DOI 10.4430/bgta0168

Multivariate geostatistics based on a model of geo-electrical properties for copper grade estimation: a case study in Seridune, Iran

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(Received: June 17, 2013; accepted: December 28, 2015)

ABSTRACT In projects involving reserves estimation, a principal aim is to reduce the variance of the estimation and related uncertainty. This requires extensive and costly drilling. Among the variety of geostatistical-based techniques used to reduce the variance of the estimation in mineral reserves modelling, multivariate geostatistical methods can be appropriate tools when a sparse pattern of drilling boreholes exist. The present work introduces collocated cokriging and kriging with an external drift as multivariate geostatistical methods to incorporate sulphide factor as a secondary correlated variable to estimate Cu grade distribution. The study area is one of the potential zones of porphyry Cu occurrences, located in the Kerman province of southern Iran. To estimate the Cu grade distribution in this region, sulphide factor data as a dense correlated geophysical variable with the primary variable was used because this incurs less cost than drilling holes. Application of these multivariate geostatistical techniques to a specific exploration such as Seridune Copper Deposit in interpolating Cu grade measurements (primary data) using weakly correlated sulphide factor (secondary data) suggests that when Cu grade is undersampled, the secondary data can contribute substantially to identifying primary data. The results show that incorporating a secondary variable leads to better results than ordinary kriging (as a univariate method) that does not incorporate sulphide factor data. The validation of leave-out samples was used to compare the performance of the methods. Based on mean absolute error, root mean square error and the correlation coefficient of the observed and estimated values, the methods of the collocated cokriging and the kriging with an external drift outperformed grade estimation in comparison with ordinary kriging.

Key words: sulphide factor, ordinary kriging, collocated cokriging, kriging with external drift, Seridune copper deposit, Iran.

1. Introduction

In some engineering and industrial fields, interpolation methods are widely used to predict a spatial variable in an unknown region, and most times, multiple variables are sampled in addition to the one used to quantify the main phenomenon under study. That variable may be correlated with the principal variable of interest, being especially useful when sampled on a larger support. Then, the main attribute can be estimated from measures of itself (primary variable) and from the additional correlated secondary variable. Incorporating a secondary variable leads to more consistent models of the phenomenon under study (Boezio *et al.*, 2006a). Various geostatistical methods have been developed for incorporating a dense secondary variable. Among these methods are COllocated CoKriging (COCK) and Kriging with an External Drift (KED). Ordinary Kriging (OK) as a prevalent univariate geostatistical technique uses only one variable of interest to make estimates at unsampled locations. When secondary data are available, cokriging is the natural extension of kriging in the case of multiple variables. It presents several theoretical advantages over kriging a single variable. Its major advantages appear when secondary data are sampled on a larger support than the primary data (Boezio *et al.*, 2006b). However, high sampling density leads to instability of the cokriging set of equations in the inversion process (Wenlong *et al.*, 1992). Consequently, cokriging can be simplified to COCK by retaining only the secondary datum collocated with the location where the primary variable is being estimated. COCK also has the advantage over other methodologies in that it incorporates a secondary variable while accounting for spatial correlation. However, the principal disadvantage of this methodology comes from modelling of the coregionalisation, where direct and cross covariances must be inferred. The classic approach is based on the linear model of coregionalisation (LMC) where all direct and cross covariances are calculated as linear combinations of the same basic structures. As positive definiteness must be observed, modelling LMC is not straightforward (Boezio *et al.*, 2006b).

An alternative methodology taken into account for an exhaustive array of secondary variables is KED (Boezio *et al.*, 2006b). KED is applied in cases in which the main variable is correlated with the dependent external variable (Fernandes and Rocha, 2010). In general, KED is used to merge two sources of variables: a primary variable that is precise but only known at a few locations; and a secondary variable that is not precise but is available everywhere in the spatial domain. The second variable is related statistically to the primary variable (Grimes *et al.*, 1999). Finally, this method can be used if: 1) the spatial trend in the secondary (external) variable is related to that of the primary property of interest; 2) the residuals from the trend of the primary property can be modeled geostatistically; and 3) the property of interest and the variable from the more intensive sampling sites are linearly related (Boxter and Oliver, 2005).

In this paper, the results of the aforementioned methods, OK, KED, and COCK, are compared in later sections. The main objective of the present work is to outperform the grade estimation maps obtained solely from a primary variable. In this framework, this paper presents, through a case study, how a dense secondary variable acquired from geo-electrical properties (sulphide factor) can be used to improve the estimation of the primary attribute (Cu grade) via COCK and KED. Comparison with the methodology of OK, which makes no use of a secondary variable, is also presented.

2. Methodologies

The methodologies applied in the present work (OK, KED, and COCK) are briefly described in this section.

2.1. Ordinary Kriging (OK)

OK is one of the most commonly used forms of the generalized kriging estimator. Kriging is a method that is often associated with the best linear unbiased estimator. OK is linear because its estimates are weighted linear combinations of the available data; it is unbiased since it tries to have the mean of prediction errors equal to 0; it is best because it aims at minimizing the variance of error (Noshadi and Sepaskhah, 2005). Assuming the stationary characteristics of the domain of interest in reserves modelling (i.e., same statistical properties are applicable on the entire domain), the general equation of the OK estimator for the stationary random variable *Z(x)* is:

$$Z^*_{OK}(x_0) = \sum_{i=1}^{n} w_i Z(x_i) \quad (1)$$

where w_i is the weighting factor, Z is the measured (or random) variable, and Z^*_{OK} is the estimated variable by OK. In order to achieve unbiased estimators in kriging, the following set of equations should be solved simultaneously:

$$\begin{cases} \sum_{j=1}^{n} w_j Cov\left[Z(x_i), Z(x_j)\right] + \mu = Cov\left[Z(x_i), Z(x_0)\right], \ i = 1,2,\ldots,n \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \quad (2)$$

and the minimal prediction variance is:

$$Var_{OK}\left[Z(x_0)\right] = Cov\left[Z(x_0), Z(x_0)\right] - \sum_{i=1}^{n} w_i Cov\left[Z(x_i), Z(x_0)\right] + \mu \quad (3)$$

where μ is the Lagrange multiplier, *Cov* and *Var* stand for covariance and variance, respectively, which are inferred from the semi-variogram models while we have: $Cov\ (Z, Z) = Var\ (Z)$.

Two assumptions are needed to use kriging, namely stationarity and isotropy. Stationarity means that statistical properties do not depend on exact locations. Therefore, the mean (expected value) of a variable at one location is equal to the mean at any other location. Isotropy means that data variance is constant in the area under investigation and the correlation (covariance or semi-variogram) between any two locations depends only on the vector that separates them, not their exact locations.

2.2. Kriging with External Drift (KED)

In ordinary kriging, the assumption of local stationarity is made, i.e., the mean value of the variable over the search area is assumed constant. But in some situations, a trend is observed in the data such that the mean varies over the search area and it is therefore no longer locally stationary. In nonstationary cases, a secondary variable can be incorporated to specify the trend as a linear function of this external variable, using kriging with an external drift.

KED enables the random variable *Z*, known at relatively few points, to be estimated from another variable, *S*, that varies smoothly and is known at all points where an estimate is required. The smooth variability (trend) of the secondary variable is assumed to be related to that of the primary or target variable, *Z(x)*, being estimated. The method merges both variable sources; it uses *S(x)* as an external drift function to estimate *Z(x)*. The aim is to increase the precision of the predictions of *Z(x)*. The secondary variable must be known at all the sample locations of the primary variable and at all places on the prediction grid (Boxter and Oliver, 2005). KED is a particular type of universal kriging. It allows the prediction of a random variable *Z*, known only at a small set of points of the study area, through another variable *S*, exhaustively known in the same area. We choose to model *Z* with a random function *Z(x)* and *S* as a deterministic variable *S(x)*. The two quantities are assumed to be linearly related, i.e., it is assumed that the expected value of *Z(x)* is equal to *S(x)* up to a constant a_0 and a coefficient b_1 (Bourennane and King, 2003):

$$E\left[Z(x)\right] = a_0 + b_1 S(x) \tag{4}$$

Here, the local mean of the primary variable $E[Z(x)]$ is assessed locally, modelled as a linear function of a smoothly varying and exhaustively sampled secondary variable *S(x)* and with coefficients a_0 and b_1 considered constant in the local neighbourhood. Then, simple kriging is performed on the residuals from the local mean. The predictor is a linear combination of the sample values at location x_i *(i = 1, . . ., n):*

$$Z^*_{KED}(x_0) = \sum_{i=1}^{n} w_i Z(x_i) \quad \text{with} \quad \sum_{i=1}^{n} w_i = 1 \tag{5}$$

The kriging weights w_i are obtained by the solution of the following system of equations (Bourennane and King, 2003):

$$\begin{cases} \sum_{j=1}^{n} w_j Cov\left[Z(x_i), Z(x_j)\right] - \mu_1 - \mu_2 S(x_i) = Cov\left[Z(x_i), Z(x_0)\right], \\ \qquad\qquad\qquad\qquad i = 1,2,\ldots,n \\ \sum_{i=1}^{n} w_i = 1 \\ \sum_{i=1}^{n} w_i S(x_i) = S(x_0) \end{cases} \tag{6}$$

where *n* is the number of points in the search neighbourhood, *Cov* is the covariance of the residue, and μ_1 and μ_2 are the Lagrange multipliers that account for the unbiasedness constraints with the minimal prediction variance

$$Var_{KED}\left[Z(x_0)\right] = Cov(0) - \sum_{i=1}^{n} w_i Cov\left[Z(x_i), Z(x_0)\right] + \mu_1 + \mu_2 S(x_0). \tag{7}$$

The external drift method thus consists of incorporating into the kriging system additional universality conditions about one or several external drift variables, $S(x_i)$, $i=1, 2, \ldots, M$, measured exhaustively in the spatial domain. The function $S(x_i)$ needs to be known at all locations x_i of the samples of $Z(x_i)$, as well as at nodes of the prediction grid (Bourennane *et al.*, 2000). To be able to use the KED, it is necessary to calculate the residual between the primary and secondary data in order to construct the variogram used in estimating the main variable. In other words, theoretically, the variogram for KED needs to be inferred from the residuals *Z(x) - m(x)*, where *m(x)* is the drift between the primary and secondary variable (or the local smooth variation of the random variable *Z(x)* at the scale of observation). But this is not simple, because neither the residuals nor the trend *m(x)* is known *a priori*. A solution for this problem can be obtained by inferring the residual component variogram (or covariance function) from pairs that are not or only slightly affected by the trend. This can be performed by computing the directional variograms and retaining only the directional variogram that shows the least evidence of the trend (Goovaerts, 1999; Boxter and Oliver, 2005; Boezio *et al.*, 2006b; Haberland, 2007). Another approach involves determining the experimental residuals by removing, from the data values, the drift determined by the linear regression of the secondary variable. Then, simple kriging is performed on these residuals, and a drift locally determined through the KED system is then added (Boezio *et al.*, 2006b; Haberland, 2007). In kriging with external drift, the secondary variable needs to be known at all points as colocalization data. So, the secondary parameters were estimated by ordinary kriging, before the kriging with external drift for the primary variable was applied.

2.3. COllocated CoKriging (COCK)

COCK is a reduced and modified form of cokriging, in which the secondary variable used for estimation is reduced so as to retain only the secondary datum in the location where the primary variable is being estimated.

The COCK estimator is given by the following expression (Kay and Dimitrakopoulos, 2000),

$$Z^*_{COCK}(x_0) = \sum_{i=1}^{n} a_i\left[Z(x_i) - \overline{Z}\right] + b\left[S(x_0) - \overline{S}\right] + \overline{Z} \tag{8}$$

where a_i and b are kriging weights, S is the value of the secondary data set at the location where Z^* is being estimated and $\overline{Z}$, $\overline{S}$ are mean values of the primary and the secondary data. The COCK weights a_i and b are obtained by solving the following set of equations,

$$\begin{aligned} &\sum_{i=1}^{n} a_i Cov\left[Z(x_i), Z(x_j)\right] + bCov\left[S(x_0), Z(x_j)\right] = Cov\left[Z^*(x_0), Z(x_j)\right] \quad j = 1,2,\ldots,n \\ &\sum_{i=1}^{n} a_i Cov\left[Z(x_i), S(x_0)\right] + bCov\left[S^*(x_0), S^*(x_0)\right] = Cov\left[Z^*(x_0), S^*(x_0)\right] \end{aligned} \tag{9}$$

Here, the S^* (x_0) is the value of the secondary variable at the location where Z^* (x_0) is being estimated. In this system we only require the inference of $Cov(Z, Z)$, as $Cov(S, S)$ is not required, and $Cov(Z, S)$, $Cov(S, Z)$ can be evaluated by,

$$Cov(Z,S) = Cov(S,Z) = \sqrt{\frac{Var(S)}{Var(Z)}} \times \rho_{ZS} \times Cov(Z,Z) \tag{10}$$

where $Var(S)$, $Var(Z)$ are the variances of the S and Z sets of data and ρ_{ZS} is the linear correlation coefficient of the collocated Z and S data sets (Kay and Dimitrakopoulos, 2000).

It is necessary to model the covariance function for the primary variable, and when there is low correlation between the secondary and primary data, the resulting Z will not be compelled to be similar to the secondary S variable map. A significant disadvantage of the mentioned algorithm is that all noncollocated secondary data are ignored when estimating the Z variable at a given location (Kay and Dimitrakopoulos, 2000). When performing ordinary collocated cokriging in attendance of non-stationarity, the trend can be considered constant within a limitary local neighbourhood.

3. Background geology

The study area is part of the Urumieh-Dokhtar (Sahand-Bazman) magmatic arc assemblage that runs from NW to SE of Iran. This belt is classified as an Andean type magmatic arc shown in Fig. 1 (Alavi, 1980; Berberian *et al.*, 1982; John *et al.*, 2010). The north-western part of the Urumieh-Dokhtar magmatic arc is the product of Tethys oceanic plate subducted under the Iranian microplate followed by continent-to-continent collision of the Arabian and Eurasian plates (Regard *et al.*, 2004; John *et al.*, 2010). Seridune porphyry copper deposit is in a granodiorite-quartz monzonite pluton. Two large deposits belonged to this area are Sarcheshmeh and Darrehzar (Abedi *et al.*, 2013).

The detailed lithological map of the Seridune prospect is shown in Fig. 2a. This deposit consists of Eocene andesite and trachyandesite intruded by upper Miocene granodiorite, which

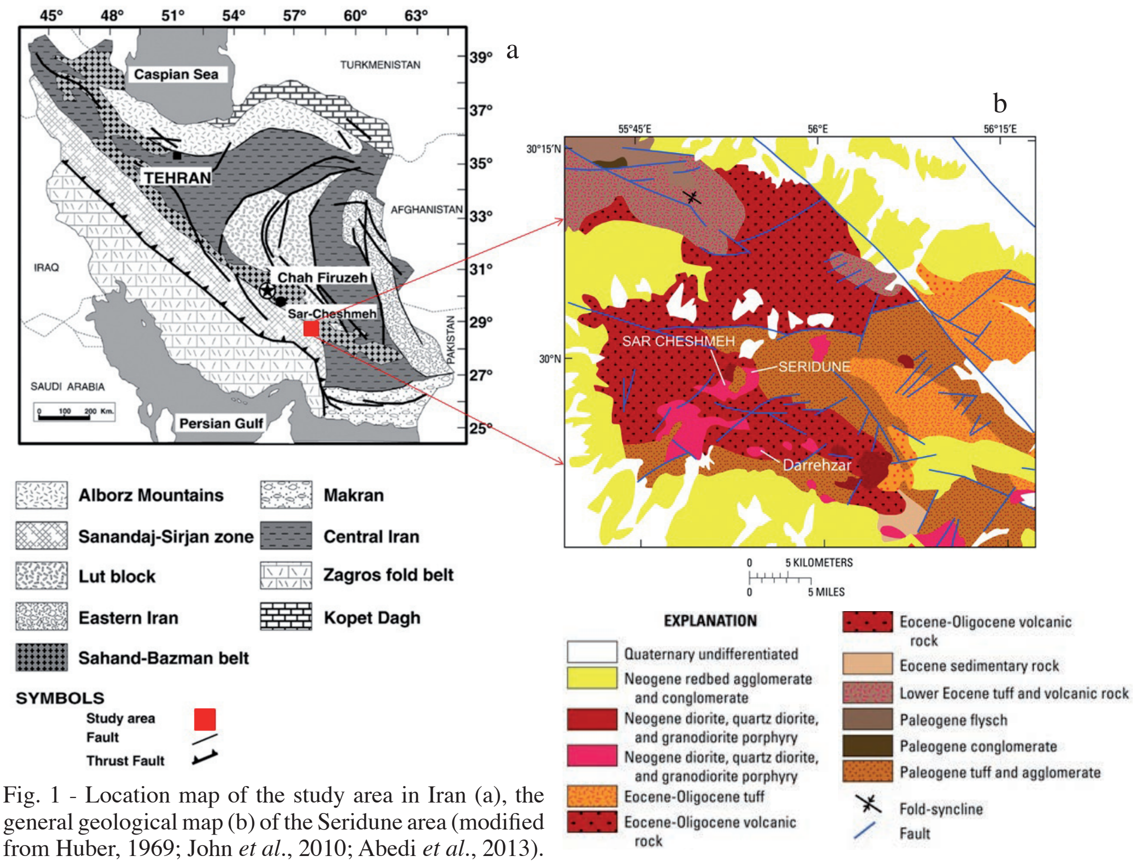


Fig. 1 - Location map of the study area in Iran (a), the general geological map (b) of the Seridune area (modified from Huber, 1969; John *et al.*, 2010; Abedi *et al.*, 2013).

is cut by quartz monzonite and granodiorite porphyry dikes (Barzegar, 2007; John *et al.*, 2010). Post mineralization Pliocene dacite and Quaternary gravels cover parts of the andesite and intrusive rocks. The granodiorites, monzonites, and andesites adjacent to the intrusive rocks contain complexly intermixed argillic and sericitic alteration zones and an area of propylitically altered rocks in the south-eastern part of the prospect. North-trending silica lithocaps cut argillic, sericitic, and propylitic alteration zones. A zone of advanced argillic-altered rocks borders the lithocaps, and quartz stockwork veins are in the central part of the prospect, Fig. 2b (Barzegar, 2007; John *et al.*, 2010; Abedi *et al.*, 2013).

4. Case study

Previous studies show that the Seridune copper deposit located in the Kerman province of Iran has high potential of copper occurrences. A fuzzy knowledge-based method which integrated various geophysical data in order to prepare a mineral prospectivity map generated a map in which high-potential zones correspond to higher fuzzy values. To prepare this map, shown in Fig. 3, different geophysical layers which derived from magnetic and electrical

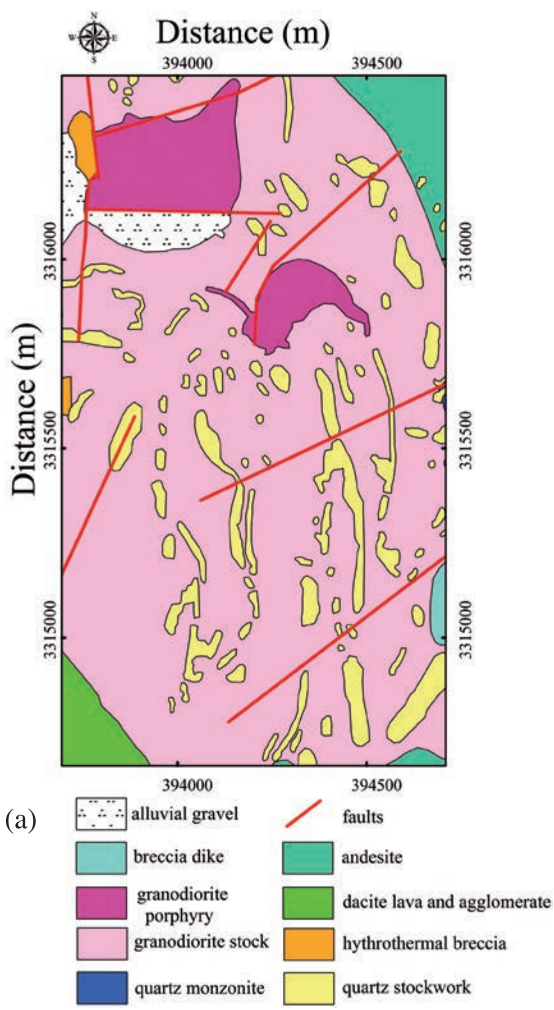


Fig. 2 - a) Detailed lithological map of the Seridune prospect; b) hydrothermal alteration map of the Seridune prospect (reproduced from Barzegar, 2007; John *et al.*, 2010; Kazemi Mehrnia *et al.*, 2011).

surveys were used to evaluate the Seridune copper deposit (Abedi *et al.*, 2013). Based on the mineral potential map, a sparse pattern of drilling was recommended. In the following, since multivariate kriging of a sparse pattern of drilling yields lower uncertainty in mineral deposit modelling, a model of geo-electrical property (i.e., the sulphide factor) as a second variable which has an acceptable level of correlation with the first variable, Cu, is used to make a model of Cu grade.

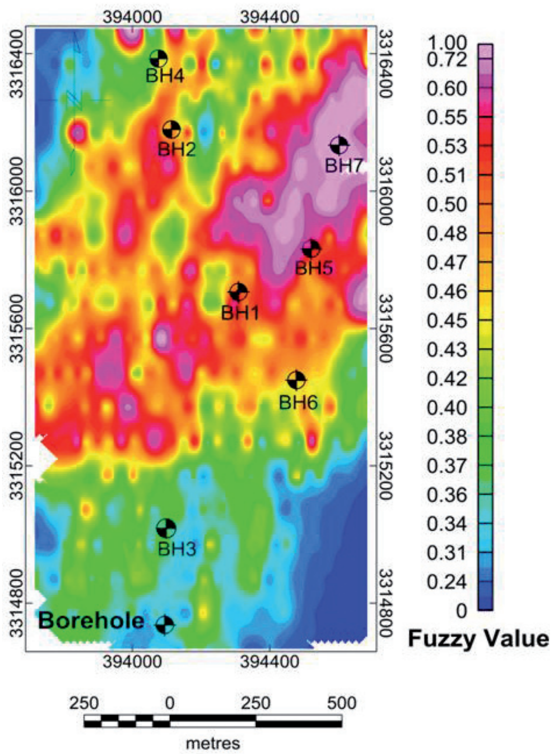


Fig. 3 - Mineral prospectivity map generated by fuzzy knowledge-based method, which indicates high-potential zones of mineral occurrences with higher fuzzy values. Drilled boreholes are shown by symbols on the map (Abedi *et al.*, 2013).

4.1. Data and methods

The primary data used in this study was Cu grade of boreholes and sulphide factor used as the secondary variable. The most common geophysical methods for exploration of sulphide deposits are electrical techniques. In this study, first Induced polarization (IP) "chargeability map" and resistivity (RS) surveys with rectangle array and two pole-dipole electrical profiles were implemented, and then with available resistivity and chargeability data, sulphide factor could be approximated by (Loke, 2010):

$$SF = \frac{M \times 2000}{6.6 \times \rho_a} , \qquad (11)$$

where M is chargeability, ρ_a is electrical resistivity, and SF is sulphide factor. The chargeability and resistivity are in terms of ms and Ω·m. Here, the 2D electrical data are inverted using RES2DINV Software as in Loke (2010). Fig. 4 shows the sulphide factor maps of rectangle

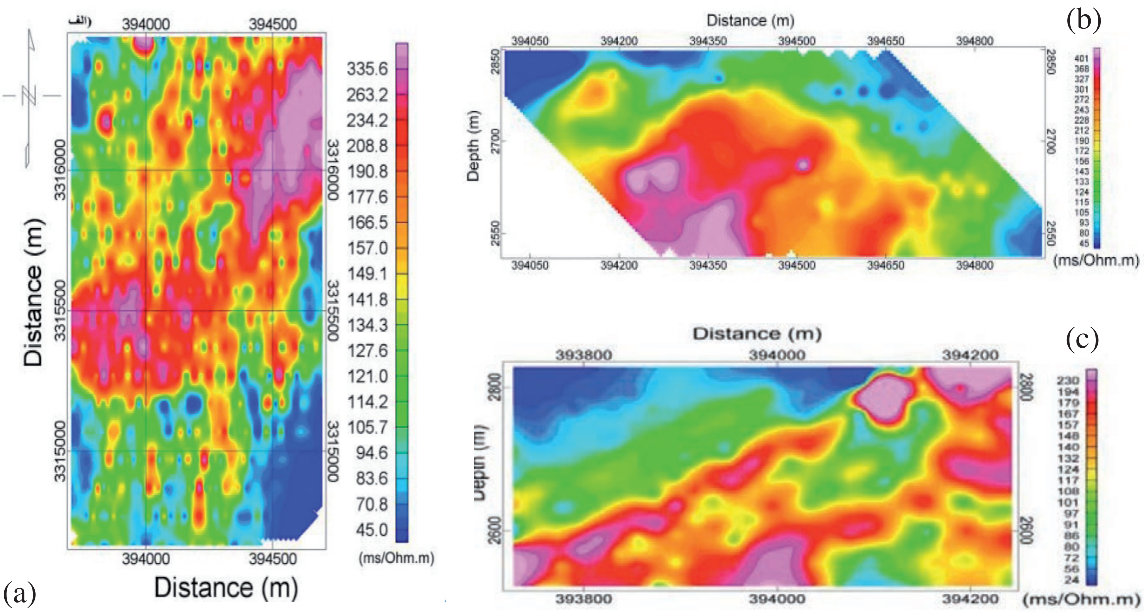


Fig. 4 - Sulphide factor maps of: a) rectangle array; b) and c) two pole-dipole profiles.

array and two pole-dipole profiles. Here a rectangular array with 1200-m space as current electrode was used such that distances between profiles and stations were 100 and 20 m. Electrode spacing for both pole-dipole profiles is 40 m.

In this survey, Cu data were composited from the sparse drilled boreholes which are shown in Fig. 5. The length of the composited boreholes is 5 m at three dimentions *x*, *y,* and *z*. A grid with specific dimensions has been designed to interpolate Cu grade for an undersampled pattern of boreholes. This grid contains all primary and secondary data (Fig. 5). Since the semi-detailed exploration drilling has been done in this study, the drilling grid is sparse and undersampled to create a 3D model of Cu grade in the desired area. The length of the boreholes varies from 250 up to 360 m for 9 drilled boreholes, which certainly yields high uncertainty when applying kriging. Therefore, incorporating a secondary variable can decrease the uncertainty of the constructed Cu grade model. The point should be noted that a high correlation between the primary variable (Cu grade) and the secondary one (*SF*) should exist to allow the application of a multivariate geostatistical approach like the COCK and KED. The estimation of the *SF* was performed by ordinary kriging. *SF* at the locations of measured Cu was also determined. This collocated data configuration allows verification of the correlation and the linear relationship between the primary and the secondary variables. The scatter plot shows that the correlation coefficient exceeds 0.466 (Fig. 6). Since the correlation coefficient is a bit low, we used the collocated values of *SF* rather than using all data. One of the main advantages of this technique is the reduction of the effect of the secondary variable when this correlation is low (Abedi *et al.*, 2015).

Two compulsory assumptions of stationarity and isotropy are required to apply any kriging methods. Here, to demonstrate that the data satisfy the “isotropy assumptions”, Fig. 7 is provided, showing the regional variable (*SF*) has isotropic behaviour in different directions (variograms are provided for four 0, 45, 90, 135 azimuths). As illustrated, the variograms in

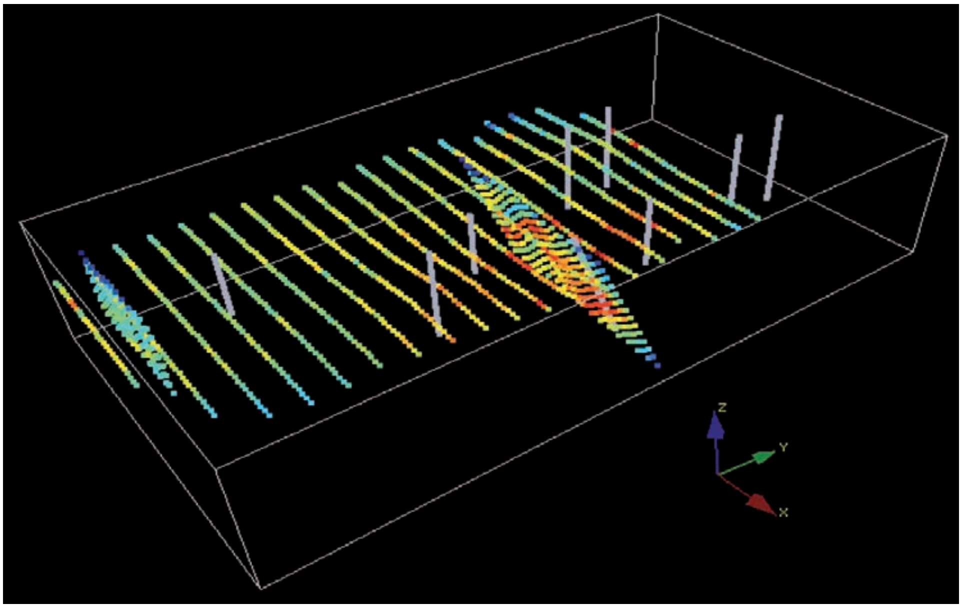


Fig. 5 - Grid for estimation of Cu grade based on the primary variable derived from composited drilled boreholes and the sulphide factor as a correlated secondary variable.

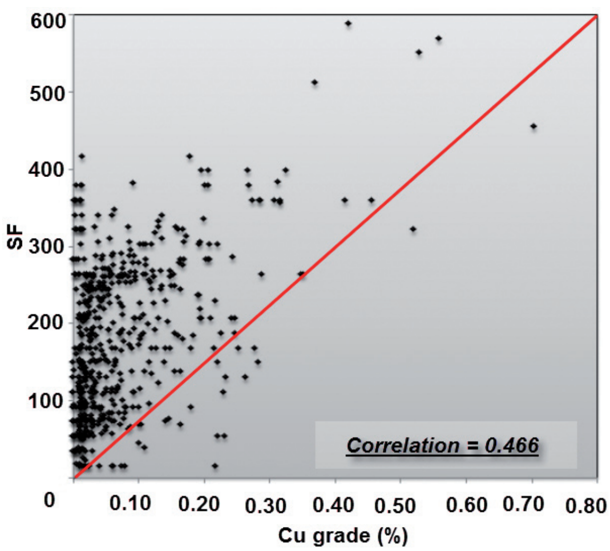


Fig. 6 - Scatter plot of Cu and *SF* variables, which shows linear correlation between the composited Cu grade data and the *SF*. Here, the correlation coefficient is equal to 0.466.

different azimuths have approximately the same ranges and sills. It is worth mentioning that in comparison with anisotropy in layered deposits, it can be ignored.

To address the stationarity of the data, scatter plots of the *SF* variable are plotted versus coordinates of *X* and *Y*, respectively. Fig. 8 indicates that there is not any significant trend in either *X* or *Y* directions. Therefore, it can be assumed that there is no meaningful trend in any direction, indicating stationarity of the studied domain.

To apply three methods, i.e., COCK, KED, and OK, we need to have information about semi-variogram models of both the primary and the secondary variable. The Experimental

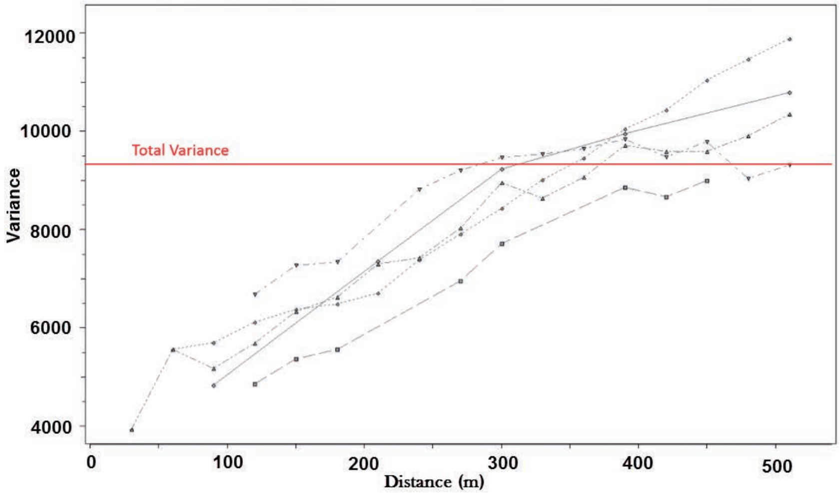


Fig. 7 - Similar variogram curves of secondary variable *SF* for four azimuths of 0, 45, 90, and 135, showing the isotropic behaviour of the studied domain.

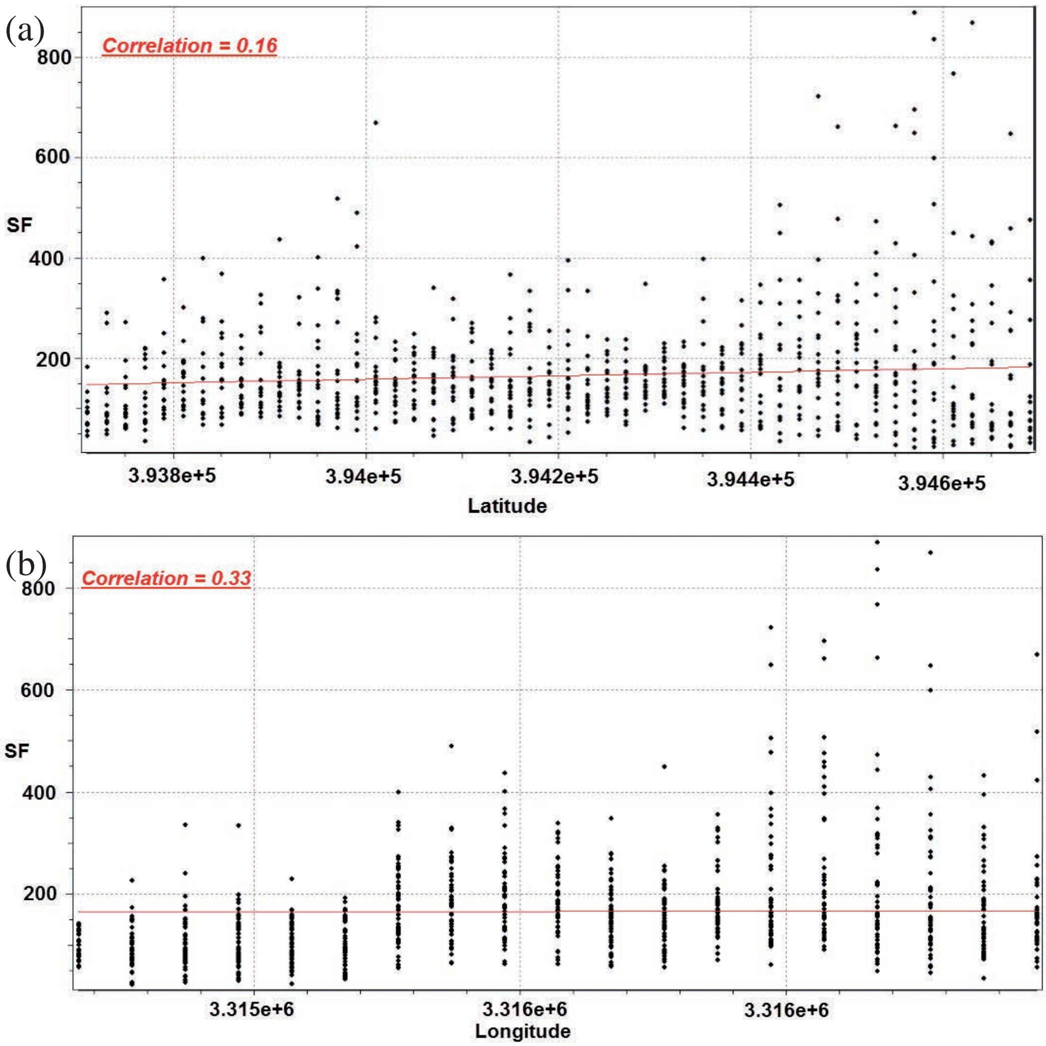


Fig. 8 - Scatter plot of *SF* versus coordinates of *X* (a), and *Y* (b), showing the stationarity behaviour of the studied domain.

omni-directional semi-variograms models for both the composited Cu grade data and the *SF* are shown in Figs. 9a and 9b, respectively. Since the kriging set of equations should be applied to a variable with normalized distribution, the normalized transform was applied for both variables. The ranges of the semi-variogram models for the composited Cu grade and the *SF* are 300 and 200 m, respectively. The experimental semi-variogram of the composited Cu grade has been modelled with a spherical variogram which has a nugget and sill values equal to 0.37 and 0.63, respectively. The spherical model has also been used to fit a model which has a nugget and sill values equal to 0.3 and 0.7, respectively, for the *SF* data. Here, we have used the Stanford Geostatistical Modeling Software (SGeMS), which is free for all users.

At first the KED and the COCK were carried out to estimate distribution of Cu grade. The kriging maps are presented in Figs. 10b and 10c. To compare, the Cu grades were also estimated

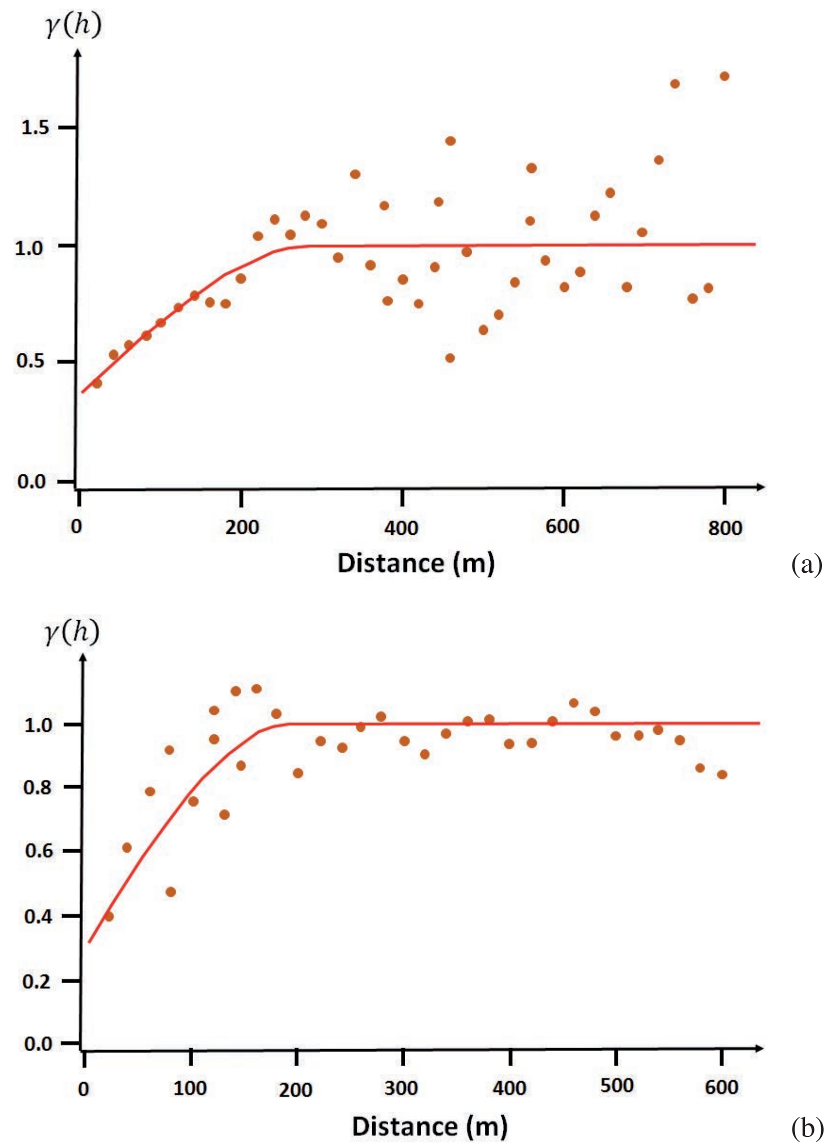
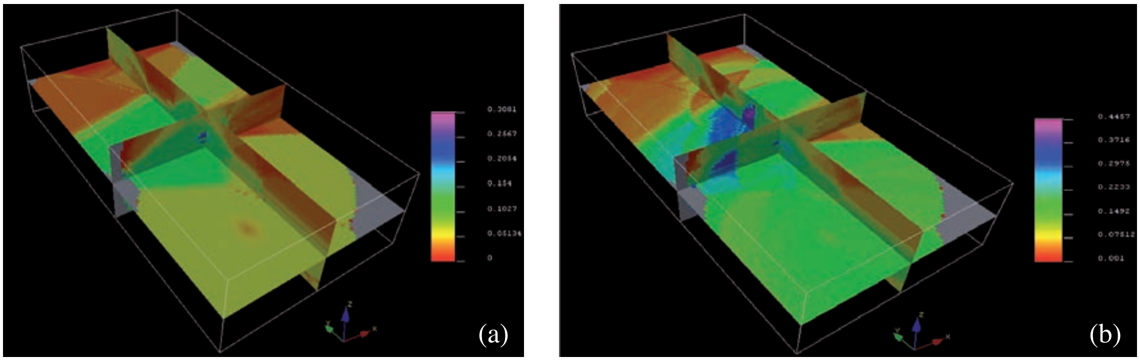


Fig. 9 - Experimental omni-directional semi-variograms and the models of: a) the composited Cu grade data; b) the *SF*.

by OK (Fig. 10a). The obtained maps of the Cu grades show the difference between the three estimators, but it is clear that estimates by the COCK and the KED are less influenced by boreholes than ordinary kriging. The maps obtained using the COCK (Fig. 10c) and the KED (Fig. 10b) reveal a greater influence of the sulphide factor compared with the excessively smooth OK estimates.



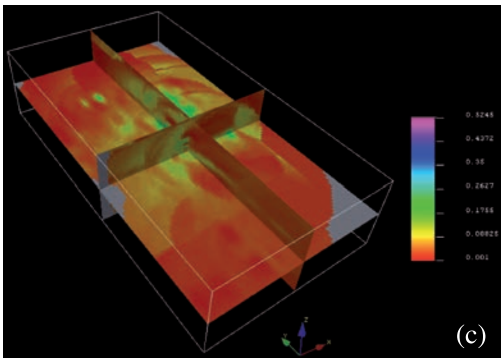
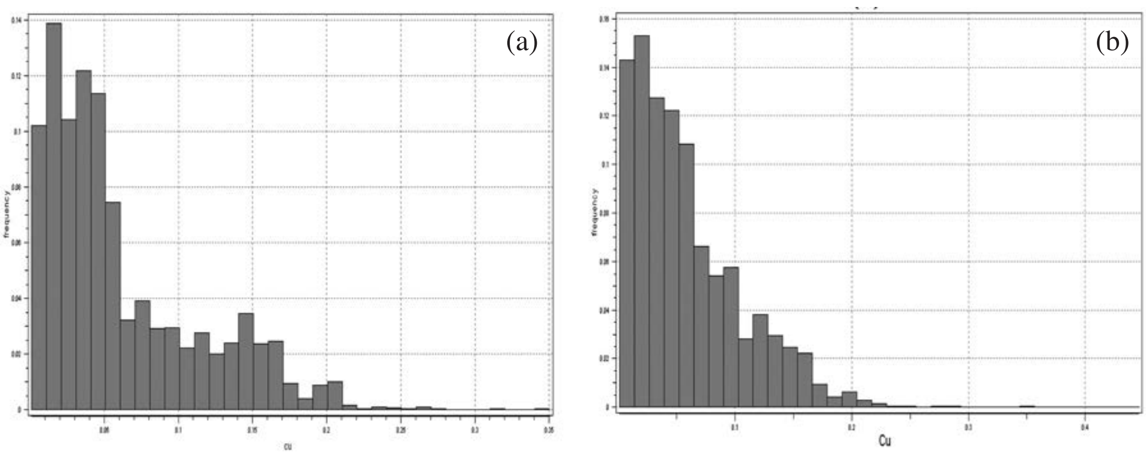


Fig. 10 - 3D interpolation of Cu grade using: a) OK; b) KED; c) COCK.

Table 1 - Statistical parameters for the methods applied in this study.

	COCK	KED	OK
Standard deviation of the estimates	0.035	0.148	0.166
Mean Cu	0.05	0.059	0.062
Standard deviation of Cu	0.045	0.046	0.052
Minimum	0.001	0.001	0.001
Maximum	0.542	0.45	0.35
Range	0.541	0.449	0.349

Table 1 summarizes the statistical parameters for methods that were used in this study. Standard deviation of the estimates with the minimum and maximum interpolated values of Cu and standard deviation of Cu with a range of interpolated values of Cu are shown in Table 1. The histograms of the estimates are presented in Fig. 11. The range of changes for Cu in COCK and KED is greater than with OK. It shows that these methods can estimate low/high values of Cu grades better than the OK. But the OK shows greater dispersion in the distribution around the mean value.



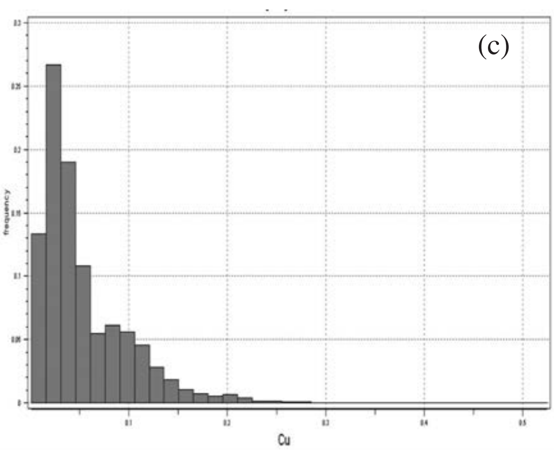


Fig. 11 - Histograms of the estimated Cu grade obtained by: a) OK; b) KED; c) COCK.

4.2. Validation

A survey validation was performed by leaving out 20% of the samples that were selected randomly from the data set and then estimated using the remaining data and methods. Using the real data value and the estimated value, based on mean absolute error (MAE), root mean square error (RMSE), and the correlation coefficient of real and estimated values (R), validation was performed. Scatter plots of real and estimated values of all methods that were used are presented in Fig. 10. The comparison of the results shows that the COCK and the KED methods by incorporating the secondary variable, i.e., *SF*, have better results. Results for validations are presented in Table 2.

Table 2 - Validation results for the methods applied in this study.

	MAE	RMSE	R
OK	0.0309	0.00488	0.8025
KED	0.02806	0.00439	0.84897
COCK	0.03218	0.00470	0.82116

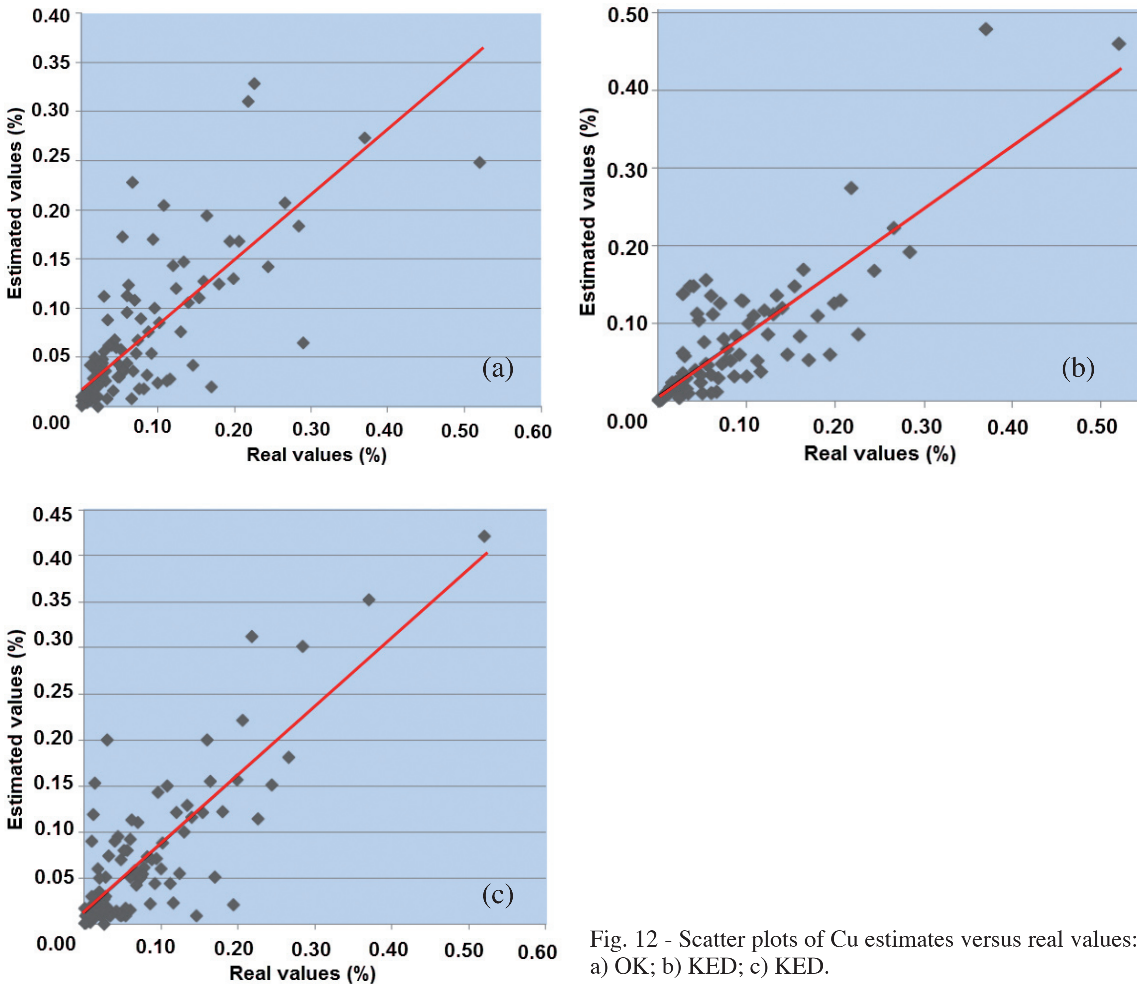


Fig. 12 - Scatter plots of Cu estimates versus real values: a) OK; b) KED; c) KED.

5. Conclusions

This paper described the application of three well-known geostatistical approaches known as ordinary kriging, collocated cokriging, and kriging with an external drift in order to estimate Cu grade distribution. To reduce the uncertainty arising from a sparse pattern of drilled boreholes in a studied area, sulphide factor (*SF*) as a correlated auxiliary geophysical variable was incorporated in Cu grade estimation, showing its effectiveness when information is lacking. Collocated cokriging and kriging with an external drift that incorporate a secondary variable based on *SF* lead to better results than ordinary kriging that does not incorporate *SF* data. The resultant maps of the Cu grades show that estimates by the collocated cokriging and the kriging with external drift are less influenced by drilled boreholes than ordinary kriging. Based on the statistical values of the mean absolute error, the root mean square error, and the correlation coefficient of the real and the estimated values acquired from the leave-out-samples, two multivariate geostatistical methods could substantially outperform Cu estimates in the studied porphyry deposit located in Iran.

Acknowledgements. The authors gratefully acknowledge the School of Mining Engineering, University of Tehran, for all of their support. We would like to thank D. Slejko, Editor-in-Chief of the Bollettino di Geofisica Teorica ed Applicata, P. Holota, Associate Editor of this journal, and two anonymous referees who contributed constructive and valuable comments, which helped us to improve the quality of our work.

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