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Application of fuzzy AHP method to integrate geophysical data in a prospect scale, a case study: Seridune copper deposit

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ABSTRACT This paper describes the usage of fuzzy knowledge based method to integrate various geophysical data in order to prepare Mineral Prospectivity Map (MPM). Different geophysical layers which are derived from magnetic and electrical surveys are used to evaluate Seridune copper deposit located in the Kerman province of Iran. Since, electrical layers involving resistivity, induced polarization and metal factor, in comparison with layers derived from magnetic survey, are more important in porphyry copper exploration, fuzzy Analytical Hierarchy Process (AHP) is used to determine the weights belonged to each of them. Four layers which are derived from magnetic data involving upward continuation, analytic signal, reduced to pole and pseudo gravity are considered in this study. Three geoscientists who are expert in geophysical prospecting are used to implement fuzzy AHP. After determination of the normalized weights of the seven evidential layers as main criteria, fuzzy operators are applied to integrate different layers. To evaluate the result of the approach, drilled boreholes are used to validate final MPM. By considering a borehole which is belonged to the suggested area for drilling, a satisfactory result was obtained. The proposed method can be a useful approach for integrating various geodata sets for MPM.

Key words: fuzzy AHP, fuzzy operators, electrical and magnetic surveys, MPM, porphyry copper deposit.

1. Introduction

Mineral exploration is a sophisticated process in which the main purpose is to discover a new mineral deposit in the region of interest. To achieve this goal, one of the main steps in mineral exploration is to demarcate prospective areas in the region. In this manner to do so, various thematic (e.g., geological, geophysical, geochemical) geodata sets should be collected, analysed and integrated for Mineral Prospectivity Map (MPM) in the region of interest. The MPM process is, in fact, a Multi-Criteria Decision-Making (MCDM) task in different scales. However, the MPM is a predictive model of outlining prospective areas.

Several approaches may be used for MPM, which can be divided into either, data-driven or knowledge-driven methods (Bonham-Carter, 1994; Pan and Harris, 2000; Carranza, 2008). In data-driven techniques, the known mineral deposits in a region of interest are used as 'training points' for establishing spatial relationships with particular geological, geochemical and geophysical features. The spatial relationships between the input data and the training points

are quantified and used to establish the importance of each evidence map and finally integrated into a single MPM (Nykänen and Salmirinne, 2007; Carranza, 2009). Examples of the empirical methods used are weights of evidence (Bonham-Carter *et al.*, 1989), logistic regression (Agterberg and Bonham-Carter, 1999), neural networks (Singer and Kouda, 1996; Porwal *et al.*, 2003, 2004), and evidential belief functions (Carranza and Hale, 2002; Carranza, 2008).

The other techniques, in which geoscientist's expert opinion is applied, are called knowledge-driven methods involving the use of Boolean logic (Bonham-Carter, 1994), index overlay (Bonham-Carter, 1994), the Dempster-Shafer belief theory (Moon, 1990), and fuzzy logic overlay (Chung and Moon, 1990; An *et al.*, 1991).

If there are databases of previous exploration projects, MPM can be a classification process for outlining new prospective areas in a region of interest. There are two types of classification: supervised and unsupervised. Supervised classification is used to categorise every location as either prospective or non-prospective based on various evidential layers and a training data vector of known deposit locations and non-deposit locations. The other type is known as unsupervised classification is based only on features statistics of evidential layers.

Selecting the best area for exploratory drilling is a MCDM problem. In this study, different evidence layers of geophysical data such as electrical and magnetic surveys are used to prepare MPM. Fuzzy Analytical Hierarchy Process (AHP) is applied to determine the weights belonged to each layer in order to integrate these evidential layers. Three Decision Makers (DMs) who have the knowledge related to geophysical prospect are used to calculate weights using fuzzy AHP. Finally, fuzzy operators are used to integrate weighted layers to generate final MPM. Real data, Seridune copper deposit located in Kerman, central part of Iran, is chosen to suggest additional drilling. Obtained results are compared to the boreholes in the study area to validate the MPM.

2. Fuzzy Knowledge based Method

Summary of the methods are described as follow.

2.1. AHP method

The AHP which was suggested and developed by Saaty (1980, 1986, 1988, 1995) is one of the well-known techniques used as MCDM approach. The AHP is a theory of relative measurement on absolute scales of both tangible and intangible criteria, based both on the judgment of knowledgeable and expert people and on existing measurements and statistics needed to make a decision. How to measure intangibles is the main concern of the mathematics of the AHP. The AHP has been mostly applied to multi-objective, multi-criteria and multiparty decisions because decision-making has this diversity (Figueira *et al.*, 2005).

The procedure of applying the AHP is based on three principles, namely: (1) construction of a hierarchy, (2) priority setting and (3) logical consistency (Macharis *et al.*, 2004). These procedures are described as follow.

2.1.1. Construction of a hierarchy

The first step to implement the AHP for a complex decision problem is structured as a hierarchy. AHP initially breaks down a complex MCDM problem into a hierarchy of interrelated

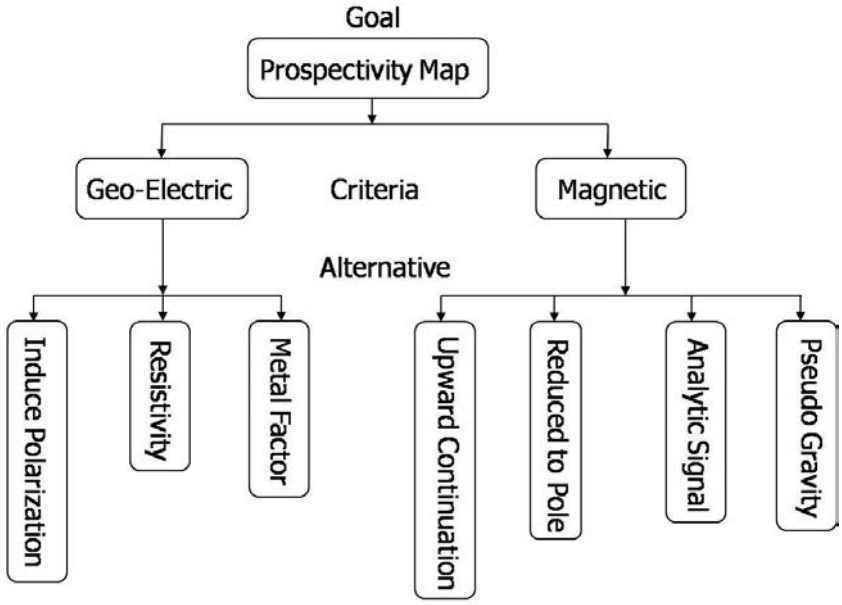

Fig. 1 - Hierarchical structure of mineral prospectivity map.

decision elements (criteria, decision alternatives). With the AHP, the objectives, criteria and alternatives are arranged in a hierarchical structure similar to a family tree. A hierarchy has at least three levels: overall goal of the problem at the top, multiple criteria that define alternatives in the middle, and decision alternatives at the bottom (Dağdeviren, 2008). The hierarchical structure that is used to prepare MPM in this study is shown in Fig. 1.

2.1.2. Priority setting

The relative ‘‘priority or weight’’ given to each element in the hierarchy is determined by comparing pairwise the contribution of each element at a lower level in terms of the criteria (or elements) with which a causal relationship exists (Macharis *et al*., 2004). The pairwise judgment starts from the second level and finishes in the lowest level, alternatives. In each level the criteria are compared pairwise according to their level of influence and based on the specified criteria in the higher level. The DM uses a standardised comparison scale of nine levels that is shown in Table 1 (Dağdeviren, 2008).

Table 1 - Nine-point intensity of importance scale and its description.

Definition	Intensity of importance
Equally important	1
Moderately more important	3
Strongly more important	5
Very strongly more important	7
Extremely more important	9
Intermediate values	2, 4, 6, 8

Let $C = \{C_j \mid j = 1,2,...,n\}$ be the set of criteria. The results of the pairwise comparison on n criteria can be summarized in an $(n \times n)$ evaluation matrix A in which every element a_{ij} $(i, j = 1,2,...,n)$ is the amount of weights of the criteria that is shown in Eq. 1 (Dağdeviren, 2008):

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \; a_{ii} = 1, \; a_{ji} = 1/a_{ij}, \; a_{ij} \neq 0. \tag{1}$$

The mathematical process starts to normalise and find the relative weights for each matrix. The relative weights are given by the right eigenvector (G) corresponding to the largest eigenvalue (λ_{max}) as:

$$AG = \lambda_{max} G \tag{2}$$

If the pairwise comparison are completely consistent, the matrix A has rank 1 and $\lambda_{max} = n$. In that case, weights can be acquired by normalising any of the rows or columns of A matrix (Dağdeviren, 2008).

2.1.3. Logical consistency

It should be noted that the quality of the output of the AHP is strictly related to the consistency of the pairwise comparison judgments. The complete consistency is defined for relation as follow:

$$a_{ij} \times a_{jk} = a_{ik} \tag{3}$$

when the pairwise comparison matrices are completely consistent, the priority (or weight) vector corresponds to the right eigenvector (G). Therefore, the highest eigenvalue (λ_{max}) is equal to n. In case the inconsistency of the pairwise comparison matrices is limited, slightly λ_{max} deviates from n. This deviation ($\lambda_{max} - n$) is used as a measure for inconsistency. This measure that is divided by (n-1) yields the average of the other eigenvectors (Macharis *et al.*, 2004). The consistency index (CI) is:

$$CI = (\lambda_{max} - n) / (n - 1) \tag{4}$$

The final Consistency Ratio (CR), on the basis of which one can conclude whether the evaluations are sufficiently consistent, is calculated as the ratio of the CI and the Random Index (RI is given in Table 2) and it corresponds to the degree of consistency that automatically arises when completing at random reciprocal matrices with the values on the 1 - 9 scale (Macharis *et al.*, 2004):

$$CR = CI / RI. \tag{5}$$

The number 0.1 is the accepted upper limit for *CR*. If the final *CR* exceeds this value, the evaluation procedure has to be repeated to improve consistency. The measurement of consistency can be used to evaluate the consistency of DMs as well as the consistency of all the hierarchy (Dağdeviren, 2008).

Table 2 - Random Index (from Macharis *et al*., 2004).

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

2.2. Fuzzy AHP method

In the fuzzy extension of AHP, the weights of the nine-level fundamental scales of judgments are expressed via the Triangular Fuzzy Numbers (TFN) in order to represent the relative importance among the hierarchy criteria (Karimi *et al*., 2011).

A fuzzy number M on R ($M \in M(R)$) is a TFN if its membership functions $\mu_M(x) : R \rightarrow [0,1]$ is equal to

$$\mu_M(x) = \begin{cases} \dfrac{x}{m-l} - \dfrac{l}{m-l}, & x \in [l,m] \\ \dfrac{x}{m-u} - \dfrac{u}{m-u}, & x \in [m,u] \\ 0, & otherwise \end{cases} \tag{6}$$

where $l \le m \le u$, l and u stand for the lower and upper value of the support of M respectively, and m gives the maximal grade of the membership function $\mu_M(x)$. Here, $M(R)$ represents all fuzzy sets, and R is the set of real numbers. The triangular fuzzy number can be denoted by (l, m, u). The support of M is the set of elements $\{x \in R \mid l < x < u\}$ (Chang, 1996).

The fuzzy AHP is a popular technique which has been applied for MCDM problems. This method was expressed by Laarhoven and Pedrycs (1983) where the fuzzy comparing judgment is represented by TFN. The steps of applying fuzzy AHP that is suggested by Chang (1996) are used in this study. The paper published by Karimi *et al*. (2011) is used to describe these steps as follow:

Step 1: a group of t Decision Makers (DM_p) is used to prepare pairwise comparison. Each DM individually will construct a Pairwise Comparison Matrix (PCM) as shown in Eq. (7) for each criterion:

$$DM_p = \begin{bmatrix} a_{11_p} & a_{12_p} & \dots & a_{1m_p} \\ a_{21_p} & a_{22_p} & \dots & a_{2n_p} \\ . & . & & . \\ . & . & & . \\ . & . & & . \\ a_{n1_p} & a_{n2_p} & \dots & a_{mm_p} \end{bmatrix} \quad p = 1,2,\dots,t, \tag{7}$$

where, m is the number of alternatives for each criterion and t is the number of DMs. For instance in Fig. 1, the number of alternatives m for magnetic criterion is 4. Then, a comprehensive PCM is constructed by integrating the grades of all DMs via Eq. (8). By this way, the PCM values of DMs are transformed into TFN to make the fuzzy evaluation matrix:

$$\begin{aligned} &l_{ij} = \min(a_{ij_p}), \qquad m_{ij} = \sum_{p=1}^{t} a_{ij_p} \Big/ t, \qquad u_{ij} = \max(a_{ij_p}) \\ &M_{ij} = (l_{ij}, m_{ij}, u_{ij}), \quad p = 1,...,t, \;\; i = 1,...,m, \;\; j = 1,...,m \end{aligned} \tag{8}$$

where, $\min(a_{ij_p})$ and $\max(a_{ij_p})$ indicate minimum and maximum values of PCMs prepared by DMs for each i, j respectively. This step is used to construct fuzzy evaluation matrices with respect to criteria and alternatives of designed hierarchy.

Step 2: the value of the fuzzy synthetic extent with respect to the i-th object of m alternatives for each criterion is defined by:

$$S_i = \sum_{j=1}^{m} M_{ij} \odot \left[\sum_{k=1}^{m} \left(\sum_{j=1}^{m} M_{kj} \right) \right]^{-1}, \; i, j, k = 1,...,m \tag{9}$$

where, all the M_{ij} are TFNs after construction of the fuzzy evaluation matrix. The symbol $\odot$ indicates the fuzzy multiplication operator. Considering two TFNs $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$, their operational laws are as follows:

$$(l_1, m_1, u_1) \odot (l_2, m_2, u_2) \approx (l_1 l_2, m_1 m_2, u_1 u_2) \tag{10}$$

$$(l_1, m_1, u_1)^{-1} \approx (1/u_1, 1/m_1, 1/l_1). \tag{11}$$

Step 3: as $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two TFNs, the degree of possibility (V) of $M_2 \geq M_1$ is defined by:

$$V(M_2 \geq M_1) = \begin{cases} 1 & if \;\; m_2 \geq m_1 \\ 0, & if \;\; l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & otherwise. \end{cases} \tag{12}$$

Fig. 2 represents evaluation of two TFNs M_1 and M_2.

Step 4: to compare M_1 and M_2, it needs to consider both values of $V(M_2 \geq M_1)$ and $V(M_1 \geq M_2)$. The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i ($i = 1, ..., k$) can be defined by:

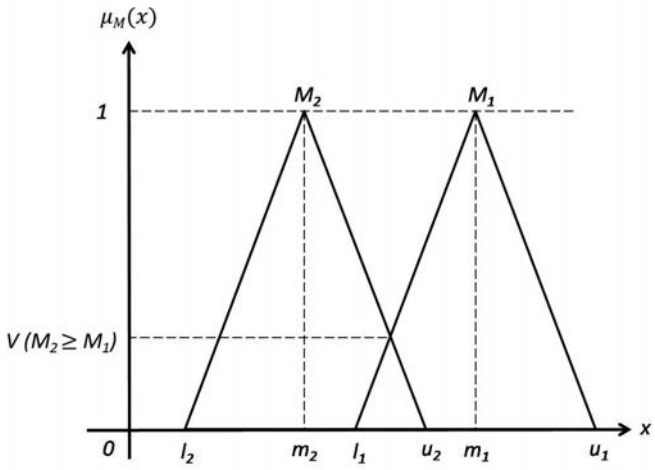

Fig. 2 - Evaluation of two TFNs M_1 and M_2.

$$V(M \geq M_1,...,M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } ... (M \geq M_k)] = \min V(M \geq M_i),\ i = 1,...,k \quad (13)$$

Assume that: $d(B_i) = \min V(S_i \geq S_k)$, $k = 1, ..., m$; $k \neq i$. Then the weight vector is given by:

$$W' = (d'(B_1), ..., d'(B_m))^T \quad (14)$$

where, $B_i (i = 1, ..., m)$ are m elements.

Step 5: via normalisation, the normalised weight vectors are:

$$W = (d(B_1), ..., d(B_m))^T \quad (15)$$

where, W is a non-fuzzy number.

The point should be noted is that the extent analysis method on fuzzy AHP is a reliable method except in cases that irrational zero weight to some useful decision criteria and alternatives are assigned. The weights determined by the extent analysis method in such cases do not represent the relative importance of decision criteria or alternatives and cannot be used as their priority. We refer interested readers to study paper published by Wang *et al*. (2008) to prevent zero weights of criteria and alternatives which cause wrong priority weights.

2.3. Using fuzzy operators

The conceptual fuzzy approach uses the expertise of the exploration geologists, geochemists, and geophysicists to define the threshold values for the evidential data sets. In classical set theory, the membership of a set is defined as true or false (1 or 0), whereas membership of a fuzzy set (μ) is expressed on a continuous scale from 0 to 1 (e.g., somewhere between 'anomalous' and 'not anomalous'). The values of fuzzy membership can be chosen based on subjective judgment of an expert (Nykänen and Salmirinne, 2007). In the Table 3, some prevalent fuzzy operators are described.

It is common to determine the weight of alternatives by knowledge-driven methods such as fuzzy AHP. In this study, fuzzy AHP method is applied to calculate the weights of each evidential layer and then fuzzy operators are used to integrate them; finally MPM is produced.

Table 3 - Fuzzy operators (Bonham-Carter, 1994; Nykänen and Salmirinne, 2007).

Description	Boolean Equivalent	Function	Operator
It works as minimum operator and creates smallest fuzzy membership values. Fuzzy AND uses when all the evidence must exist for a mineral occurrence.	AND	$\mu_{FA} = MIN(\mu_1, \mu_2, ..., \mu_N)$ *	Fuzzy AND
It works as maximum operator and creates largest fuzzy membership values. Fuzzy OR uses when every of evidence could be sufficient for a mineral occurrence.	OR	$\mu_{OR} = MAX(\mu_1, \mu_2, ..., \mu_N)$	Fuzzy OR
It tends to reduce effect of multiplying several numbers smaller than, or equal to, the smallest contributing membership value.		$\mu_{FAP} = \prod_{i=1}^{n} \mu_i$	Fuzzy Algebraic Product
It tends to increase effect of multiplying several numbers larger, or equal to, the largest contributing membership value.		$\mu_{FAS} = 1 - \prod_{i=1}^{n} (1 - \mu_i)$	Fuzzy Algebraic Sum
It combines the effect of fuzzy algebraic product and the fuzzy algebraic sum.		$\mu_{FG} = (\prod_{i=1}^{n} \mu_i)^{1-\gamma} \cdot (1 - (\prod_{i=1}^{n} 1 - \mu_i))^{\gamma}$	Fuzzy Gamma

* μ_1, μ_2,..., μ_N are, respectively, the input fuzzy evidential scores at a location in evidence map 1, evidence map 2, … evidence map n.

The summary procedure of applying fuzzy method to prepare MPM is shown in Fig. 3.

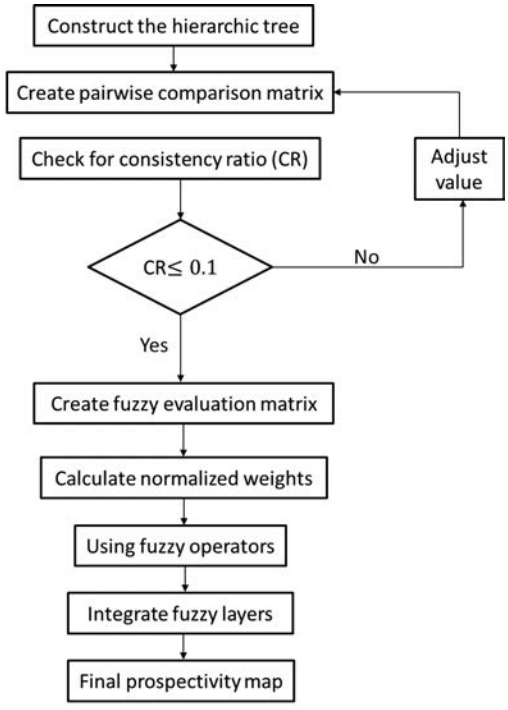

Fig. 3 - The procedure of applying fuzzy method.

3. Geology of the study area

The study area (Fig. 4) is part of the Urumieh-Dokhtar magmatic arc assemblage that runs from NW to SE of Iran. This belt is classified as an Andean type magmatic arc (Alavi, 1980; Berberian *et al*., 1982; John *et al*., 2010). The north-western part of the Urumieh-Dokhtar magmatic arc is the product of Tethys oceanic plate subducted under the Iranian microplate followed by continent-to-continent collision of the Arabian and Eurasian plates (Regard *et al*., 2004; John *et al*., 2010). Seridune porphyry copper deposit is in a granodiorite-quartz monzonite pluton. Two large deposits belonged to this area are Sar Cheshmeh and Darrehzar.

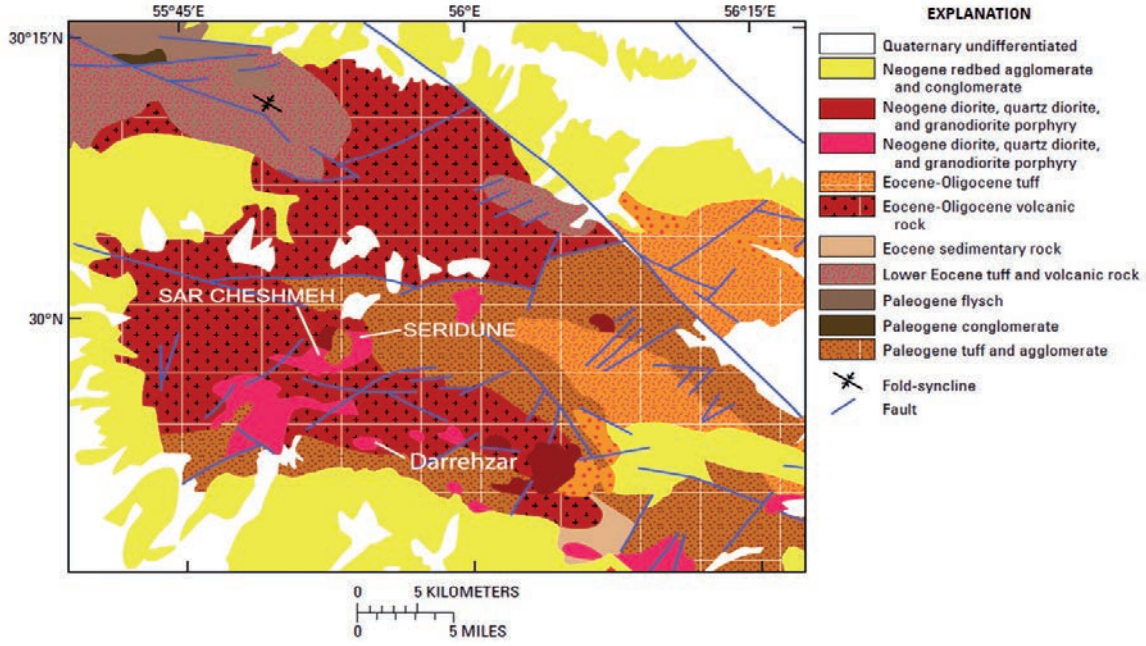

Fig. 4 - Geological map of the Seridune area (modified from Huber, 1969; John *et al*., 2010).

The detailed lithological map of the Seridune prospect is shown in Fig. 5a. This deposit consists of Eocene andesite and trachyandesite intruded by upper Miocene granodiorite, which is cut by quartz monzonite and granodiorite porphyry dikes (Barzegar, 2007; John *et al*., 2010). Post mineralization Pliocene dacite and Quaternary gravels cover parts of the andesite and intrusive rocks. The grano-diorites, monzonites, and andesites adjacent to the intrusive rocks contain complexly intermixed argillic and sericitic alteration zones and an area of propylitically-altered rocks in the south-eastern part of the prospect. North-trending silica lithocaps cut argillic, sericitic, and propylitic alteration zones. A zone of advanced argillic-altered rocks (Fig. 5b) borders the lithocaps, and quartz stockwork veins are in the central part of the prospect, (Barzegar, 2007; John *et al*., 2010).

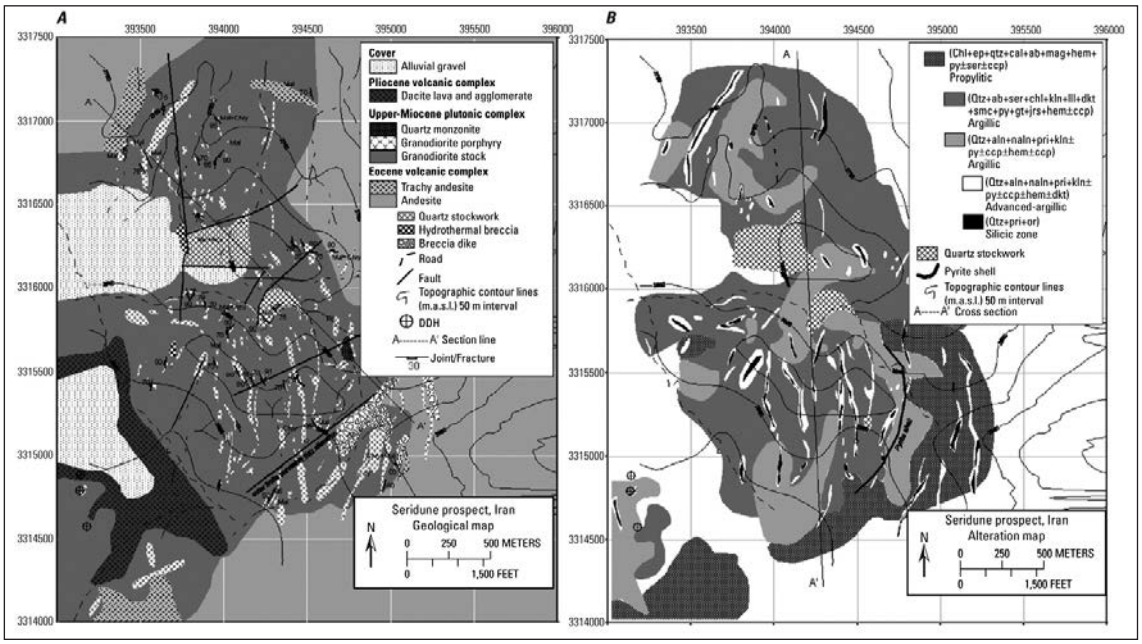

Fig. 5 - a) Detailed lithological map of the Seridune prospect (Barzegar, 2007). Mal=malachite, Azu=azurite, Chry=chrysocolla, Hem=hematite, Lm=limonite, m.a.s.l=meter above sea level; b) Hydrothermal alteration map of the Seridune prospect (John *et al.*, 2010).

4. Application of fuzzy AHP to Seridune copper deposit

4.1. Criteria for MPM

Two common geophysical methods to prospect porphyry copper deposit are magnetic and electrical surveys. Magnetic methods are used in the exploration and characterization of porphyry copper deposits worldwide. The primary control on bulk magnetic properties of host rock and magnetic intrusions is the partitioning of iron between oxides and silicates (Clark, 1999), although sulphide minerals associated with hydrothermal alteration also provide fundamental, localized geophysical targets (John *et al.*, 2010). Simple models for porphyry copper deposits involve contrasting zones of alteration centered about the deposit. Magnetic anomalies, at least in principle, reflect the location of these zones: weak local magnetic highs over the potassic zone, low magnetic intensity over sericitic zones, and gradually increasing intensities over the propylitic zone (Thoman *et al.*, 2000). As an example, Fig. 6 shows the magnetic anomaly over a hypothetical but geologically plausible porphyry copper deposit (John *et al.*, 2010).

The resistivity method is one of the oldest techniques in geophysical exploration. Resistivity is a measure of the ability of electrical charge to form currents that move through the geological section. Minerals and rocks associated with hydrothermal alteration often have anomalous electrical properties, and thus geophysical methods that detect and model such properties are mainstays in the exploration for and characterization of porphyry copper deposits. Like the distribution of magnetic minerals, electrical properties reflect the type and degree of hydrothermal alteration. Hydrothermal minerals relevant to geophysical exploration are pyrite,

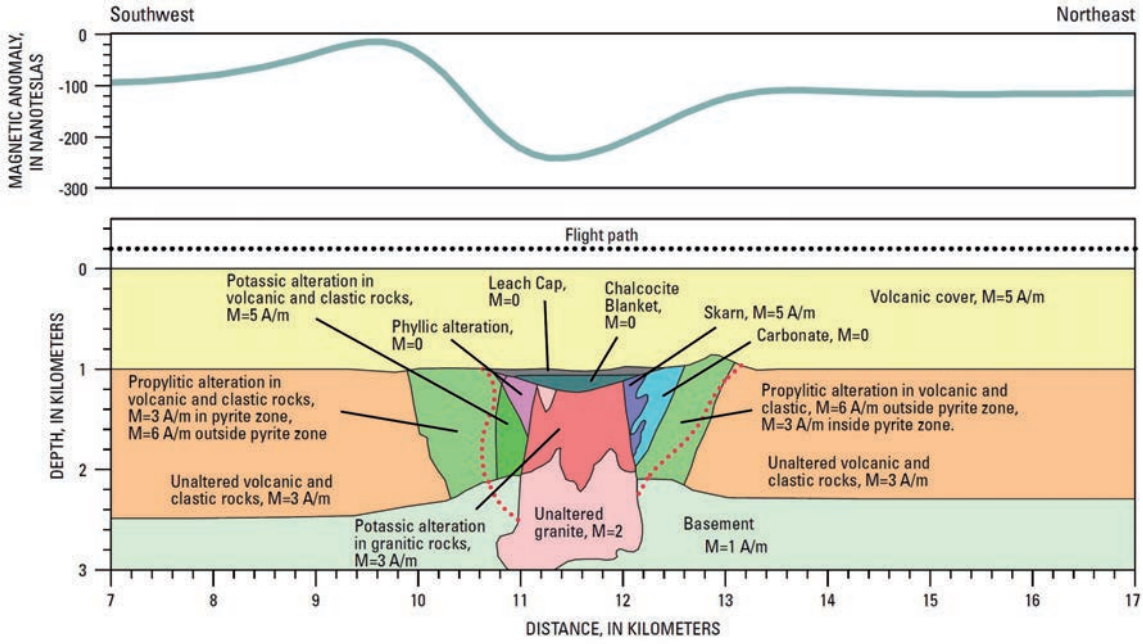

Fig. 6 - Magnetic anomaly caused by a hypothetical porphyry copper deposit. Magnetic field parameters are assumed as inclination 58.3° and declination 11.6°. M indicates magnetizations in amperes per meter (A/m) (from John *et al.*, 2010).

chalcopyrite, chalcocite, biotite, and sericite. As with magnetic anomalies, we would expect to see the intensity and type of alteration reflected in resistivity anomalies, with lowest resistivity centered on sericitic alteration that is developed in zones of most fracturing and fluid flow (Thoman *et al.*, 2000; John *et al.*, 2010). The dispersed nature of sulphide minerals in porphyry systems is particularly suitable for Induced Polarization (IP) methods (Sinclair, 2007). Indeed, the IP method was originally developed for the exploration of porphyry copper deposits (Brant, 1966) and still is commonly used. IP is a complex phenomenon. In simplest terms, IP anomalies reflect the ability of a mineral, rock, or lithology to act as an electrical capacitor. In porphyry copper deposits, the strongest IP responses correlate with quartz-sericite-pyrite alteration (Thoman *et al.*, 2000; John *et al.*, 2010).

Typically, the zone of potassic alteration in the core of the deposit is low in total sulphide minerals, the surrounding zone of sericitic alteration has high sulphide content, including pyrite, and the distal zone of propylitic alteration has low pyrite. Thus, the sericitic zone of alteration is an important IP target (John *et al.*, 2010). Therefore, various layers of information are derived from magnetic and electrical survey to produce MPM.

4.2. Using fuzzy knowledge based method

In this study, seven layers of geophysical data are used to prepare prospectivity map. The most common geophysical methods for exploration of sulfide deposits exploration are electrical techniques. In this study, Resistivity (RS), IP "chargeability map", and metal factor map (as a ratio of chargeability to resistivity) are used. Rectangular array with 1200 m space as current electrode was used in the study area such that distances between profiles and stations were 100

and 20 m. These maps are shown in Fig. 7.

Ground-based magnetic survey was done in the area, whereby distances between profiles and stations were 100 and 20 m respectively. The geomagnetic field is 45770 nT (inclination = 46.4°, declination = 2.3°). Analytical upward continuation was used because it is suspected that copper deposit exists at depth. This method calculates the magnetic field further to the source and consequently results in a better map of deep deposit and reduces the effect of shallow structures with high frequency. Map of residual magnetic data that were upward continued to 20 m (UP20) is shown in Fig. 8a.

A general filter operation applied to magnetic data is Reduced To the Pole (RTP), which is a technique that converts magnetic anomaly to symmetrical pattern that would have been observed with vertical magnetization. RTP technique eliminates the dipolar nature of magnetic anomalies and converts its asymmetric shape to symmetric shape (Ansari and Alamdar, 2009). The RTP map of Seridune prospect is shown in Fig. 8b.

Many filters are available to enhance magnetic field data, such as downward continuation, horizontal and vertical derivatives, and other forms of high-pass filters. One of these techniques is the analytic signal method. The basic concept of the analytic signal method for magnetic data was extensively discussed by Nabighian (1972, 1974, 1984). Fig. 8c shows the analytic signal map.

Pseudo gravity transformation of magnetic data is based on Poisson relation that transforms magnetic to gravity. The assumption is that both the magnetic and gravity signals are caused by the same anomalous body (with the same geometry) and that magnetic anomalies are entirely induced by the present geomagnetic field (no remanent magnetization). This filter can show the boundary of anomalies better than magnetic data, and can be a simple tool to interpret by geologist. Interested readers are referred to Blakely (1995) for additional details of this transformation. The pseudo gravity map is shown in Fig. 8d.

Three DMs who are expert in geophysical prospect must be used at least to apply fuzzy AHP procedure. In this study, three DMs were used to construct pairwise comparison matrices. After forming the decision hierarchy for MPM, the criteria to be used in evaluation process are assigned weights by using AHP method. Three DMs are given the task of forming individual PCM by use of scale in Table 1.

All consistency ratios obtained from the PCM for implementing AHP are presented in Table 4. All of them are less than 0.1. So, the results are used to make fuzzy evaluation matrices. Fuzzy evaluation or pairwise matrices with respect to criteria of geophysical prospecting, geo-electrical alternatives, and magnetic alternatives constructed by DMs which are transformed into TFN are brought in Tables (5 to 7).

Table 4 - CR of the pairwise comparison matrix.

CI	Criteria		Geo-Electrical Alternatives			Magnetic Alternatives			
	Electric	Magnetic	Resistivity	Induced Polarization	Metal Factor	UP-warded	Reduced to Pole	Analytic Signal	Pseudo Gravity
DM1	0		0.0158			0.0899			
DM2	0		0.0032			0.0263			
DM3	0		0.0739			0.0038			

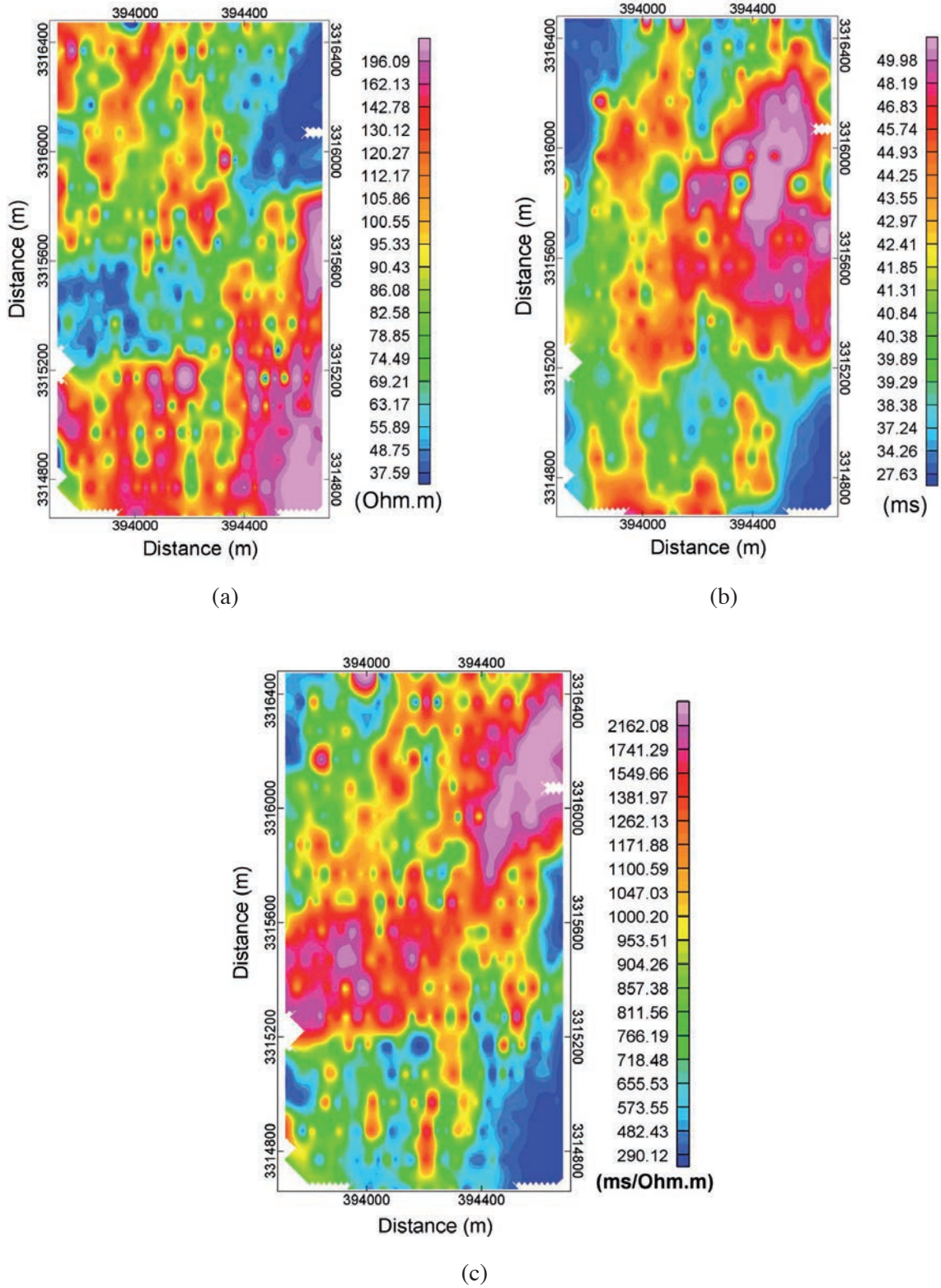

Fig. 7 - Geo-electrical layers, a) RS map, b) IP "chargeability map", c) metal factor.

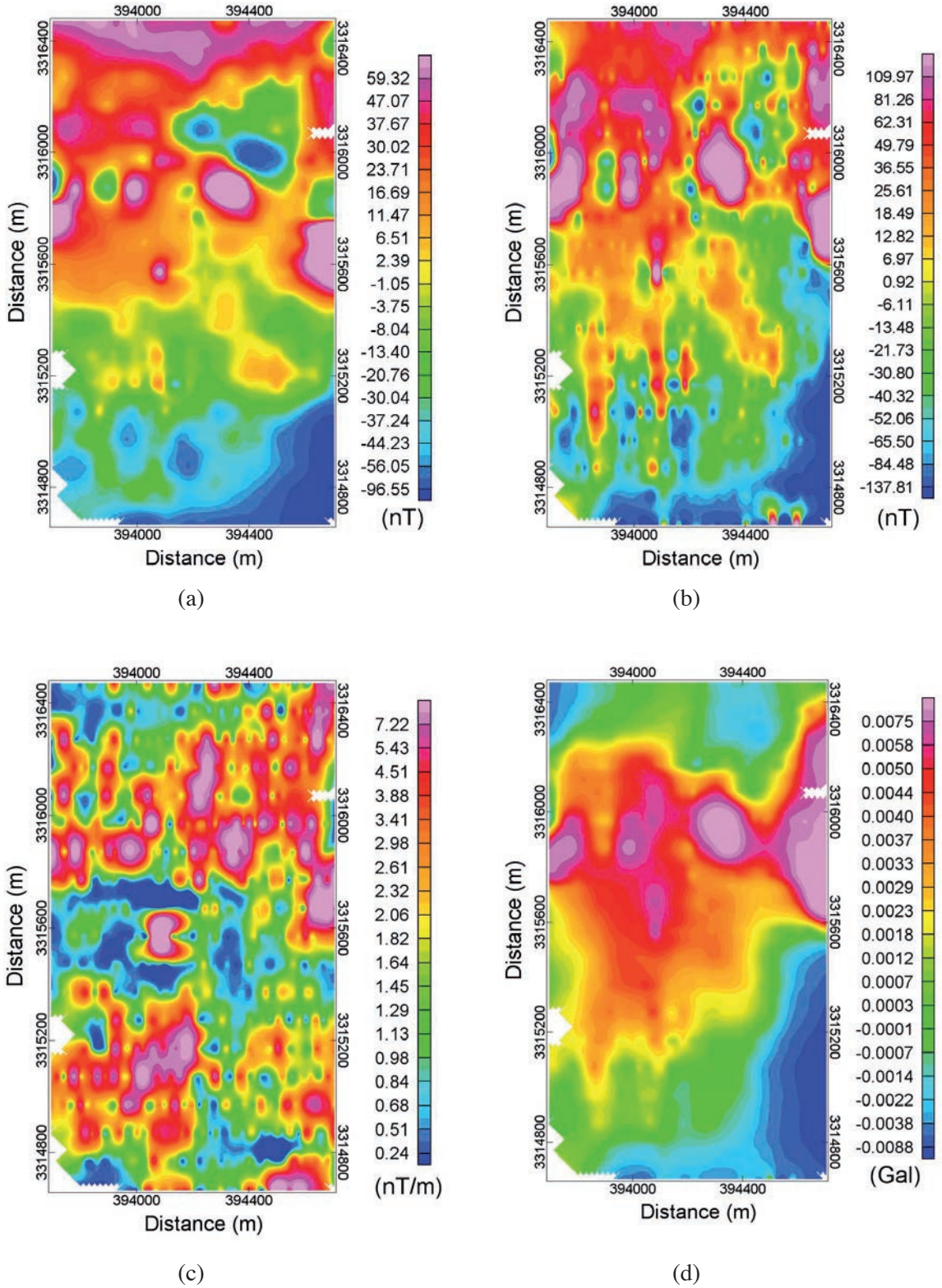

Fig. 8 - Magnetic layers, a) UP20, b) RTP of magnetic data, c) analytic signal of magnetic data, d) pseudo gravity.

Table 5 - Fuzzy evaluation matrix with respect to criteria of geophysical prospecting.

	Magnetic	Electric
Magnetic	(1, 1, 1)	(0.2, 0.84, 2)
Electric	(0.5, 2.83, 5)	(1, 1, 1)

Table 6 - Fuzzy evaluation matrix with respect to geo-electrical alternatives.

	IP	RS	MF
IP	(1, 1, 1)	(0.5, 2.83, 5)	(0.33, 0.94, 2)
RS	(0.2, 0.84, 2)	(1, 1, 1)	(0.17, 0.46, 1)
MF	(0.5, 1.83, 3)	(1, 4, 6)	(1, 1, 1)

Table 7 - Fuzzy evaluation matrix with respect to magnetic alternatives.

	RTP	A.S	UP	Pseudo-gravity
RTP	(1, 1, 1)	(0.33, 1.44, 3)	(0.5, 2.5, 4)	(0.5, 1.7, 2)
A.S	(0.33, 1.44, 3)	(1, 1, 1)	(0.33, 2.11, 4)	(0.33, 1.44, 3)
UP	(0.25, 0.86, 2)	(0.25, 1.25, 3)	(1, 1, 1)	(0.25, 0.86, 2)
Pseudo-gravity	(0.5, 1.17, 2)	(0.33, 1.44, 3)	(0.5, 2.5, 4)	(1, 1, 1)

Weights of seven evidential layers of information can be determined by fuzzy AHP. For instance, the procedure of obtaining normalized weights from fuzzy evaluation matrix with respect to magnetic alternatives is illustrated. From Table 7, the value of the fuzzy synthetic extent is calculated from Eq. (9) as follow:

$$\begin{aligned} S_{RTP} &= (2.33, 6.64, 10) \odot \left(\frac{1}{39}, \frac{1}{22.71}, \frac{1}{8.41}\right) = (0.0597, 0.2924, 1.1891) \\ S_{A.S} &= (2, 5.99, 11) \odot \left(\frac{1}{39}, \frac{1}{22.71}, \frac{1}{8.41}\right) = (0.0513, 0.2638, 1.3080) \\ S_{UP} &= (1.75, 3.97, 8) \odot \left(\frac{1}{39}, \frac{1}{22.71}, \frac{1}{8.41}\right) = (0.0449, 0.1748, 0.9512) \\ S_{P.G} &= (2.33, 6.11, 10) \odot \left(\frac{1}{39}, \frac{1}{22.71}, \frac{1}{8.41}\right) = (0.0597, 0.2690, 1.1891). \end{aligned} \tag{16}$$

Eq. (12) is used to compare these fuzzy values as follow:

$$\begin{aligned} &d(S_{RTP} > S_{A.S}) = 1 && d(S_{RTP} > S_{UP}) = 1 && d(S_{RTP} > S_{P.G}) = 1 \\ &d(S_{A.S} > S_{RTP}) = 0.9776 && d(S_{A.S} > S_{UP}) = 1 && d(S_{A.S} > S_{P.G}) = 0.9959 \\ &d(S_{UP} > S_{RTP}) = 0.8835 && d(S_{UP} > S_{A.S}) = 0.91 && d(S_{UP} > S_{P.G}) = 0.9044 \\ &d(S_{P.G} > S_{RTP}) = 0.9797 && d(S_{P.G} > S_{A.S}) = 1 && d(S_{P.G} > S_{UP}) = 1 \end{aligned} \tag{17}$$

Then, priority weights are calculated by using Eq. (13):

$$\begin{aligned} d'(RTP) &= (1,1,1) = 1 \\ d'(A.S) &= (0.9776,1,0.9959) = 0.9776 \\ d'(UP) &= (0.8835,0.91,0.9044) = 0.8835 \\ d'(P.G) &= (0.9797,1,1) = 0.9797. \end{aligned} \tag{18}$$

The weights vector is $W = (1, 0.9776, 0.8835, 0.9797)$. Finally, the normalised weights vector is calculated as $W = (0.2604, 0.2545, 0.23, 0.2551)$. This procedure is applied on all evaluation matrices to obtain final normalized weights vector. Table 8 shows the normalized weights for all geophysical alternatives. These weights show that metal factor as a ratio of IP to resistivity data has the highest amount, so it has significantly effect to prospect copper ore deposit.

Table 8 - Weight of each criteria and alternative to evaluate prospectivity map.

Criterion	Weight	Alternative	Weight	Final Weight
Geo-Electrical Method	0.5803	Resistivity	0.2535	0.1471
		Induced Polarization	0.3530	0.2048
		Metal Factor	0.3935	0.2283
Magnetic Method	0.4197	UP-warded	0.2300	0.0965
		Reduced to Pole	0.2604	0.1093
		Analytic Signal	0.2545	0.1068
		Pseudo Gravity	0.2551	0.1071

Final map of MPM is integrated by fuzzy operators. Before applying the operators, final weights obtained by fuzzy AHP must be multiplied in each alternative (evidence layers). Normalized real values of geophysical layers are used to be multiplied by final weights. Instead of resistivity values that can be low to prospect copper deposit in most of cases, other geophysical layers must have high values over suitable zone for drillings. Therefore, normalised reverse values of real resistivity data are used in this study. Fuzzy sum operator is separately applied for both of magnetic and electrical alternatives. Finally, fuzzy Gamma operator is used to integrate maps of magnetic and electrical criteria in order to prepare final MPM. The amount of Gamma is considered equal to 0.9. The map of MPM is shown in Fig. 9. High values of fuzzy scores are related to high potential zone, and exploratory boreholes must be drilled in this zone. Results of seven boreholes that were drilled in the study area are used to evaluate the capability of the fuzzy knowledge based method to prioritise the region of interest for copper exploration.

Copper concentration analyses along drilled boreholes show that just borehole 7 has slightly valuable amount of copper, and other boreholes have not shown economically values of copper. Fig. 10 shows variation of copper concentration along boreholes 1, 5, 7. By considering the map of mineral potential, location of borehole 7 is corresponded to highest values of fuzzy prospectivity map. In fact, prospectivity map has reasonably matching with drilled boreholes. As a consequence, using fuzzy knowledge based technique can prioritise high potential zones

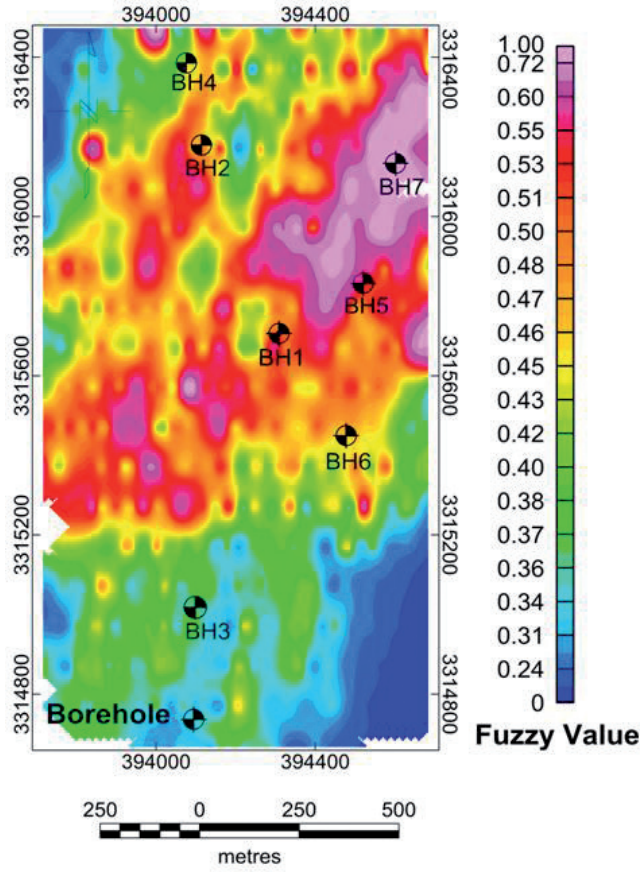

Fig. 9 - Final map generated by fuzzy knowledge based method.

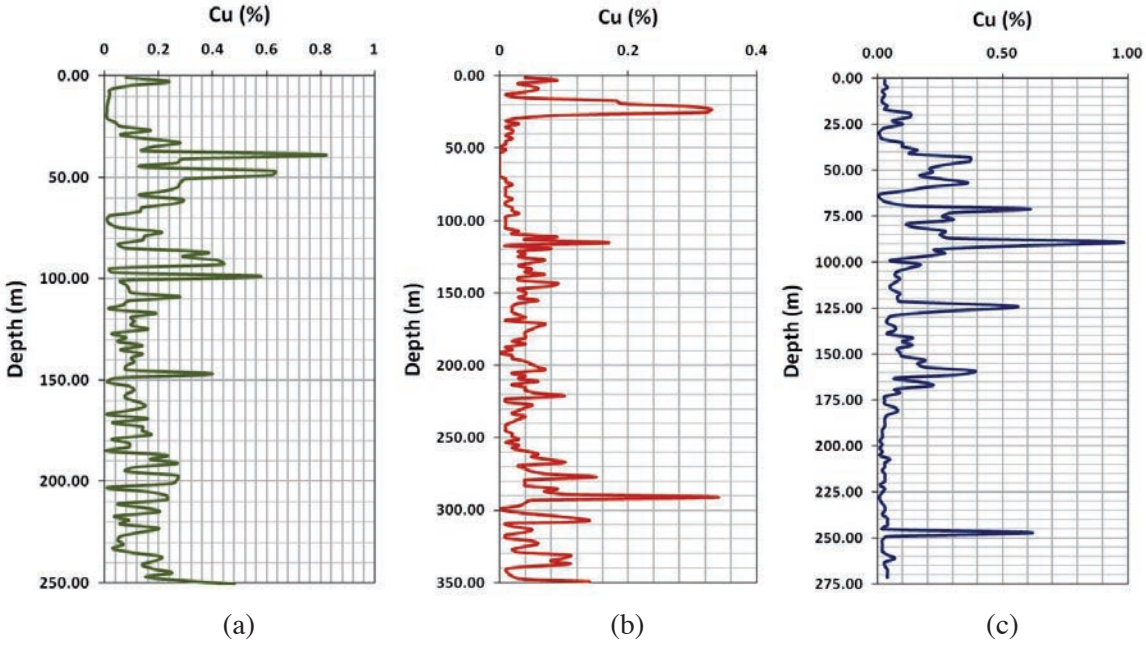

Fig. 10 - Variation of Cu concentration along boreholes, a) 1, b) 5, c) 7.

in mineral prospect, and prevents drilling in wrong locations of prospect area. Authors suggest using geological and geochemical evidential layers simultaneously to integrate them for final MPM. The authors also had no access to all evidential layers for considering in this study.

5. Conclusion

Application of fuzzy knowledge based method is considered in this paper. Various geophysical layers derived from magnetic and electrical data are used to prepare map of mineral potential. AHP method is applied to construct PCMs for geophysical layers as alternatives to produce final prospectivity map. Fuzzy AHP technique is carried out to reduce the vagueness and uncertainty of the PCMs. In fact, fuzzy AHP uses the DMs knowledge to determine final normalised weights of each geophysical layer. Subsequently, these weights are used to integrate normalised real values of them. Finally, fuzzy operators are applied to produce prospectivity map for copper exploration. Results of drilled boreholes in the region of interest showed that the prospectivity map is significantly matched with boreholes locations. As a consequence, fuzzy knowledge based method can be a useful tool in mineral potential mapping to integrate various data layers, and reduce the uncertainty.

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