blocks (v), respectively; Z^* and Z_c are estimated grade and cutoff grade; and v and V denotes the small block and panel, respectively. As indicated in section 1, the accurate rankings of blocks in the panels can be inferred from the major spatial distribution trends, which are often recognized by geologists in such deposits. Also, in complicated deposits, kriging of the small blocks using the sparse input data is obviously different from the real grades of blocks. A more precise ranking leads to a more accurate final LUC modelling.

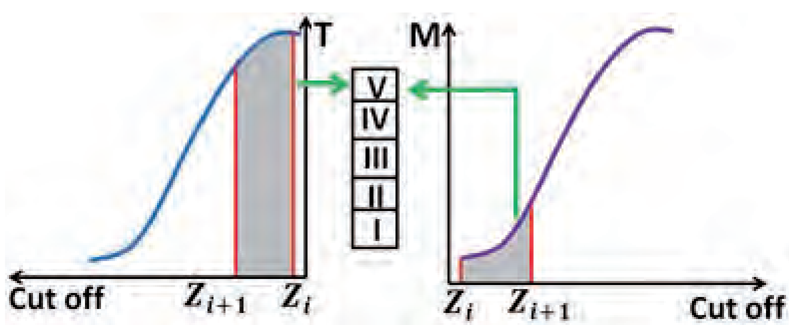


Fig. 1 - Schematic definition of the grade classes and assigning the grades to the SMU blocks.

2.2. Direct block multivariate simulation

The direct block simulation algorithm randomly visits all the blocks in the domain and simulates the N discretizing points for each block and for each factor (orthogonalization), $m^{l*}(ui)$, $i = 1, \ldots, N$; $l = 1, \ldots, K$ conditional to point and block data. For each block, the simulation is performed pointwise with the generalized (group) sequential Gaussian simulation (GSGS) approach (Dimitrakopoulos and Luo, 2004). The simulated point-support values are averaged into the simulated block support, the simulated points are not used in other conditioning and are discarded. For block simulation, the simulated discretizing point values conditional to the neighboring point and block data are obtained with joint localized uniform (LU) simulation (Verly *et al.*, 1984). In the present case, let $\mathbf{C}^l_{IIVV}$ be the covariance matrix of all the conditioning information and $\mathbf{C}^l_{pIV}$ be the covariance matrix between the discretizing point and the neighboring point and block data for the lth factor:

$$\mathbf{C}^l_{IIVV} = \begin{pmatrix} \mathbf{C}^l_{II} & \mathbf{C}^l_{IV} \\ \mathbf{C}^l_{VI} & \mathbf{C}^l_{VV} \end{pmatrix} \qquad \mathbf{C}^l_{IIVV} = \begin{pmatrix} \mathbf{C}^l_{pI} & \mathbf{C}^l_{pV} \end{pmatrix} \tag{6}$$

where $\mathbf{C}^l_{II}$,$\mathbf{C}^l_{IV}$, and $\mathbf{C}^l_{VV}$ are respectively the point-to-point covariance matrices, points to block, and block-to-block covariance, for the lth factor; $\mathbf{C}^l_{pI}$ and $\mathbf{C}^l_{pV}$ are respectively the covariance between the discretizing points and the point data and block data, for the lth factor. The covariance values between data on different supports are generated from regularizing the point-support covariance $C(u_i,u_j)$, where u_i, i = 1, . . . , N are discretizing point-support variables. The point-to-block and the block-to-block covariances are obtained, respectively, with:

$$\bar{C}(u_\alpha, v) \approx \frac{1}{N}\sum_{i=1}^{N} C(u_\alpha, u_i), u_i \in v \tag{7}$$

$$\bar{C}(v_1, v_2) \approx \frac{1}{N_1 N_2}\sum_{j=1}^{N_1}\sum_{i=1}^{N_2} C(u_j, u_i), u_i \in v_1, u_j \in v_2 \tag{8}$$

where weight vector $\mathbf{w}_{IV}$ is obtained by solving:

$$m^l \approx L^l_w \begin{pmatrix} m^l_{IV} \\ m^{l*}_p \end{pmatrix} = \begin{pmatrix} L_{IIVV}{}^l & 0 \\ L_{pIV}{}^l & L_{pp}{}^l \end{pmatrix} = \begin{pmatrix} w_{IV} \\ w_p \end{pmatrix} \tag{9}$$

where $\mathbf{m}^l_{IV}$ is the vector containing the neighboring point. The block data for factor l and $\mathbf{m}^{l*}_p$ is a vector with the discretizing points for factor l to be simulated. The matrices $\mathbf{L}^l_{IIVV}$, $\mathbf{L}^l_{pIV}$, and $\mathbf{L}^l_{pp}$ are obtained from the following (Davis, 1987):

$$\begin{pmatrix} \mathbf{C}^l_{IIVV} & \mathbf{C}^{l^T}_{pIV} \\ \mathbf{C}^l_{pIV} & \mathbf{C}^l_{pp} \end{pmatrix} = \begin{pmatrix} L^l_{IIVV} & 0 \\ L^l_{pIV} & L^l_{pp} \end{pmatrix} \begin{pmatrix} L^{lT}_{IIVV} & L^{lT}_{pIV} \\ 0 & L^{lT}_{pp} \end{pmatrix} \tag{10}$$

Finally, the vector of N simulated values $\mathbf{m}^{l*}_p$ for factor l is obtained from:

$$m^{l*}_p = L_{pIV}{}^l (L^l_{IIVV})^{-1} m^l_{IV} + L^l_{pp} w_p \tag{11}$$

and the block value can be obtained through averaging the back-transformed points discretizing the block. Once the neighboring point and block data are found, the process is split into two

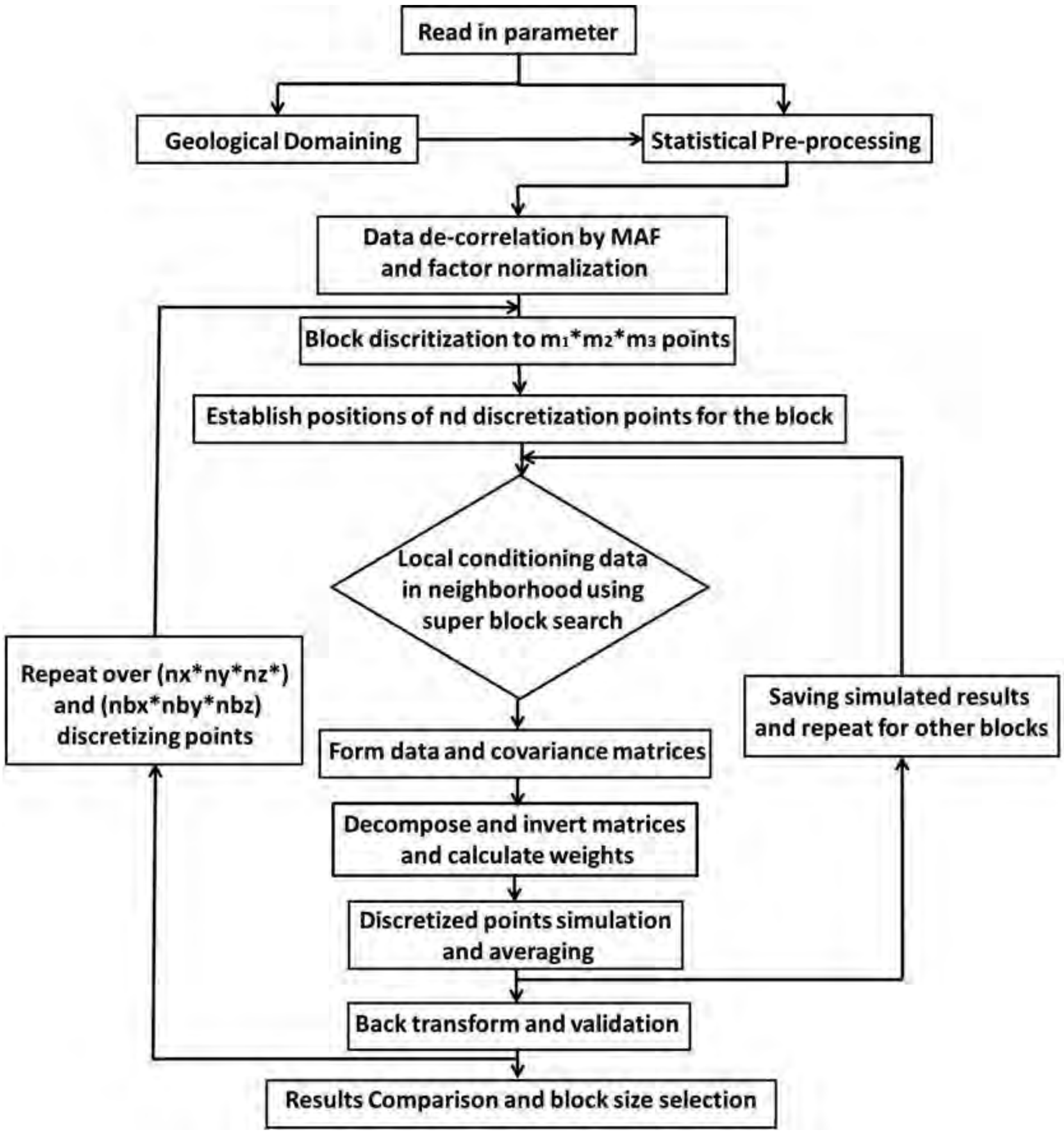


Fig. 2 - Flow diagram for the DBSIM program: nx, ny, and nz are the numbers of blocks in three dimensions.

parts: 1) the block values for the factors are achieved by averaging the simulated factors on point support, and 2) the internal points are rotated back into the normal score transform and, subsequently, back transformed into the data space and, finally, averaged to generate the desired block value. The value from part (1) is saved for further conditioning, while the value of part (2) is the final output simulated value for the block, and is written in a file. Fig. 2 depicts the flow diagram for the DBSIM algorithm.

2.3. Modified localized multivariate uniform conditioning

In the original implementation of the LUC algorithm, a ranking of the small block is done based on its kriged grades. When the variogram of the studied variable is characterized by a large nugget effect, the block ranks produced by kriging can significantly differ from their "true"

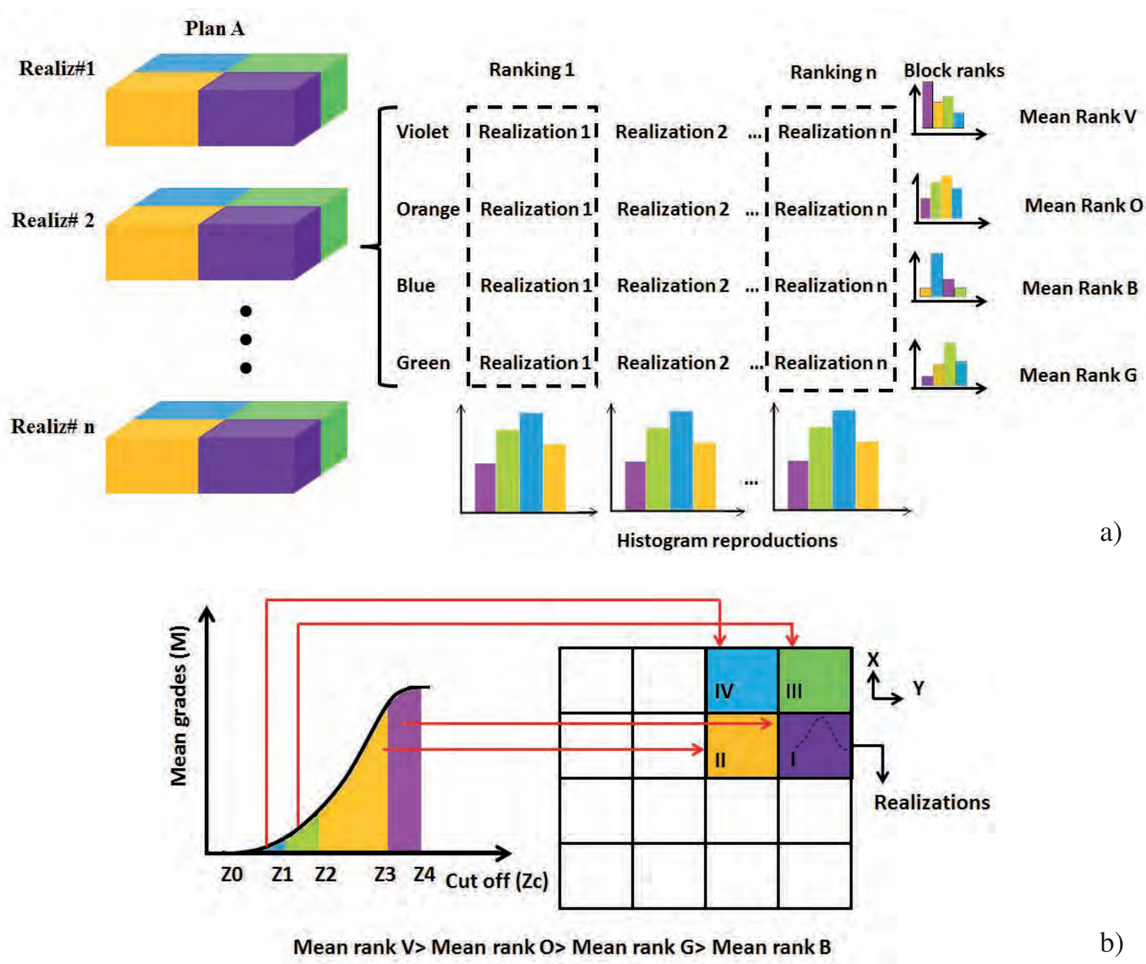


Fig. 3 - a) Illustration of proposed ranking method; b) sketch presenting the grade classes and assigning the grade values to the blocks.

distribution (Abzalov, 2014). To overcome this challenge, a step involving modified ranking of LUC post-processing is suggested in the current paper. For this purpose, the DBSIM method was used to model the blocks within a panel. First, the simulated block was arranged in an increasing order, considering each of the realizations. Then, the mean of assigned ranks to a block (based on different realizations of the block) was presented as the final rank of the mentioned small block. Fig. 3 is shown schematically for better understanding of the algorithm in the general form. It presents the mean grade (M) and considered cut-off of the curve, which is divided into several classes with respect to different cut-offs. As can be inferred from the figure, the highest mean grade class is assigned to the green block, based on the mean ranks of 50 realizations of the block. This procedure is performed to assign a specific rank to other blocks.

The algorithm code of the LUC-modified version is written in MATLAB software. This modification can provide a more reliable grade assignment and subsequently a more robust recoverable resource model.

Fig. 4 shows the flow diagram of the proposed algorithm, which is executed on the real data, and its outcomes are presented in section 4.2.

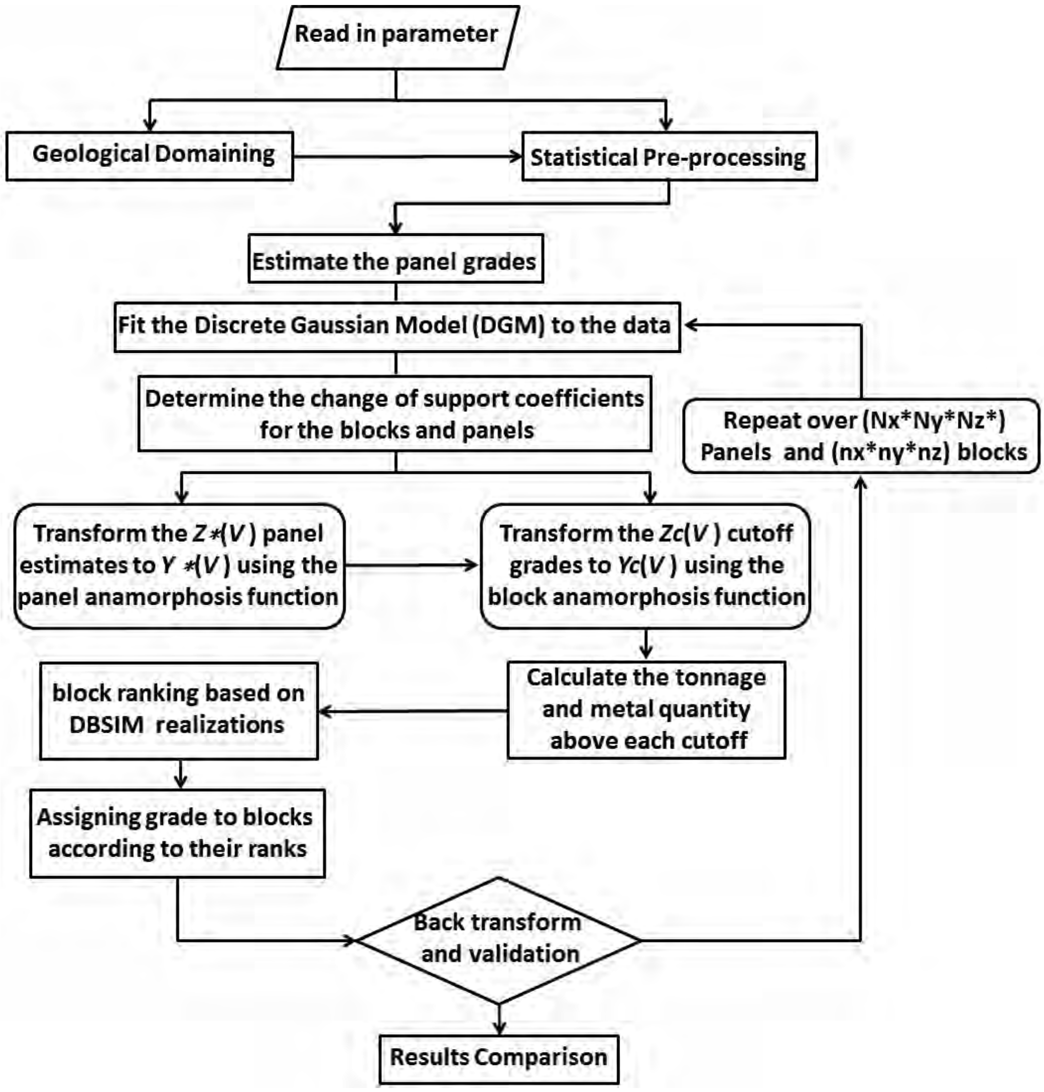


Fig. 4 - Flow diagram of the modified LMUC algorithm.

3. Case study: Sungun copper deposit

3.1. Regional geology

The Sungun porphyry copper deposit is located 100 km NE of Tabriz, in east Azerbaijan, NW of Iran. This porphyry system is situated on the Urumieh-Dokhtar magmatic arc. The Urumieh-Dokhtar arc contains the main copper-bearing regions in Iran, such as Sarcheshmeh, Sungun, Meiduk, Darehzar, Sarkuh, and Daralu (Hezarkhani *et al.*, 1998; Hezarkhani, 2006).

The porphyry Sungun deposit is formed by hydrothermal processes, but the arrangement of alterations and mineralization domains does not follow the simple models of porphyry systems (Hezarkhani *et al.*, 1998; Hezarkhani, 2006; Asghari and Hezarkhani, 2008). There are four types of hypogene alterations in Sungun (Hezarkhani, 2006): i) potassic; ii) potassic–phyllic; iii) phyllic; and iv) propylitic.

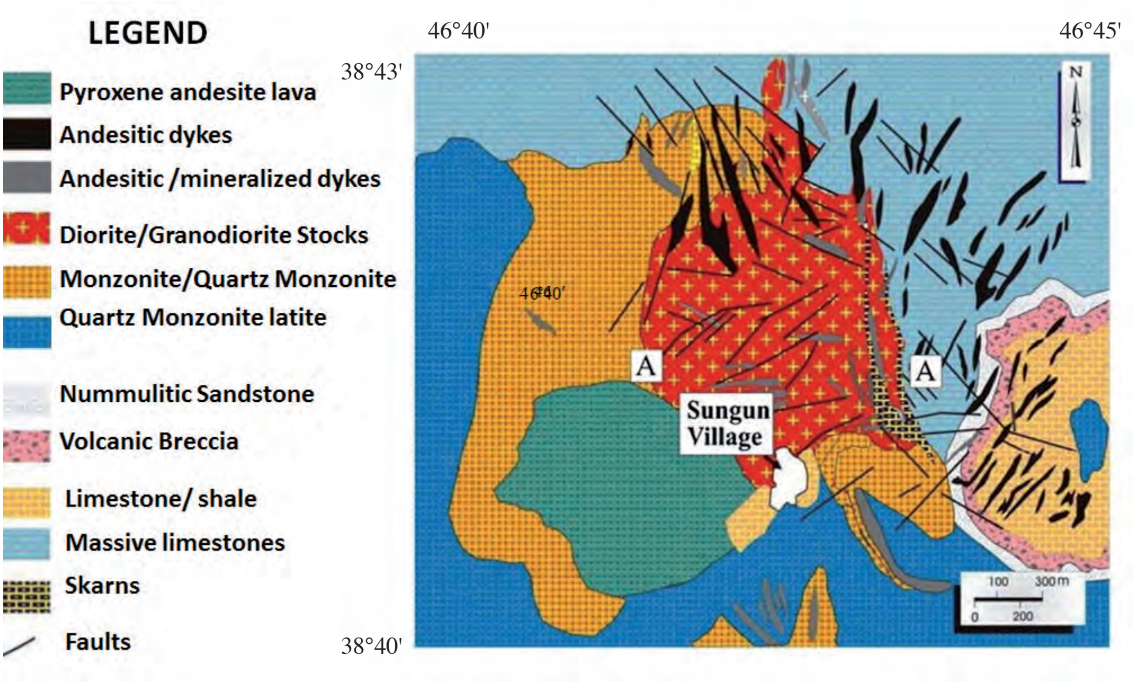


Fig. 5 - Geological map of Sungun deposit.

Types i, ii, and iii are principally developed within porphyry stock, whereas the Type iv alteration is restricted to the porphyry stocks and some of the early dike series. Hypogene mineralization is characterized by the introduction of sulfides (Hezarkhani *et al.*, 1998; Hezarkhani, 2006). Fig. 5 shows the geological map of the Sungun deposit. In this deposit, pyrite is the most abundant sulfide and chalcopyrite is the dominant copper ore, which is accompanied by minor amounts of molybdenite. Previous studies on the geological features of this deposit revealed three main rock type domains controlling the copper grade distribution (Hezarkhani *et al.*, 1998): Sungun porphyry (SP) stock, skarn mineralization (SK), and late injected dikes (DK). In this research, geological domaining is performed and the potassic alteration on the hypogene zone i was selected to delineate the homogeneous areas of data for prediction purposes. The case study area is illustrated in Fig. 6.

3.2. Presentation of the data set

The main spatial variation through a site can result in a violation of the stationary assumption and can create biased estimates (Journel and Huijbregts, 1978; Dagdelen and Turner, 1996). Based on the stationary theory, the mean behavior of the studied area should represent a spatial variation, assuming the neighboring data samples are stationary. Therefore, geological domaining is performed to divide the estimation area into an optimal number of zones, and a part of the potassic alteration is selected as the area of interest. This domain covers an area of approximately 2.5 km (E-W) by 1.7 km (N-S) and 1.2 km below the surface. To evaluate recoverable resources, the exploratory data set includes the grade of copper, oxide copper, molybdenum, and iron oxide. Some 262 drill holes (880 samples selected from potassic alteration of the hypogene zone) intersect the case study area. Analysis of Cu, CuO, and Mo exist in all drill holes, which are

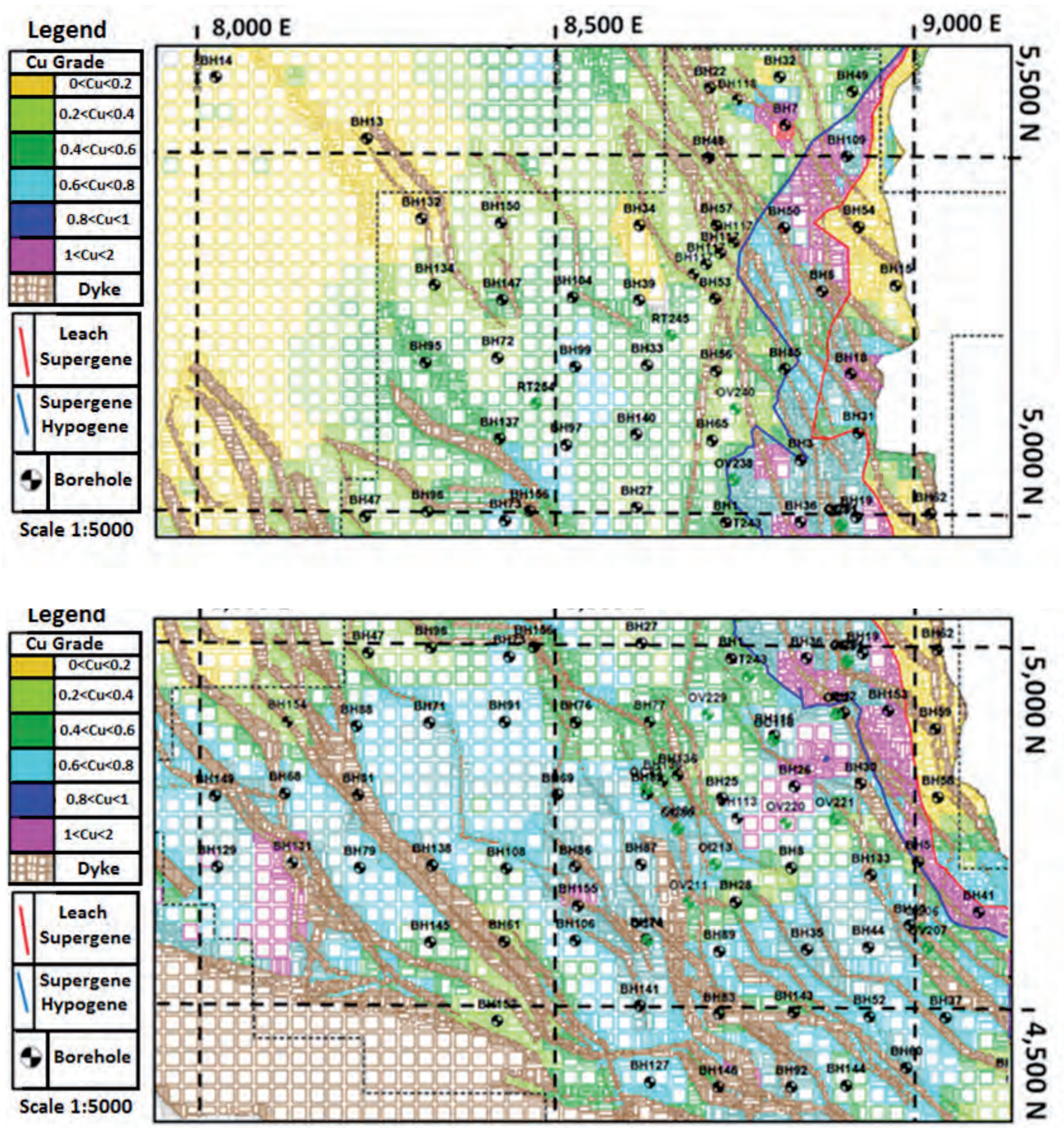


Fig. 6 - Different types of copper mineralization in Sungun porphyry copper deposit at an elevation of 1,850 m. The red line determines the leach-supergene contact, and the blue one determines the supergene-hypogene zones (modified from Pars Olang Engineering Consultant Company, 2006).

composited to 2-m downhole length before geostatistical analysis. Fig. 7 shows the location map and value of available data. The Pearson correlation coefficients (r) are as follows: between Mo and Cu is 0.415 and between Cu and CuO is 0.594. The multivariate LUC model can be created based on the correlation coefficients between these variables and the Cu variable. About 25% of the total data are randomly selected as actual data from the different parts of the case study area. In order to compare the actual data set with the estimated value, the target variable at a test point is temporarily discarded. Next, an estimation value of this variable at this point is calculated and compared to the discarded actual value (Efron, 1982).

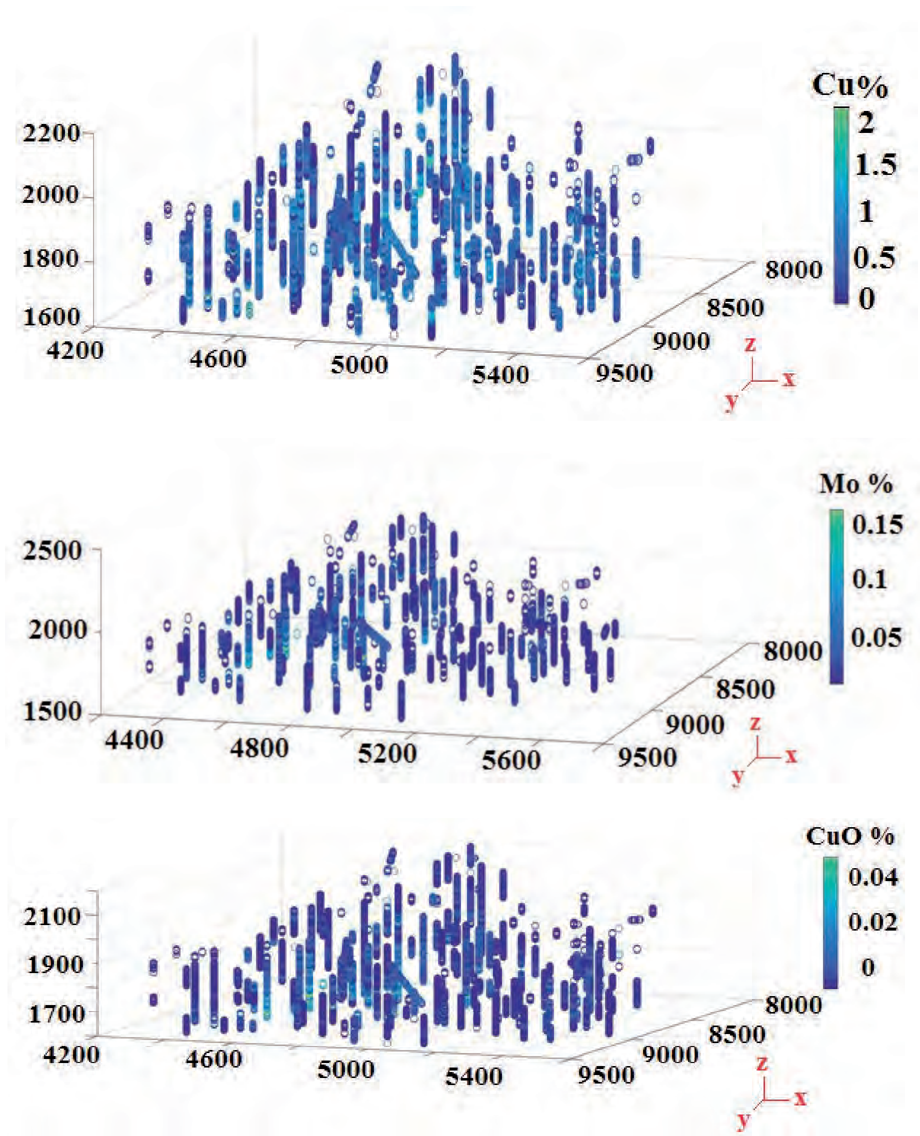


Fig. 7 - Location map and value of available data.

4. Discussion

This section shows and discusses the results of the geostatistical processing, the classic method of estimation, and the proposed algorithm. Before presenting the results of the proposed algorithm, we describe the steps of geostatistical preprocessing to generate the multivariate models. Fig. 8 illustrates the frequency histograms of the Cu, CuO, and Mo grades. The total Cu values are more continuous, compared to the CuO and Mo, so the Cu value was selected as the main variable. In this regard, CuO and Mo are considered as the secondary variables. The localized multivariate uniform conditioning model and its modified version are created considering the previously mentioned correlation coefficients. The grades were estimated using ordinary kriging with panel sizes of 80×80×40 m^3, 60×60×30 m^3, and 50×50×25 m^3, with an approximate drill spacing of 100 m.

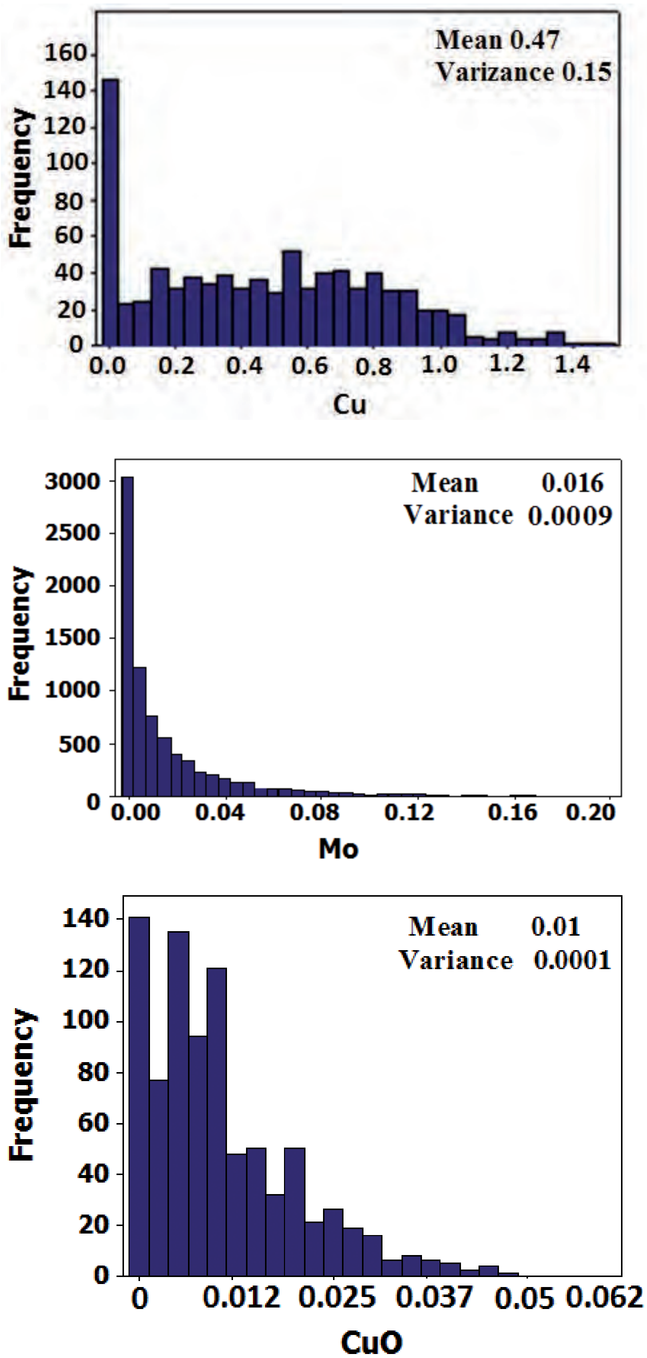


Fig. 8 - Frequency histogram of grades.

To determine the geometric anisotropy of considered variables, directional variograms were computed in different dips and azimuths with a 22.5° tolerance. According to the directional variography for all aforementioned variables, most fitted models of variograms show relatively the same parameters. Thus, an isotropic model with a 2/3 variogram range as search radius was used instead of an anisotropic ellipsoid. Fig. 9 demonstrates the omnidirectional and some directional variogram/cross variogram models of the Cu, CuO, and Mo grades. The parameters and formulation of exponential variogram models fitted to variables are presented in Table 1 and Eq. 12. Sill is the variance value at which the variogram levels off; range denotes the lag distance at which the semivariogram (or semivariogram component) reaches the sill value; nugget indicates the semivariogram value at the origin (Bohling, 2005). Using *h* to represent lag distance, *a* to represent (practical) range, *C* to represent sill and C_0 to represent nugget effect, the five most frequently used models are as follows:

$$\gamma_h = C_0 + C[1 - \exp(-\frac{3h}{a})] \tag{12}$$

Table 1 - Parameters of fitted variogram models.

	Nugget	Sill	Range
Cu	0.02	0.15	110
Mo	0.0001	0.0009	160
CuO	0.00002	0.0001	150
Cu/CuO	0.00001	0.0022	90
Cu/Mo	0.0005	0.0045	150

4.1. Grade estimation

In the present study, the LMUC and modified LUC estimation methods were applied to provide SMU grade estimates with the minimal conditional bias and high correlation with true data. The UC method, as the base of both the classic and proposed approaches, requires the calculation of the support coefficients on the SMU supports and panel sizes. The support correction depends on the variance computed from the raw data variogram model, based on the theoretical dispersion variance. The distribution of 2-m composites was modelled using a Gaussian anamorphosis function and then decomposed into Hermite polynomials. After determining the changes of support on SMUs, the coefficients of the Hermite polynomials were calculated based on the selected main variable. The interpreted coefficients applied during estimation are presented in Table 2. In the current research, support sizes are presented as large (50×50×25 m^3), medium (60×60×30 m^3), and small (80×80×40 m^3) panels for grade estimation at 25×25×12.5 m^3, 30×30×15 m^3 and 20×20×10 m^3 SMU block sizes, respectively, conforming to the Sungun production plan for SMU block size.

The global grade-tonnage curve within the mineralized domain, obtained using modified LUC models, demonstrated the different support sizes (Fig. 10). It can be inferred from the output grade-tonnage curves that the transformation from large- to medium-sized blocks is accompanied by a greater loss of selectivity and metal for a given ore tonnage. The total tonnage of ore and metal quantity for 30×30×15 m^3 and 25×25×12.5 m^3 SMU are respectively highest and lowest for all considered cut-offs. The greatest impact of the support effect is concentrated on the low-grade range, for which there is a greater loss of metal content for a given ore tonnage. Mixed with ore, this waste decreases the mineral grade and increases its tonnage. The following activities are considered for performing a uniform conditioning that is the basis of the LUC and modified LUC methods:

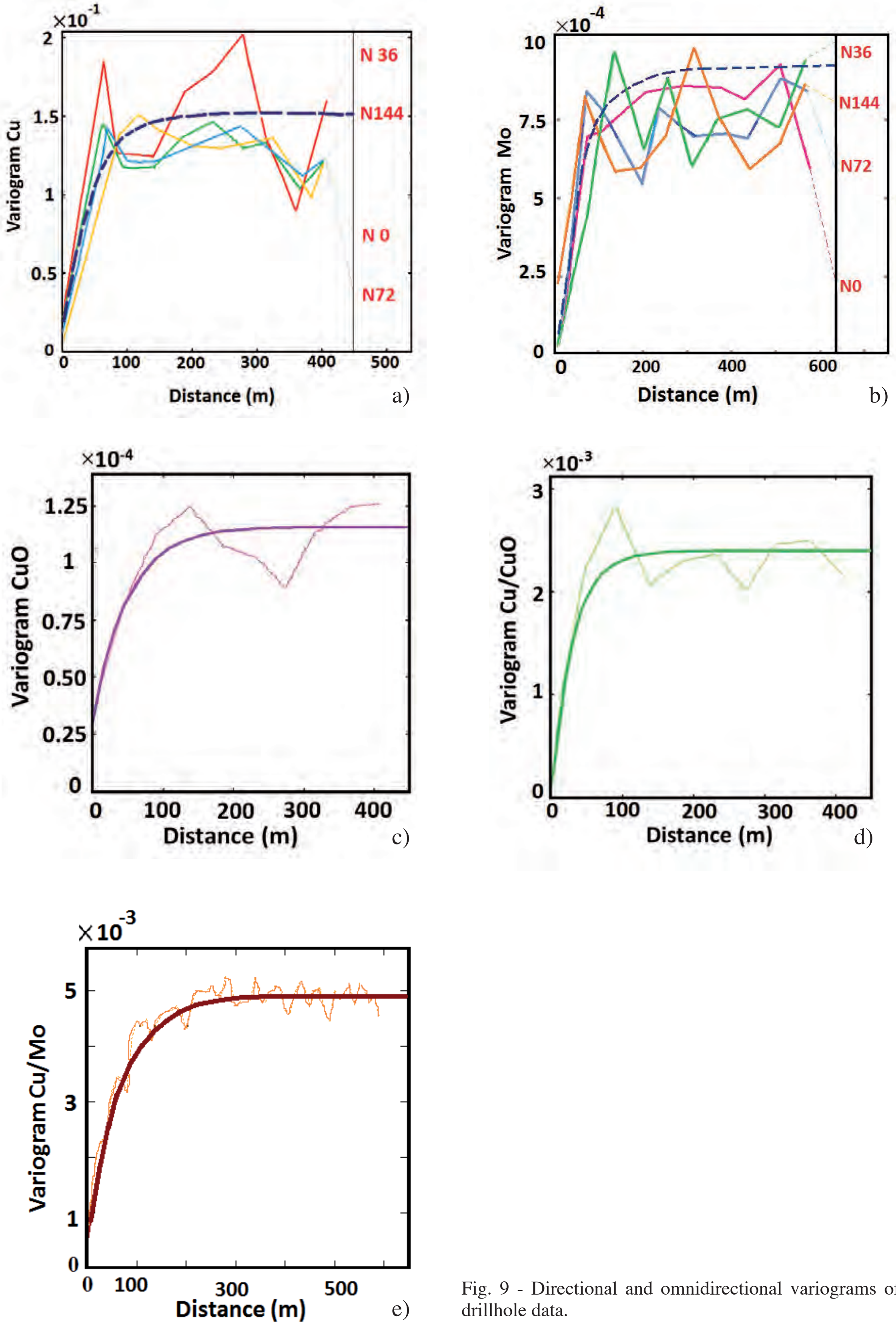


Fig. 9 - Directional and omnidirectional variograms of drillhole data.

Table 2 - Change-of-support coefficients on different supports.

Panel Size (m3)	80×80×40	60×60×30	50×50×25
Variable	Cu	Cu	Cu
Punctual Variance	0.13	0.13	0.13
Variogram Sill	0.13	0.13	0.13
Gamma(V,v)	0.03	0.03	0.04
Real Block Variance	0.09	0.1	0.08
Real Block Support Correction	0.87	0.88	0.83
Kriged Block Support Correction	0.87	0.88	0.83
Kriged Panel Support Correction	0.31	0.32	0.22
Zmin Block	0	0	0
Zmax Block	1.49	1.49	1.49
Block Support Correction	CuO=0.82 Mo=0.88	Mo=0.86 CuO=0.78	CuO=0.83 Mo=0.83

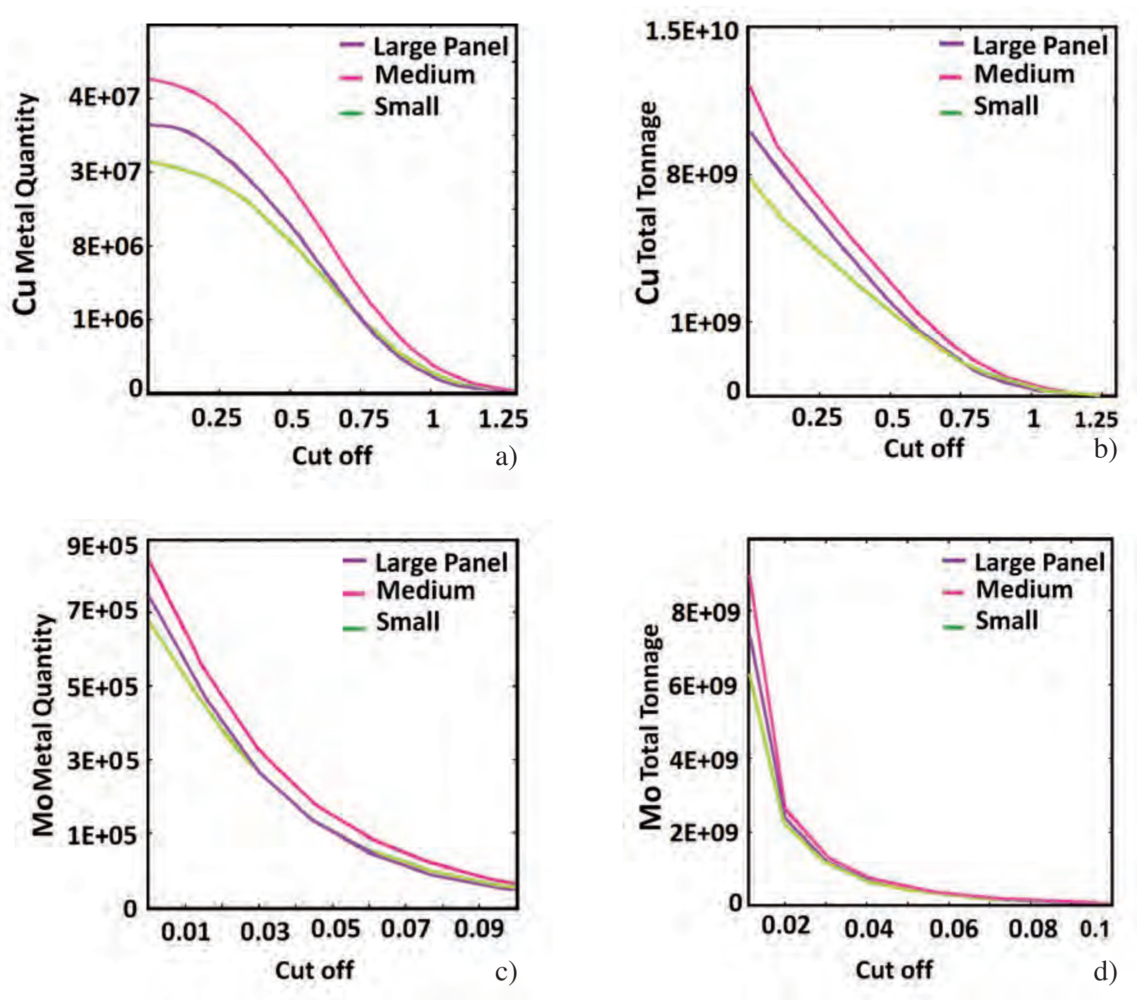


Fig. 10 - Grade-tonnage curve for Cu and Mo elements.

- transform the raw variables into Gaussian space;
- appropriate cell declustering weights are applied to obtain unbiased histograms and variograms;
- a uniform conditioning is applied to all supports in order to obtain the probability above the cut-off grade ($P(Z(u)) \geq z_c$) and probability below the cut-off grade ($P(Z(u)) < z_c$). The tonnes of waste (Tw), tonnes of ore (T_O), and the grade of ore are calculated with respect to the panels;
- the small blocks within a panel are arranged in an increasing order considering the ordinary kriging (LUC) and direct block multivariate simulation (DBMS) realization results (modified LUC). For the latter, the average of ranking values (based on the histograms of SMU realizations) is presented as the final rank of the mentioned small block;
- the results are compared with true values;
- a sensitivity analysis is conducted on SMU sizes.

4.2. Grade estimation using the modified LUC method

The schematic illustration and algorithm of the modified LMUC method are demonstrated in Figs. 3 and 4. To rank the modified version of this method, the DBSIM is executed after the maximum/minimum autocorrelation factor (MAF) approach at the block support scale. This procedure is performed as follows: a) transformation of (normal transformed) data to MAF using a 20-m lag, b) independent conditional simulation of each MAF with the DBSIM algorithm; and c) subsequent back-transformation of the generated realizations using the MAF coefficients (Desbarats and Dimitrakopoulos, 2000). The three factors of MAF_1, MAF_2, and MAF_3 are shown at below:

$$MAF_1 = 0.408 \text{ Cu} - 0.170 \text{ Mo} + 0.864 \text{ CuO} \tag{13}$$

$$MAF_2 = 0.994 \text{ Cu} + 0.503 \text{ Mo} + 0.873 \text{ CuO} \tag{14}$$

$$MAF_3 = 0.106 \text{ Cu} + 0.673 \text{ Mo} + 0.473 \text{ CuO} \tag{15}$$

The above additional variables MAF_1, MAF_2, and MAF_3 are Gaussian with zero mean and unit variance and have zero covariance at lag 0 and at lag 20. The direct block multivariate simulation was applied to generate 50 realizations of the three orthogonalized factors. The simulation method was independently performed for each factor on the three block sizes. The simulation results must be back-transformed to the original units of data. Finally, 50 realizations of the Cu, CuO, and Mo in blocks were considered for ranking. The simulated blocks within a panel were arranged in an increasing order through each of 50 realizations. Then, the average of all assigned ranks to a block was considered as the final rank of the mentioned block. A number of basic checks must be performed prior to applying the ranking results. Simulations should consider the input information, including the data values at their locations and the data distributions (Leuangthong *et al.*, 2003).

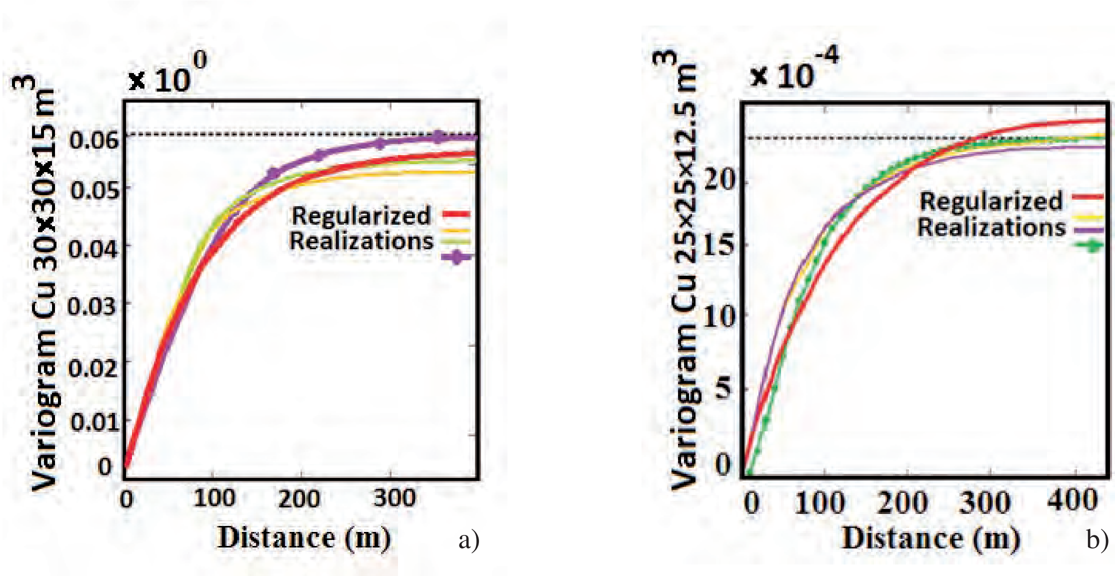
The multivariate relations are important and must be taken into account. The correlation of back-transformed simulated values shows a good reproduction of the bivariate distribution of input data. Histograms and variograms of the simulated values after back transformation, as well, are important factors to be investigated. These distributions should be similar to the input data histogram, with comparable statistics. Table 3 summarizes the statistics of some realizations considered for histogram reproduction. For each realization, there is a mean and a variance associated with the resulting distribution. Fig. 11 presents the consistency between the regularized variogram models and the DBSIM block simulations. The block conditioning certainly produces a good variogram reproduction of the block supports. It can be inferred that the histograms and the variograms of simulation realizations are subsequently and satisfactorily reproduced on the large and small block sizes.

Table 3 - Reproduction of statistical parameter by DBSIM.

		Mean	Median	Skewness	Variance
Cu	Raw data	0.42	0.37	1.1	0. 6
	30 m-Realization 15	0.40	0.35	0.91	0.51
	25 m-Realization 5	0.39	0.34	1.03	0.53
	20 m-Realization 20	0.39	0.34	1.01	0.54
Mo	Raw data	0.015	0	0.027	1.83
	30 m Realization 15	0.012	0.01	0.020	1.81
	25 m Realization 5	0.013	0.01	0.022	1.82
	20 m Realization 20	0.012	0.01	0.021	1.81
CuO	Raw data	0.011	0.01	1.23	0.00011
	30 m Realization 15	0.010	0.01	1.14	0.00008
	25 m Realization 5	0.011	0.01	1.08	0.00009
	20 m Realization20	0.011	0.01	1.08	0.00009

4.3. Comparison between the results of the standard and new methods

Figs. 12 to 14 present the 3D model of LMUC, modified LMUC, and E-type value of DBSIM outputs for Cu variable on the considered block supports. E-type value of simulation outputs can be explained as the mean value of realizations in a block. According to the geological maps of the case study (Fig. 6), the minimum and the maximum estimated copper grades are in the leached and supergene zones, respectively. Both estimation and simulation of the 3D model demonstrated high values of Cu on the north-western part of the area, which is superposed on the high-grade zone. As highlighted earlier, the main advantage of the proposed approach is to derive a non-smoothed block model through the high variability area. The performance of the simulation technique was presented by plotting the scatter diagrams between output grades and true variables (Fig. 15). The trend line between estimated Cu variables and true data for large block size shows a slope of less than 1, reflecting significant biases and underestimation. The slope of regressions for medium and small blocks, however, is close to unity if the model is conditionally unbiased and robust. It would also mean a high level of correlation between the results and true variables. Furthermore, the performance of the LMUC and modified LMUC techniques can be compared using the correlation coefficient between the outputted grades and true variables. The Pearson correlation coefficient (r) between the outputs of applied methods and the true data are presented in Table 4. Based on this table, the r between modified LUC results and true data for Cu, CuO, and Mo in a small SMU size is more than that for medium and large SMU sizes. The table can



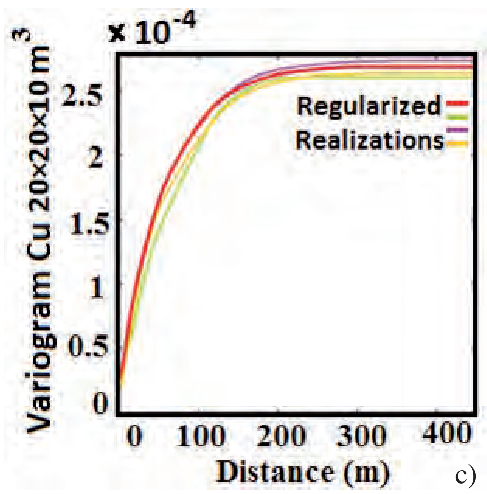


Fig. 11 - Regularized variogram of Cu variable and block simulation realizations.

Table 4 - Pearson correlation coefficient between the results of different methods and true data.

	Modified LUC			DBMS			LUC		
SMU (m^3)	30×30×15	25×25×12.5	20×20×10	30×30×15	25×25×12.5	20×20×10	30×30×15	25×25×12.5	20×20×10
Cu	0.73	0.75	0.76	0.66	0.77	0.78	0.73	0.72	0.72
Mo	0.73	0.78	0.71	0.72	0.71	0.73	0.68	0.70	0.70
CuO	0.66	0.68	0.70	0.63	0.69	0.70	0.67	0.62	0.67

generally depict the strength of the results of a 20×20×10 m^3 block model, and the match between the outputs of the proposed method to those of standard methods and actual data. Based on the above descriptions and the reliable results of the block simulation as the basis of the ranking approach, it can be deduced that the introduced method benefits from the advantages of both simulation and nonlinear methods.

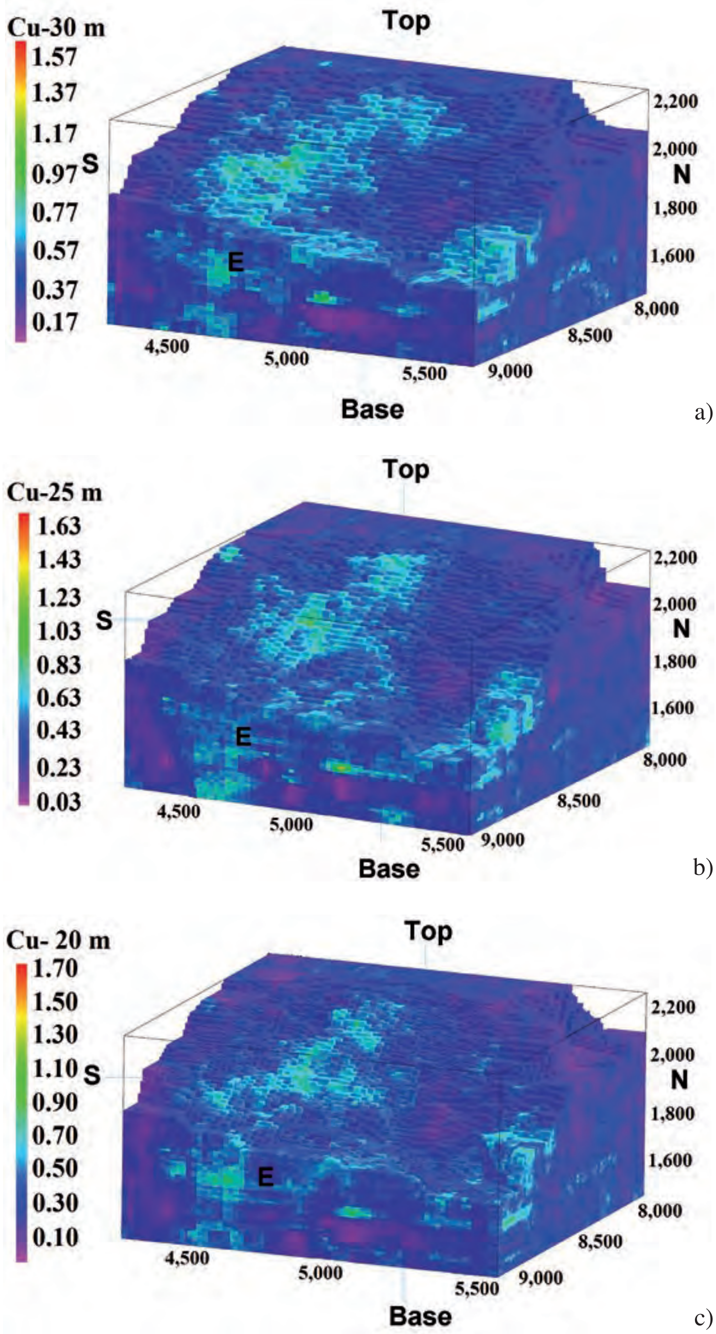


Fig. 12 - 3D plan of Cu modified LMUC estimation assigned to: a) 30×30×15 m^3; b) 25×25×12.5 m^3; and c) 20×20×10 m^3 SMU.

5. Conclusions

At the exploration stage of new mining projects, drilling data are distributed on a relatively large grid which is typically larger than the SMUs. Hence, direct estimates of blocks will then be smoothed due to the information effect and the high error variance. Any capital project decision

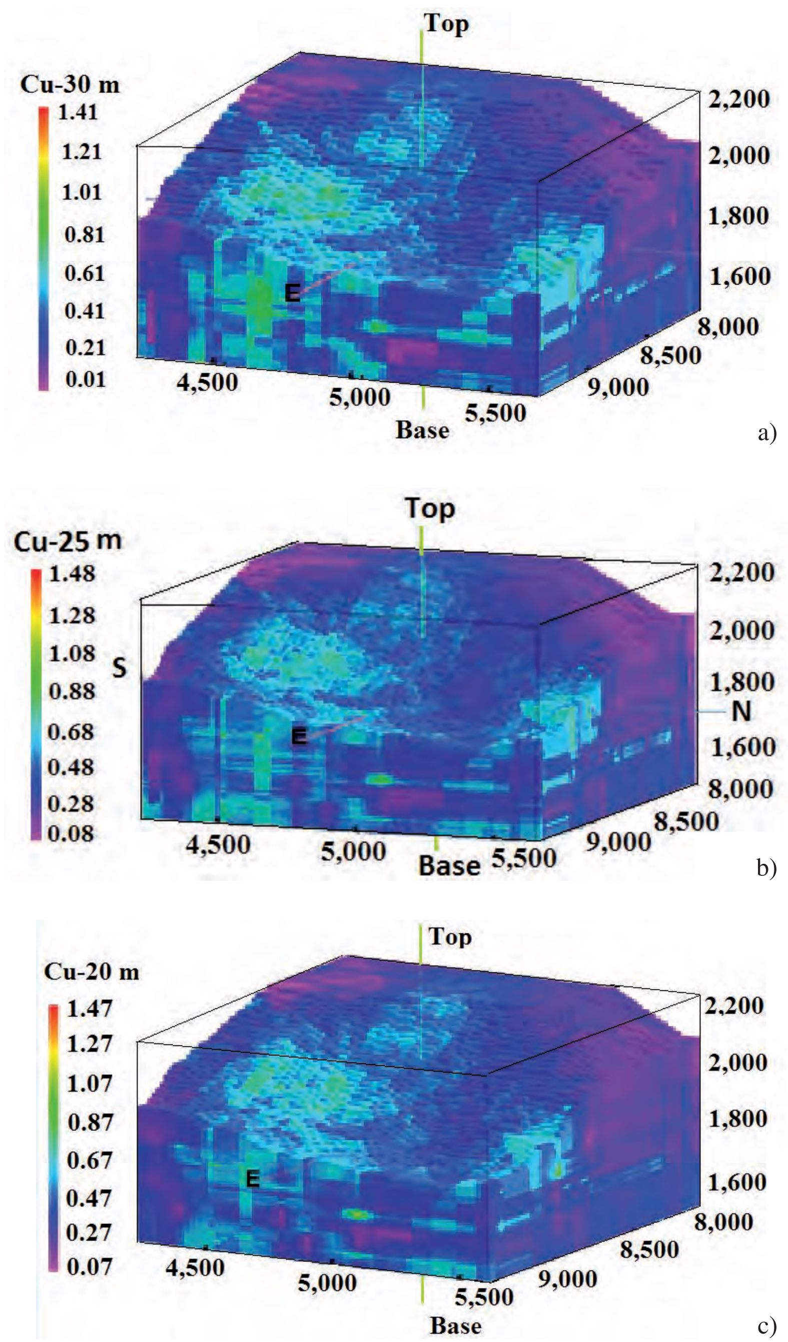
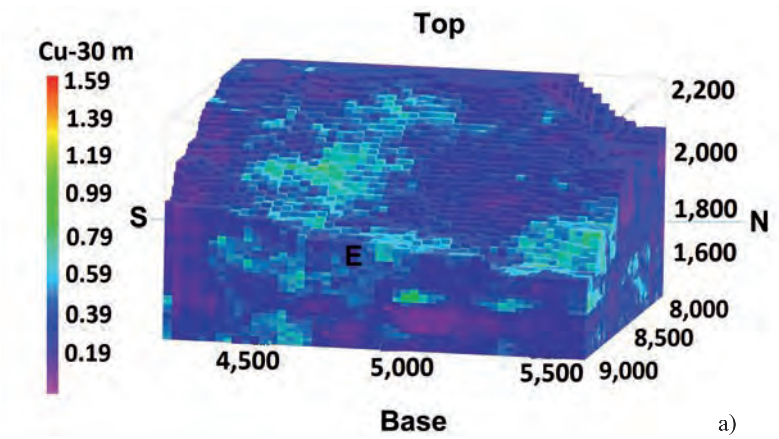
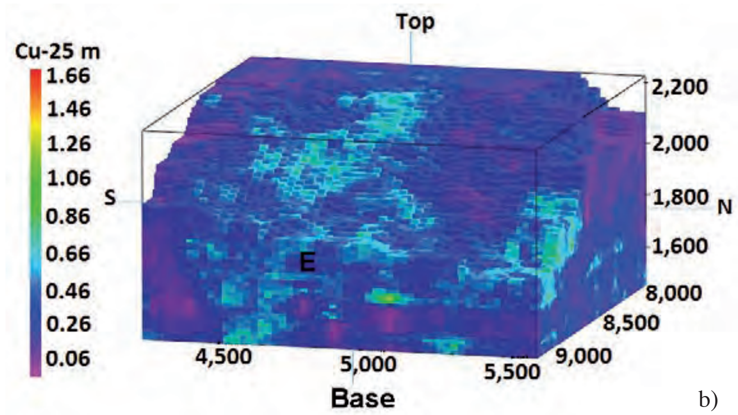


Fig. 13 - 3D plan of Cu E-type (DBMS) assigned to: a) 30×30×15 m^3; b) 25×25×12.5 m^3; and c) 20×20×10 m^3 SMU.

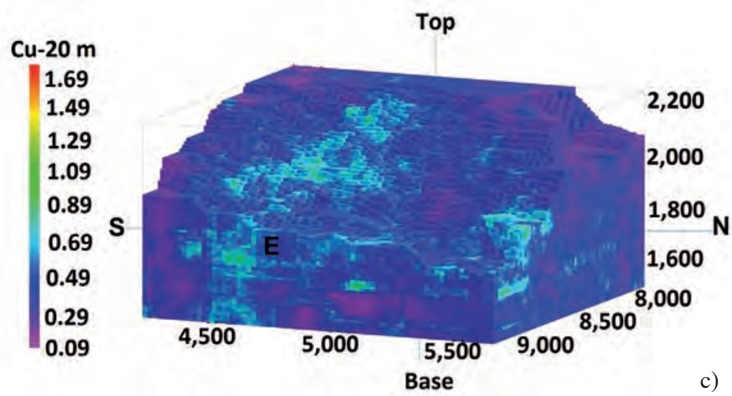
made on the basis of the smoothed estimates is likely to misrepresent the economic values of the project or operation. The LMUC technique provides a consistent framework to represent the economic values more accurately. The accuracy of the local estimation through this method depends on the block ranking techniques. The idea proposed in this study is the application of DBSIM realizations for ranking. The applied method efficiently preserves the spatial variation



a)



b)

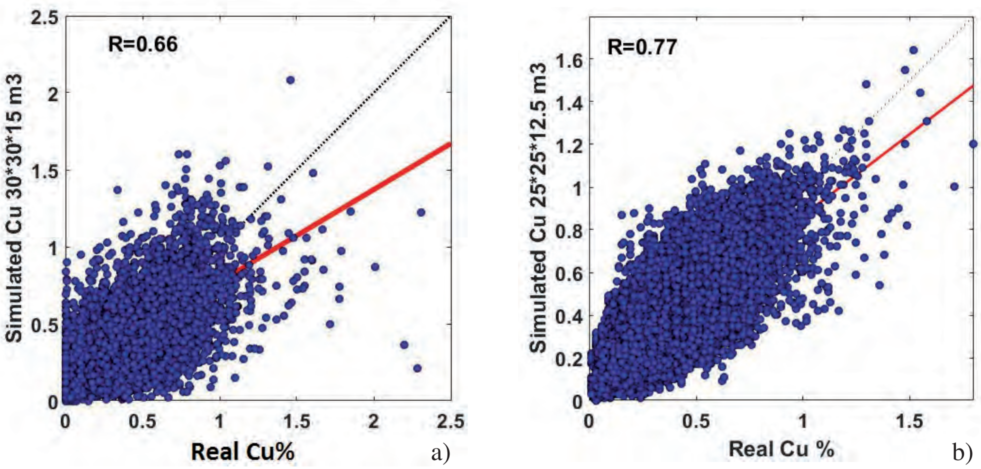


c)

Fig. 14 - 3D plan of Cu LMUC estimation assigned to: a) 30×30×15 m^3; b) 25×25×12.5 m^3; and c) 20×20×10 m^3 SMU.

of attributes and provides an accurate ranking and robust recoverable resource estimation. The grades of total Cu, CuO, and Mo were estimated in the case study area using LUC and the modified LMUC methods. Also, a sensitivity analysis was performed on the out-puts of different block sizes. The performance and evaluation are briefly described in the following:

- estimation/simulation methods yield SMU grade estimates with minimal conditional bias and high correlation with true data (r>0.7 for all the block estimations in this paper);



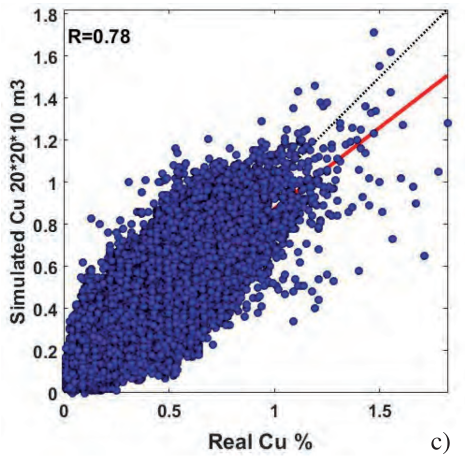


Fig. 15 - Scatter plot of Cu DBMS E-type and true variable for different supports.

- the support effect generates a loss of selectivity as the block size increases from a medium to a large panel and decreases from a large to a small panel. This variation is more critical when passing from a model with medium block size to one with a large block size than when passing from the medium to small block size. Accordingly, the support effect is more critical for smaller blocks;
- the greatest impact of the support effect is concentrated on the low-grade range, for which there is a greater loss of metal content for a given ore tonnage;
- the case study shows that the results from modified LUC are superior or equal to those from other methods. The similarity and adjustment between the outputs of modified LUC and standard methods and the real data demonstrates the robustness and efficiency of the modified method.

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