

A dynamic inversion method for enhancing spatial resolution in UAV-borne gamma-ray spectrometry measurements

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(Received: 6 July 2025; accepted: 17 March 2026; published online: 14 October 2026)

ABSTRACT Unmanned aerial vehicle-borne gamma-ray spectrometry (UAV-GRS) faces significant challenges in accurately delineating the boundaries of high-radioactivity areas and detecting weak radiation signals. To address these issues, this study proposes a dynamic inversion technique for improving the spatial resolution of UAV-GRS measurements. The method involves developing a dynamic measurement model, discretising the acquisition time and geological body into differential elements, and constructing a position sensitivity matrix. Regularised least squares are, then, applied to perform concentration inversion calculations. The results demonstrate that the dynamic inversion technique significantly enhances the clarity of high-radioactivity boundaries, improves the detection of weak radiation signals, and reduces data dispersion. Specifically, the relative standard deviation of the data decreases from 54.56% to 31.03% after dynamic inversion, indicating a substantial improvement in data consistency. This technique provides higher-precision data for UAV-GRS, offering a solid foundation for its widespread application in geological exploration and environmental monitoring.

Key words: Unmanned aerial vehicle (UAV), gamma-ray spectrometry, dynamic inversion technique, radiometric exploration.

1. Introduction

Unmanned aerial vehicle-borne gamma-ray spectrometry (UAV-GRS) has gained increasing attention in recent years due to its efficiency and versatility in a wide range of applications, including mineral exploration, environmental monitoring, nuclear emergency response, and radiation hazard assessment (Hegab, 2024). Unmanned aerial vehicle-mounted gamma-ray detectors offer several advantages over traditional ground-based methods, including high mobility, cost-effectiveness, and the ability to cover large and hard-to-reach areas quickly (Xi *et al.*, 2024). However, despite the significant advancements in UAV-GRS technology, several challenges still remain in improving the spatial resolution and accuracy of the measurements, particularly in areas with weak radiation signals or complex geological features (Xiong *et al.*, 2024).

The primary issue with UAV-GRS lies in the sampling overlap effect, which occurs when the UAV flight path and the detector spatial coverage lead to overlap between consecutive sampling footprints. This overlap can cause inaccuracies in the spatial delineation of high-radioactivity

anomaly boundaries and reduce the clarity of weak radiation signals (Druker, 2017). The current UAV-GRS measurement models typically assume a static or uniform detection area for each sampling point, but the reality of UAV motion introduces significant temporal and spatial variation that is not always adequately accounted for. Thus, a key challenge in UAV-GRS research is to develop methods that can more accurately model the dynamic movement of the UAV and its detector, thereby reducing the impact of sampling overlap and improving spatial resolution (Zhang, 2010).

In terms of data processing, spectral line processing and energy window processing are the two main techniques employed in UAV-GRS. Spectral line processing typically includes background subtraction, noise reduction, and peak deconvolution. Methods such as the Fourier Transform Stepwise Approximation have been employed to effectively reduce scattered background noise (Eilers and Boelens, 2005). Additionally, asymmetric least-squares methods have gained popularity for their speed and ability to preserve signal integrity during baseline correction (Sun *et al.*, 2018). These methods have been successfully applied to improve the accuracy of GRS by removing noise and correcting for interference from atmospheric factors such as radon (Gu *et al.*, 2014).

Energy window processing, on the other hand, focuses on the quantitative analysis of gamma-ray counts within specific energy ranges. Various inversion techniques have been developed to correct for variations in UAV flight altitude and terrain ruggedness. For example, one-dimensional inversion methods have been used to model and correct for terrain variations (Zhang, 2010), while methods like the logarithmic barrier approach have been introduced to address false negative values in the data (Weihermann *et al.*, 2021). Despite these advancements, the dynamic effects of UAV motion, including changes in speed and altitude, are still not adequately accounted for, leading to errors in both the boundary delineation of radioactive anomalies and the identification of weak radiation signals (Xiong *et al.*, 2024).

In recent years, several dynamic models have been proposed to account for UAV flight paths and varying detector positions. For instance, Druker (2012) proposed a two-dimensional inverse problem framework to process airborne GRS data, which incorporates the effects of UAV altitude and flight path variations. Other studies have focused on developing methods to optimise UAV flight paths in real-time to minimise overlap effects and improve the overall measurement quality. However, these approaches often rely on static models or require real-time adjustment of UAV parameters, which can be difficult to implement in practice due to the complexity of UAV systems and the unpredictability of environmental conditions (Liu *et al.*, 2021).

To address these issues, this study proposes a dynamic inversion technique that integrates time-dependent modelling with numerical optimisation algorithms to reduce the sampling overlap effect and enhance the spatial resolution of UAV-GRS measurements. The dynamic inversion technique involves discretising the acquisition time and geological body into differential elements, constructing a position sensitivity coefficient matrix, and applying regularised least-squares inversion to improve the accuracy of concentration estimates. This method significantly enhances the clarity of high-radioactivity boundaries, improves the detection sensitivity of weak radiation signals, and reduces data dispersion (Zhang *et al.*, 2010). Existing inversion frameworks typically employ a 'static approximation', assuming that the detector is stationary during the sampling interval. This assumption ignores the continuous motion of the UAV, leading to significant 'motion blur' and spatial averaging in the resulting data. The core novelty of this study is the development of a fully dynamic inversion framework that explicitly incorporates the UAV flight velocity (V) and sampling time (T) into the position sensitivity matrix. Unlike static models, our approach mathematically reconstructs

the detector's moving footprint, effectively deconvolving the motion effect to recover high-resolution ground concentrations.

This paper is structured as follows: Section 2 provides the theoretical foundations of the dynamic measurement model for UAV-GRS, including the discretisation of time and space and the construction of the position sensitivity matrix. Section 3 presents the details of the dynamic inversion algorithm and its implementation, followed by experimental results in Section 4 that demonstrate the effectiveness of the proposed method in improving spatial resolution and detection sensitivity. Finally, Section 5 discusses the conclusions and potential future directions for this research.

2. Theoretical foundations

2.1. Dynamic measurement of unmanned aerial vehicle-borne gamma-ray spectrometry

The physical basis of UAV-GRS lies in the airborne detector receiving gamma rays emitted from the Earth-atmosphere interface. These gamma rays primarily originate from the spontaneous decay of naturally occurring radionuclides in the Earth's crust, such as uranium-238 (U-238), thorium-232 (Th-232), and potassium-40 (K-40). The accuracy of UAV-GRS in terrestrial detection refers to the system's ability to identify and characterise the spatial distribution of surface radioactive element concentrations. This accuracy is primarily quantified by spatial resolution or ground sampling distance (GSD). GSD is defined as the spatial distance between adjacent measurement points and its value is inversely proportional to spatial resolution. Specifically, a smaller GSD value corresponds to higher spatial resolution, thereby enhancing the system's ability to characterise the spatial heterogeneity of surface radioactive element concentrations.

Within a single sampling cycle, the detector records a time-integrated spectrum, which represents the cumulative distribution of energies of all incident gamma photons within a specific time window. This measurement approach reflects the spatial integral characteristics of the terrestrial gamma radiation field within the detection area, providing statistically averaged information on the distribution of regional radioactive elements. However, since this method employs an integral measurement mode, it cannot capture the incident direction or time-resolved characteristics of individual gamma photons. This limitation somewhat restricts the spatial resolution capability of the measurement results.

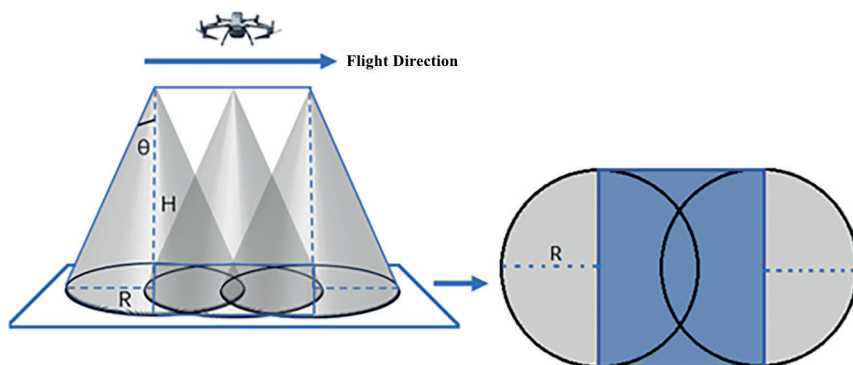


Fig. 1 - Schematic diagram of single sampling.

As shown in Fig. 1, during the dynamic measurement process of UAV-GRS, the detection range of the gamma-ray detector mounted on the UAV is a circular area centred on the ground projection of the detector, with the ground effective detection length as its diameter. When the UAV flies along a straight trajectory, the effective detection area within a unit acquisition time, ΔT , can be modelled as a geometric combination of a rectangle and two semicircles. The length component of the rectangle is determined by the flight distance of the UAV during time interval ΔT , while the width component corresponds to the effective detection diameter of the detector. The two semicircular regions represent the detection coverage at the starting and ending points of the flight path, respectively. These regions achieve spatial coverage during the UAV course adjustment process.

Assuming the UAV flight altitude is H , the UAV flight speed is V , the effective detection radius of the detector is R , and the detection angle of the detector is θ , the area S swept and detected by the detector within a detection time T can be expressed as:

$$S = \int_0^T 2RVdt + \pi R^2 = 2VTH \tan \theta + \pi H^2 \tan^2 \theta. \quad (1)$$

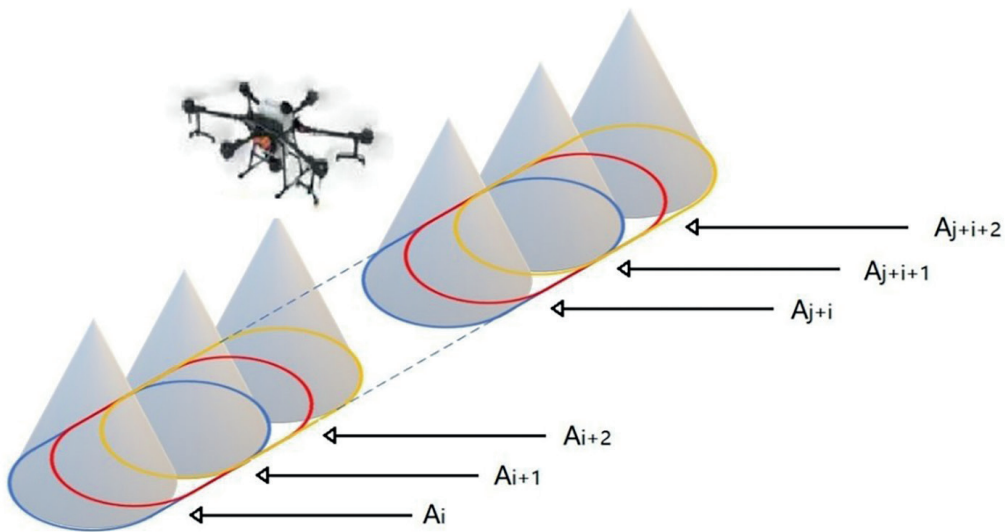


Fig. 2 - Schematic diagram of dynamic measurement.

As shown in Fig. 2, the blue, red, and yellow regions correspond to the detection ranges of the instrument spectral counts A_i , A_{i+1} , and A_{i+2} , respectively. Observations indicate that when a high-concentration geological body of radioactive elements with a radius R (represented by the conical shaded area in the figure) is present, its radiation signal may produce a superposition effect on multiple consecutive measurement points. This phenomenon is primarily manifested in the measurement results as follows: the apparent spatial distribution range of the high-radioactivity region expands, the boundary clarity decreases, and the measured values may be lower than the actual concentrations. The mechanism behind this error can be attributed to the repeated collection of radiation signals from the same geological body by adjacent measurement points during dynamic measurement. This leads to spatial dispersion of radioactive counts, thereby reducing the spatial resolution and quantitative accuracy of the measurement data.

2.2. Impact of different speeds on measurement results

According to the technical requirements (National Nuclear Safety Administration, 2020), the maximum flight speed must be determined based on key parameters such as the crystal size of the spectrometer to ensure that the terrestrial detection accuracy meets the specified standards. This requirement stems from the significant correlation between the detection efficiency of the gamma-ray spectrometer and the crystal size: an increase in crystal size can expand the detectable energy range but may lead to a reduction in energy resolution. Therefore, during dynamic measurements, flight parameters must be optimised based on the performance parameters of the spectrometer (including, but not limited to, energy resolution, detection efficiency, and dead time) to ensure that the GSD meets the required precision standards, thereby achieving high-accuracy acquisition of measurement data.

During the practical experimental process of UAV-GRS, the dynamic variation in the UAV flight speed leads to differences in the surface area covered per unit time. This results in changes in the dwell time of the detector over a fixed micro-unit measurement area, which is inversely proportional to the covered area. Consequently, the gamma-ray spectrum value obtained by the detector is essentially a spatially weighted average of the gamma radiation field within the measurement region. To accurately simulate the measurement process, the flight path of the UAV within a measurement interval can be discretised into multiple micro-units. The average gamma-ray spectrum value for each micro-unit is calculated separately and, then, the gamma-ray spectrum measurement value for the measurement point is obtained through summation and averaging.

For a measurement profile in a specific survey area, under the condition of a fixed flight direction, simulated flight measurements were conducted by setting different flight speeds V . The experimental results are shown in Fig. 3.

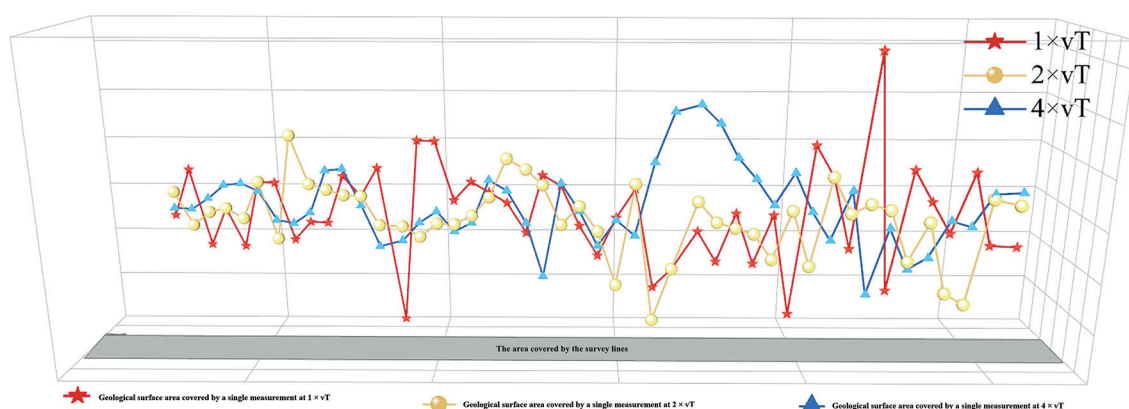


Fig. 3 - Influence of different velocities (V) on measurement results.

As shown in Fig. 3, the variation in flight speed V has a significant impact on the spatial resolution characteristics of the measurement results. When the flight speed V is low, the spatial coverage of the detector within a single sampling cycle is smaller, resulting in higher spatial resolution in the measurement results. Specifically, the spatial fluctuation characteristics of radioactive element concentrations are pronounced and the spatial boundaries of high-

concentration regions are clearly distinguishable, accurately reflecting the spatial heterogeneity of the gamma radiation field within the survey line area. Conversely, when the flight speed V is high, the spatial coverage of the detector within a single sampling cycle expands, leading to a reduction in the spatial resolution of the measurement results. In this case, the spatial fluctuations in radioactive element concentrations become smoother, the spatial boundaries of high-concentration regions become blurred, and the measurement results struggle to effectively capture local fine variations in the gamma radiation field.

3. Research methodology

3.1. Construction of the dynamic model

Based on the aforementioned theoretical analysis, the spatial heterogeneity of radionuclides within geological bodies and their relative positional changes with respect to the detector are key factors influencing dynamic measurement results. The effective detection area of the detector can be modelled as a circular region centred directly below the detector with a radius R . However, due to the high complexity of integrals involving circular regions in mathematical calculations, this study adopts a geometric approximation method to simplify the computational model. Specifically, the circular geological body surface source is approximated as a square surface source with a side length of $2R$. This approximation significantly reduces computational complexity while maintaining calculation accuracy. As shown in Fig. 4, the relative positional relationship between the detector and the geological body surface source during dynamic measurement is reconstructed based on this simplified model, providing a geometric foundation for subsequent quantitative analysis.

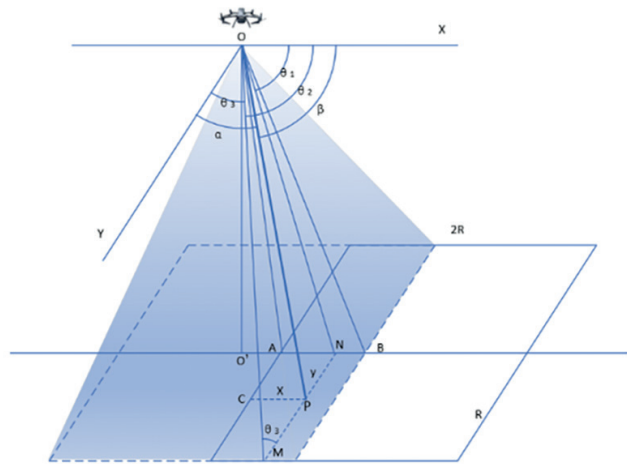


Fig. 4 - Geometric schematic of dynamic measurement.

As shown in Fig. 4, when detecting the target geological body within the solid-line region, the overlapping area between the detector's effective detection region (indicated by the dashed line) and the geological body constitutes the effective region of radiation contribution. Taking the current position of the UAV as the coordinate origin, O , a Cartesian coordinate system is established and the following geometric parameters are defined: let $\angle XON = \angle ONO' = \theta_1$,

$\angle XOA = \angle OAO' = \theta_2$, $\angle YOM = \angle OMP = \theta_3$, $CP = x$, $NP = y$, $AO' = R_{left}$ and $BO' = R_{right}$, then, the following relationships hold:

$$\begin{cases} \tan \theta_1 = \frac{H}{R_{right}} \\ \tan \theta_2 = \frac{H}{R_{left}} \\ \tan \theta_3 = \frac{H}{R} \csc \theta_1 \end{cases} \Rightarrow \begin{cases} \theta_1 = \tan^{-1} \left(\frac{H}{R_{right}} \right) \\ \theta_2 = \tan^{-1} \left(\frac{H}{R_{left}} \right) \\ \theta_3 = \tan^{-1} \left(\frac{H}{R} \csc \theta_1 \right) \end{cases} . \quad (2)$$

When calculating the full-energy peak count n_f of the detector above the geological body surface, the non-orthogonality between the detector's base plane and the incident direction of gamma photons must be considered. To accurately describe the incident geometric relationship of gamma photons, corrections for the incident angles in the X-axis and Y-axis directions are required. Let α be the angle between the incident photon and the normal to the detector's base plane in the XOZ plane, and β be the corresponding angle in the YOZ plane. Based on geometric optics principles, the following correction relationships can be established:

$$\begin{cases} x = H \times \cot \beta \\ y = H \times \cot \alpha \times \csc \beta \\ r = H \times \csc \alpha \times \csc \beta \\ dx = H \times d(\cot \beta) \\ dy = H \times \csc \beta \times d(\cot \alpha) \end{cases} . \quad (3)$$

Based on Eqs. (3), the full-energy peak count n_f of the detector above the geological body surface can be simplified as:

$$n_f = \int_{\theta_1}^{\theta_2} \int_{\beta_3}^{\frac{\pi}{2}} A e^{-\mu H \csc \alpha \csc \beta} \sin \alpha d\alpha d\beta \quad (4)$$

where A is a constant value specific to the scenario influenced by various factors (such as UAV flight altitude, overburden thickness, and gamma-ray energy).

Since Eq. (4) does not have an analytical solution for n_f , the W -function (Lu, 1979) is introduced:

$$\begin{aligned} W(x, y, \varphi_0) &= \int_{\varphi_0}^{\frac{\pi}{2}} \int_{\tan^{-1}(y \csc \varphi)}^{\frac{\pi}{2}} e^{-x \csc \theta \csc \varphi} \sin \theta d\theta d\varphi \\ \Rightarrow W(x, y, \theta_1) - W(x, y, \theta_2) &= \int_{\theta_2}^{\theta_1} \int_{\beta_3}^{\frac{\pi}{2}} e^{-x \csc \alpha \csc \beta} \sin \alpha d\alpha d\beta \\ \Rightarrow n_f &= A(W(x, y, \theta_1) - W(x, y, \theta_2)) \end{aligned} \quad (5)$$

where:

$$\begin{cases} x = H \times \cot \beta = \mu H \\ y = H \times \cot \alpha \times \csc \beta = \frac{H}{R} \\ \theta_1 = \tan^{-1} \left(\frac{H}{R_{right}} \right) \\ \theta_2 = \tan^{-1} \left(\frac{H}{R_{left}} \right) \end{cases} . \quad (6)$$

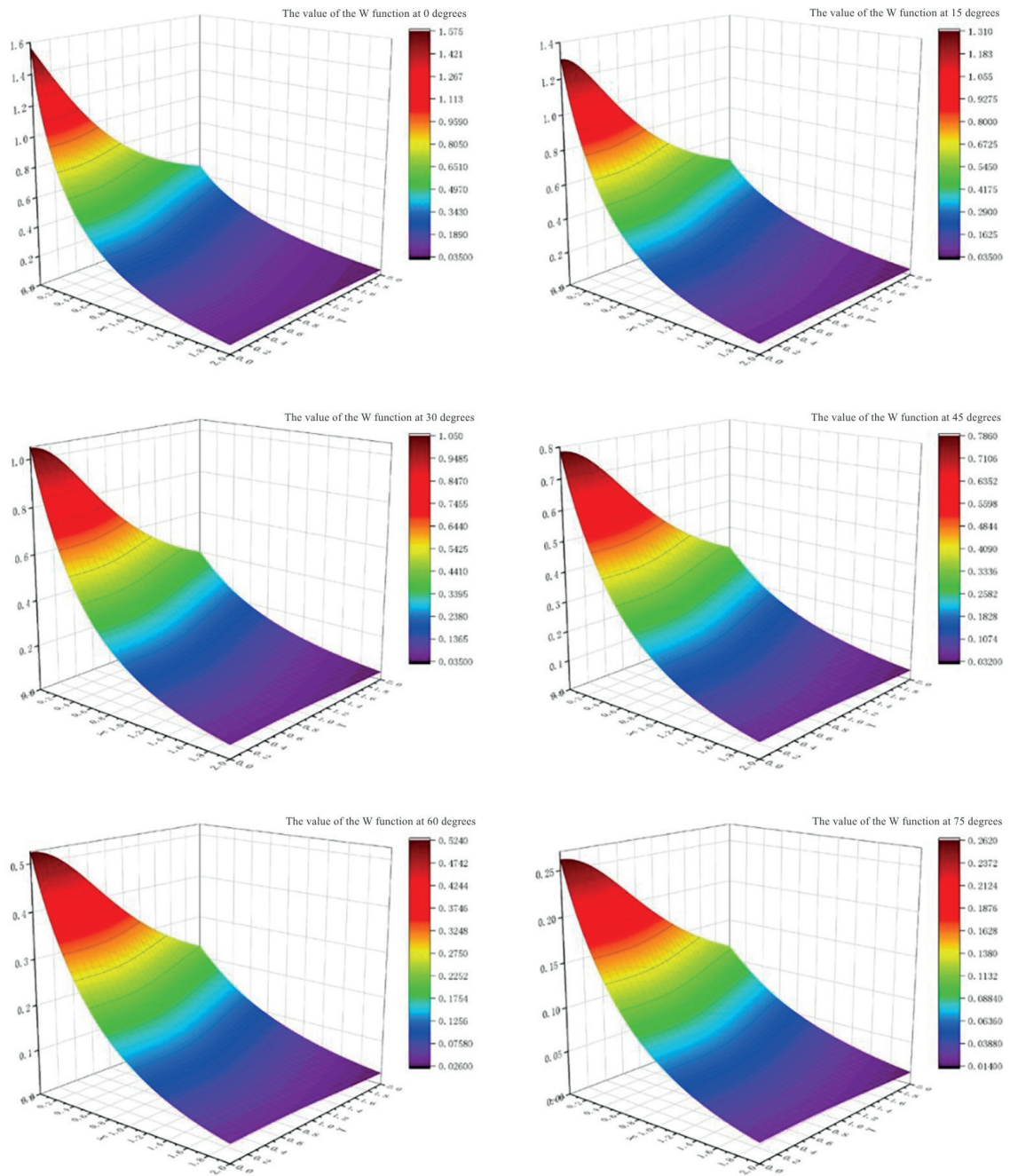


Fig. 5 - Plot of partial values of the W -function.

As shown in Fig. 5, the distribution characteristics of the W -function values, with respect to different angles, are clearly illustrated. From top to bottom, the figure presents the response curves of the W -function values as a function of x and y coordinates under six different angle conditions: 0° , 15° , 30° , 45° , 60° , and 75° . It can be observed that as the angle increases from 0° to 75° , the W -function values exhibit a significant decay trend. When the angle reaches

90°, the W -function values completely converge to 0, indicating no effective response at this point.

During the dynamic detection process, the area of the detector's counting contribution region is time dependent, dynamically adjusting in size as the relative position between the detector and the geological body changes. Based on the spatial relationship between the detector and the geological body, the counting contribution region can be divided into the following four geometric scenarios:

1. the detector is located outside the left boundary of the geological body;
2. the detector is located between the left boundary and the left side of the centre point of the geological body;
3. the detector is located between the right side of the centre point and the right boundary of the geological body;
4. the detector is located outside the right boundary of the geological body.

The spatial relationship between the detector and the geological body exhibits central symmetry, meaning Scenario 1 and Scenario 4 are symmetric, as are Scenario 2 and Scenario 3. Leveraging this symmetry, the integral intervals for α and β can be determined by analysing only Scenario 1 and Scenario 2, thereby significantly reducing computational complexity.

After simplification, Scenario 1 can be expressed as:

$$n_f = 2A[W(x, y, \theta_1) - W(x, y, \theta_2)]. \quad (7)$$

After simplification, Scenario 2 can be expressed as:

$$n_f = 2A[W(x, y, \theta_1) + W(x, y, \theta_2)]. \quad (8)$$

3.2. Forward model for dynamic measurement

Based on the aforementioned analysis, this study adopts a micro-element discretisation approach, dividing the geological body into multiple strip-shaped micro-elements along the survey line direction to enhance measurement accuracy. Furthermore, since the relative position between the detector and the geological micro-elements changes over time during dynamic measurement, the acquisition time is also discretised into micro-elements to construct a precise forward model.

Assume a rectangular surface source with a length of $a(m)$ and a width of $b(m)$ in a measurement profile, where the full-energy peak count of the UAV gamma-ray spectrum is n_{f_x} ($x = 1, 2, 3, \dots, n$), and n is the number of groups in the rectangular surface source. This rectangular surface source is divided into strip-shaped surface sources $part_y$ ($y = 1, 2, 3, \dots, m$), each with a length of $a/m(m)$ and a width of $b(m)$, and a radioactive concentration of rad_k .

After discretisation into micro-elements, the full-energy peak count n_{f_x} of the measurement profile can be expressed as the sum of the counts n_{f_x} from the m strip-shaped surface sources. Let the angles between the detector and each micro-element be $\theta_{1-x,y}$ and $\theta_{2-x,y}$. Combining with Eq. (5), we obtain:

$$n_{f_x} = \sum_{y=1}^m \left(\int_{\theta_{1-x,y}}^{\theta_{2-x,y}} \int_{\frac{\pi}{2}}^{\pi} A e^{-\mu H \csc \alpha \csc \beta} \sin \alpha d\alpha d\beta \right). \quad (9)$$

The sensitivity $\sigma_{x,y,z}$ of the strip-shaped surface source $part_y$ at n_{fx} is given by:

$$\sigma_{x,y,z} = \int_{\theta_{1,x,y}}^{\theta_{2,x,y}} \int_{\theta_3}^{\frac{\pi}{2}} \frac{A}{rad_k} e^{-\mu H \csc \alpha \csc \beta} \sin \alpha d\alpha d\beta. \quad (10)$$

Therefore, the full-energy peak count can be expressed as:

$$n_{fx} = \sum_{y=1}^m (\sigma_{x,y,z} \times rad_k). \quad (11)$$

To accurately describe the dynamic measurement process, the acquisition time is also discretised into micro-elements. Assume the UAV flight speed is 6.7 m/s, the flight altitude is 40 m, and the acquisition time is 6 s, which is divided into 80 time micro-elements. Within each time micro-element, the UAV travels a distance of 0.5025 m, which is significantly smaller than the flight altitude. Thus, the relative positional relationship between the detector and the geological body can be considered unchanged. Therefore, the positional sensitivity $\sigma_{x,y}$ for each time micro-element is:

$$\sigma_{x,y} = \sum_{k=1}^Z \left(\int_{\theta_{1,x,y}}^{\theta_{2,x,y}} \int_{\theta_3}^{\frac{\pi}{2}} A e^{-\mu H \csc \alpha \csc \beta} \sin \alpha d\alpha d\beta \right). \quad (12)$$

Further transforming into matrix form:

$$\vec{n}_f = Mat \times \vec{rad} \quad (13)$$

where $\vec{n}_f$ is the count vector recorded by the instrument (dimension $n \times 1$), $\vec{rad}$ represents the radioactive concentration vector of the micro-elements (dimension $m \times 1$), and Mat is the position sensitivity coefficient matrix (dimension $n \times m$):

$$Mat = \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,y} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,y} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{x,1} & \sigma_{x,2} & \cdots & \sigma_{x,y} \end{bmatrix}. \quad (14)$$

In the construction of the forward simulation model, the most critical step is determining the position sensitivity coefficient matrix, Mat .

After discretisation into micro-elements, the spatial geometric relationship between the detector and a strip-shaped surface source micro-element at a specific moment can be categorised into three distinct scenarios: the detector is located above the left boundary, directly above, or above the right boundary of the micro-element. Based on the aforementioned geometric relationships and theoretical analysis, the expression for the position sensitivity coefficient, $\sigma_{x,y}$, of the micro-element can be derived as:

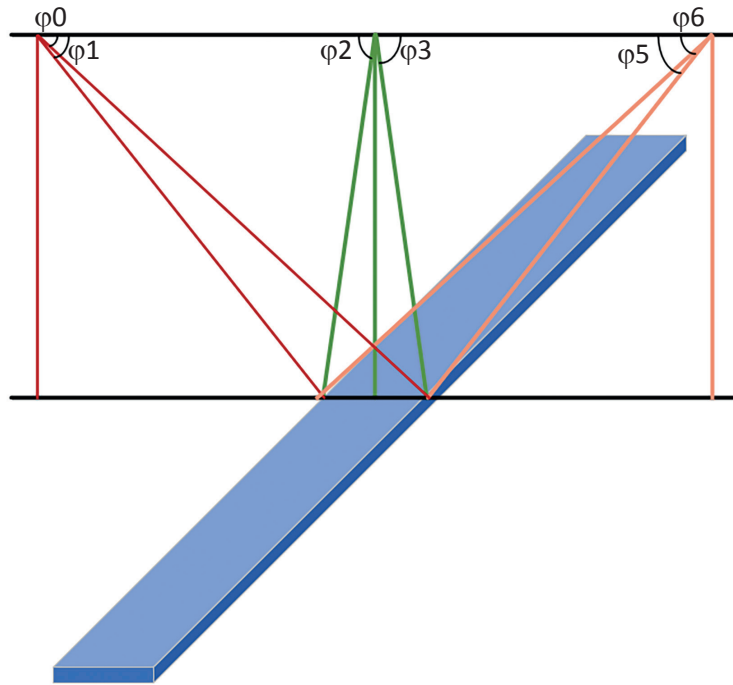


Fig. 6 - Schematic diagram of the detector and a differential element at a specific moment.

$$\begin{aligned}
 \text{Above left: } \sigma_{x,y} &= A(W(x,y,\theta_1) - W(x,y,\theta_2)) \\
 \text{Directly above: } \sigma_{x,y} &= A(W(x,y,\theta_1) + W(x,y,\theta_2)) \\
 \text{Above right: } \sigma_{x,y} &= A(W(x,y,\theta_2) - W(x,y,\theta_1)).
 \end{aligned} \tag{15}$$

In Eq. (15), θ_1 and θ_2 are determined by the coordinate positions of the detector and the micro-element.

In the process of constructing the position sensitivity coefficient matrix *Mat*, appropriate left and right extensions of the acquisition time and strip-shaped surface source micro-elements are necessary to effectively handle boundary effects. Based on the relative positional relationship between the detector and the strip-shaped surface source micro-elements, the construction of matrix *Mat* can be divided into the following three scenarios:

- 1) contribution of strip $part_1$ to the count n_{fx} .

For strip $part_1$, it is necessary to extend p micro-elements to the left, denoted as $part_r$ ($part = 1, 2, \dots, p$), and assume their radioactive concentrations are all rad_m . The position sensitivity coefficient $\sigma_{x,y}$ can be expressed as:

$$\sigma_{x,y} = \sum_{r=1}^p \sum_{k=1}^z part_{x-r,k} + \sum_{k=1}^z part_{x,k}; \tag{16}$$

- 2) contribution of strip $part_y$ to the count.

For strip $part_y$, no left or right extension is required. The position sensitivity coefficient $\sigma_{x,y}$ is given by:

$$\sigma_{x,y} = \sum_{k=1}^z part_{x,y,k}; \tag{17}$$

3) contribution of strip $part_m$ to the count n_{j_x} .

For strip $part_m$ it is necessary to extend p micro-elements to the right, denoted as $part_r$ ($part = 1, 2, \dots, p$), and similarly assume their radioactive concentrations are all rad_m . The position sensitivity coefficient $\sigma_{x,y}$ is given by:

$$\sigma_{x,y} = \sum_{r=1}^p \sum_{k=1}^z part_{x,r,k} + \sum_{k=1}^z part_{x,m,k}. \quad (18)$$

In addition, various environmental factors (such as temperature, air pressure, etc.) may cause systematic deviations between the theoretical predicted values of the model parameters and the actual observed values. Such deviations can lead to significant differences between the simulated count rates and the field-measured values. To eliminate these deviations, this study introduces a correction factor re into the position sensitivity matrix Mat . The revised model expression is:

$$\vec{n}_f = re \times Mat \times \vec{rad}. \quad (19)$$

The correction factor re is determined using the least-squares method, with the objective of minimising the L2 norm difference between the measured count rate vector $\vec{n}_f$ and the simulated count rate vector $\vec{cps}$:

$$re = \arg\min \|\vec{n}_f - \tau \times \vec{cps}\|. \quad (20)$$

The specific calculation formula is:

$$re = \frac{\vec{n}_f^T \times \vec{cps}}{\vec{cps}^T \times \vec{cps}}. \quad (21)$$

3.3. Inversion model for dynamic measurement

After constructing the forward model for dynamic inversion, the surface radionuclide concentration vector $\vec{rad}$ can be inverted using the instrument-recorded count rate $\vec{n}_f$ and Eq. (14). Direct inversion of the constructed linear system [Eq. (13)] poses a significant challenge because the problem is mathematically ill-posed. The position sensitivity matrix often has a high condition number, meaning that small fluctuations in the measured count rates, caused by random noise or instrument instability, can lead to drastically amplified, unrealistic oscillations in the calculated ground concentrations. To address this stability issue, we selected the regularised least-squares (RLS) method for solving the inverse problem. The RLS approach introduces a smoothness constraint ($\lambda\delta_{smo}$) that balances data fitting accuracy with model stability. This methodological choice is critical for suppressing noise artefacts while preserving the sharp boundaries of radioactive anomalies, offering a robust solution compared to simple least-squares estimation. This method ensures the positive definiteness and stability of the solution by minimising the weighted sum of errors between the objective function of the forward model and the actual observed values:

$$\min \delta = \delta_{dis} + \lambda\delta_{smo} (0 \leq m_y \leq \max_m). \quad (22)$$

In Eq. (22), the parameters m_y and $\max_m$ are introduced to ensure the positive definiteness of

the solution and provide numerical stability. Here, δ_{dis} represents the degree of data mismatch, δ_{smo} represents the model smoothness, and λ is the regularisation parameter. δ_{dis} is defined using the L2 norm as:

$$\delta_{dis} = \|\text{Mat}_w \times (\text{Mat} \times \overrightarrow{rad} - \overrightarrow{n_f})\|^2. \quad (23)$$

In Eq. (23), Mat_w is the data weighting matrix, used to quantify the noise level in the data. Assuming the noise of each measured value follows a Gaussian distribution $\frac{1}{\text{var}_i}$ and the noise events are mutually independent, Mat_w can be constructed as a diagonal matrix composed of the reciprocals of the measured data variances Var_i , i.e. $\text{Mat}_w \text{diag} \left[\frac{1}{\text{var}_1} \quad \frac{1}{\text{var}_2} \quad \cdots \quad \frac{1}{\text{var}_i} \right]$.

Eq. (23) can be further expressed as:

$$\delta_{dis} = [\text{Mat} \times \overrightarrow{rad} - \overrightarrow{n_f}]^T \text{Mat}_w^{-1} [\text{Mat} \times \overrightarrow{rad} - \overrightarrow{n_f}]. \quad (24)$$

The model smoothness δ_{smo} is calculated based on the least-squares smoothing criterion and is defined as:

$$\delta_{smo} = \gamma_n \int_0^a (\overrightarrow{rad} - \overrightarrow{rad}_0)^2 dx + \gamma_m \int_0^a \left\{ \frac{\partial (\overrightarrow{rad} - \overrightarrow{rad}_0)}{\partial x} \right\}^2 dx. \quad (25)$$

In Eq. (25), the known radionuclide concentration vector of the geological body micro-elements is $\overrightarrow{rad}_0$. The parameters γ_n and γ_m are weighting factors used to control the error between the inverted concentrations and the known concentrations, where the value of γ_m should be significantly larger than γ_n to ensure the stability of the inversion results. Discretising Eq. (25) into matrix form:

$$\delta_{smo} = (\overrightarrow{rad} - \overrightarrow{rad}_0)^T (\gamma_n \text{Mat}_n^T \text{Mat}_n + \gamma_m \text{Mat}_m^T \text{Mat}_m) (\overrightarrow{rad} - \overrightarrow{rad}_0) = \|\text{Mat}_w (\overrightarrow{rad} - \overrightarrow{rad}_0)\|^2. \quad (26)$$

In Eq. (26), Mat_n is a diagonal matrix whose values are the square roots of the lengths of the geological body micro-elements, i.e. $\text{Mat}_b = \text{diag} \left[\left(\frac{1}{\text{var}_1} \right)^2 \quad \left(\frac{1}{\text{var}_2} \right)^2 \quad \cdots \quad \left(\frac{1}{\text{var}_i} \right)^2 \right]$;

$$\text{Mat}_m = \begin{bmatrix} -\left(\frac{1}{\text{var}_1}\right)^2 & \left(\frac{1}{\text{var}_1}\right)^2 & 0 & \cdots & 0 \\ 0 & -\left(\frac{1}{\text{var}_2}\right)^2 & \left(\frac{1}{\text{var}_2}\right)^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -\left(\frac{1}{\text{var}_i}\right)^2 & \left(\frac{1}{\text{var}_i}\right)^2 \end{bmatrix}.$$

Therefore, Eq. (22) can be further expressed as:

$$\overrightarrow{rad} = (\text{Mat}^T \text{Mat}_w^T \text{Mat}_b \text{Mat} + \lambda \text{Mat}_w^T \text{Mat}_m^T \text{Mat}_w)^{-1} \text{Mat}^T \text{Mat}_b^{-1} \text{Mat}_b \overrightarrow{n_f}. \quad (27)$$

4. Experimental results and analysis

4.1. Data acquisition and experimental setup

To validate the proposed dynamic inversion method, a field experiment was conducted over a survey profile with a total length of 4 km. The experimental dataset consists of both airborne GRS data and ground-based validation data.

4.1.1. Ground-based data (Ground Truth)

Ground measurements were carried out using an IED-2000 portable gamma-ray spectrometer (Chengdu Zhongfu Technology Co., Ltd.). The instrument is equipped with a $\phi 75 \times 75$ mm NaI(Tl) scintillation crystal. The detection limits for potassium (K), uranium (U), and thorium (Th) are 0.2%, 0.5 $\mu\text{g/g}$, and 0.2 $\mu\text{g/g}$, respectively. The spectrometer underwent rigorous energy calibration and standard model calibration, with all performance indicators meeting the requirements of the Specification for Ground Gamma-ray Spectrometry Measurement (EJ/T 363-2012). The ground survey strictly followed operational norms to ensure data reliability. The sampling interval for adjacent ground points was approximately 40 m, providing high-precision “ground truth” values for validating the inversion results.

4.1.2. Airborne data

The airborne GRS data were collected along the same profile. The flight parameters were set as follows: a flight altitude of 40 m, a flight speed of 6.7 m/s, and an acquisition time of 6 s. The raw airborne data were pre-processed with altitude correction and atmospheric radon correction.

4.1.3. Inversion parameters

In the dynamic inversion process, the geological body was discretised into elements with a width of 20 m. The acquisition time was subdivided into 10 micro-time elements to accurately model the dynamic measuring process. For the purpose of this analysis, noise in the measured profile data was ignored, and the ground GRS results were treated as the true values for comparative analysis.

4.2. Experimental results and discussion

After acquiring the UAV-GRS measurement data, the data processing software is used to perform kriging interpolation to generate a data matrix, followed by dynamic inversion of each survey line. This study, based on the MATLAB-2021 platform and combined with the regularised least-squares principle, designs a dynamic inversion algorithm. The key steps include parameter setting, position sensitivity matrix construction, and inversion model solving. The specific workflow is as follows:

- 1) model parameter setting.

The model parameters and their values are defined, the specific variable A is defined, and the number of divisions for the measurement profile is determined as well as the coordinate positions of each division point;

2) position sensitivity matrix construction.

The position sensitivity coefficient matrix Mat is calculated using a custom function “Matrix”. The input parameters for this function include ground GRS measurement data (c_{st}), measured count rates (d), measured point coordinates (x), geological body micro-element endpoint coordinates (x_0), geological body micro-element centre coordinates (xoc), geological body micro-element size (dx), flight altitude (H), number of geological body micro-elements (M), number of measured data points (N), acquisition time (T), time subdivision count (q), and the detector’s maximum detection range (rx_{max}). The function returns the position sensitivity coefficient matrix Mat , the concentration data after geological body micro-element discretisation (m_{st}), the forward-simulated count rates (d_{st}), and the correction factor (re).

Here, the W -function values in Eqs. (5) are calculated using a custom function “ $W_function$ ”. The input parameters include the angle between the detector and the geological body micro-element (φ), $\mu H(a)$, and the detector’s maximum detection angle (b). The function returns the integral result res .

The row index x and column index y of the position sensitivity coefficient matrix Mat range from 1 to n and 1 to m , respectively. The calculation proceeds sequentially for time micro-elements, geological body micro-elements, and sampling point micro-elements, gradually constructing the complete matrix. Based on the relative positional relationship between the geological body micro-elements and the detector, the detection angles are determined, and the corresponding calculations are performed;

3) inversion model solving.

The MATLAB built-in function “pcg” is used for inversion calculations. Based on the regularised least-squares method, parameters such as the maximum number of iterations, weighting factors γ_n and γ_m are set, and the weighting matrix Mat_w and data weighting matrix Mat_b are constructed. The optimal regularisation parameter is iteratively calculated using generalised cross-validation, and the inversion model is solved iteratively using the regularised least-squares method;

4) concentration inversion for other profiles.

Using the constructed position sensitivity coefficient matrix Mat and the count rate data from other profiles, concentration inversion calculations are performed.

As shown in Fig. 7e, by comparing the concentration profile results obtained from dynamic inversion and the stripping ratio method, it can be observed that the concentration data derived from the stripping ratio method are generally slightly higher than the actual measured values. In contrast, the dynamic inversion results exhibit a high degree of consistency with the ground GRS measurement data in terms of variation trends, particularly in the detailed characterisation of concentration distribution. To further validate the credibility of the overall comparison, this study randomly selected four survey lines for localised comparative analysis (as shown in Figs. 7a, 7b, 7c, and 7d). The results indicate that the dynamic inversion method has significant advantages in terms of expanding the range of concentration distribution and reducing the extent of high-concentration areas, more accurately reflecting the spatial distribution characteristics of the surface radiation field. Specifically, the dynamic inversion method not only effectively increases the range of concentration distribution, making the differences between high-concentration and low-concentration areas more pronounced, but also significantly reduces the extent of high-concentration areas, avoiding the issue of overly large high-concentration areas observed in the stripping ratio method. Additionally, the dynamic inversion method demonstrates superior detection capabilities in low-concentration regions compared to the stripping ratio method,

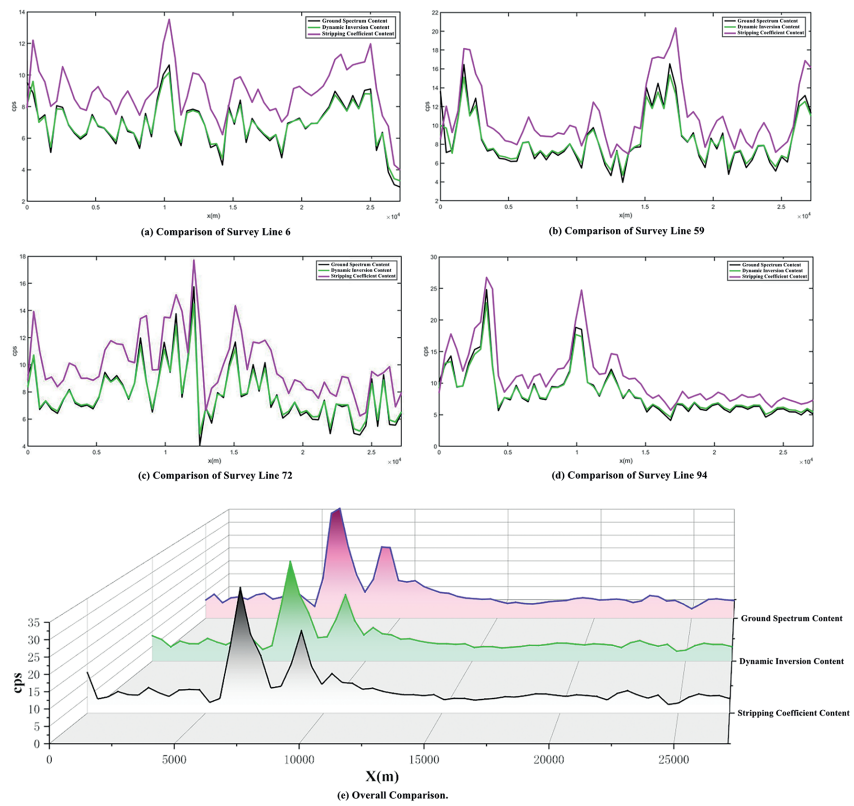


Fig. 7 - Comparison of concentration profiles. The black line represents the Ground Truth, obtained from ground-based measurements using the IED-2000 spectrometer. The green line represents the result of the proposed dynamic inversion, and the magenta line represents the traditional stripping method.

more clearly capturing the variation trends of weak radiation signals and thereby improving the detection accuracy of such signals.

By performing dynamic inversion processing on each survey line and reorganising the UAV-GRS data, a comparison of the UAV gamma-ray spectra before and after inversion was obtained, as shown in Fig. 8. Comparative analysis reveals that the dynamic inversion method significantly optimises the expression of spatial distribution characteristics in the UAV-GRS data. The post-inversion spectrum exhibits clearer boundary delineation in high-concentration areas and more pronounced distribution features of anomalous signals, particularly at the upper edge of the figure, where the identification capability of anomalous regions is notably enhanced. Furthermore, the dynamic inversion method effectively improves the data resolution in low-radiation areas, bringing out weak radiation information that was previously obscured, thereby increasing the detection sensitivity in low-concentration regions. These improvements not only optimise the description of spatial distribution characteristics in UAV-GRS data but also provide more reliable data support for the detailed interpretation of radionuclide concentrations, further enhancing the application value of UAV-GRS in geological exploration and environmental monitoring.

To further quantify the improvement effects of the dynamic inversion technique, this study employs the relative standard deviation (*RSD*) to conduct a quantitative analysis of the data before and after inversion. *RSD*, expressed as a percentage by dividing the standard deviation by the mean, effectively evaluates the dispersion of the data and the stability of the measurement results. Its calculation formula is:

$$RSD = \left(\frac{\sigma}{\mu} \right) \times 100\% \quad (28)$$

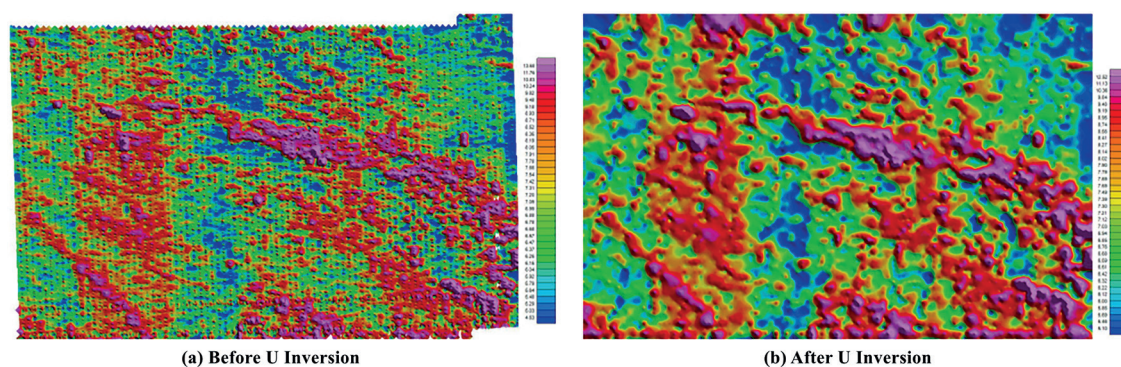


Fig. 8 - Comparison of results before and after dynamic inversion.

where σ is the standard deviation and μ is the mean. A smaller *RSD* value indicates lower data dispersion and better repeatability and stability of the measurement results. After calculation, the *RSD* value of the data processed by the dynamic inversion technique is 31.03%, significantly lower than the pre-processing value of 54.56%. This result demonstrates that the dynamic inversion technique effectively reduces data dispersion and significantly improves the stability and consistency of the measurement results. The reduction in *RSD* is primarily attributed to the dynamic inversion technique's optimisation of boundary clarity in high-concentration areas and its enhanced ability to extract weak radiation information, leading to a more concentrated data distribution and reduced impact from random and systematic errors. Additionally, the lower *RSD* value reflects the advantages of the dynamic inversion technique in terms of data repeatability and reliability, providing higher-quality data support for the precise interpretation of radionuclide concentrations.

5. Conclusions

This study addresses the issues of boundary effects in radioactive anomaly fields and the insufficient precision in describing weak radiation information in dynamic UAV-GRS measurements by proposing a dynamic inversion technique based on micro-element discretisation and regularised least-squares methods. Through theoretical modelling and experimental validation, the research demonstrates that this technique significantly optimises the delineation of high-concentration area boundaries and enhances the spatial resolution of anomaly fields. Quantitative analysis results show that the *RSD* of the data decreases from 54.56% to 31.03% after dynamic inversion, confirming the technique's effectiveness in reducing data dispersion and improving measurement stability. Furthermore, the dynamic inversion technique enhances the extraction capability of signals in low-radiation regions, providing a reliable means for the precise description of weak radiation information. This achievement not only improves the data accuracy and interpretation capability of UAV-GRS but also offers important technical support for its applications in geological exploration, environmental monitoring, and other fields.

Acknowledgments. This document is the results of the research project funded by the National Natural Science Foundation of China Youth Fund (No. 12205044), and the National Natural Science Foundation of China Regional Fund (No. 12265003).

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