

Determination and validation of the Turkish Geoid Model-2020 (TG-20)

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ABSTRACT A new hybrid geoid model for Turkey entitled Turkish Geoid Model-2020 (TG-20) has been determined and validated using historical and recently acquired terrestrial gravity and GPS/levelling data as well as the latest global geopotential and digital elevation models. Highly qualified new gravity data have enabled us to detect and correct some gross errors such as datum inconsistencies in the historical gravity data sets. The Least-Squares Modification of Stokes' formula with the Additive Corrections Method has been applied to compute gravimetric geoid model. Transformation to hybrid model has been carried out via 4-parameter fit to homogeneously distributed historical GPS/levelling data. Quality assessment of the fit residuals has let us to find out some unrevealed outlying data in the historical GPS/levelling data set, which had already been utilised in the previous geoid model computations. Validation of the TG-20 over the seven different new GPS/levelling profiles has suggested standard deviations ranging between 1.2 and 6.3 cm, that is at least twice more accurate than the previous official geoid model. The results reveal that the recent measurement campaigns and the latest global models bring about a significant and substantial amount of improvement in computing and validating a high quality national geoid model.

Key words: Turkish Geoid Model-2020, Least-Squares Collocation, Least-Squares Modification of Stokes' Formula with Additive Corrections, TR-GravNet, GPS/levelling.

1. Introduction

The advances in satellite positioning technologies particularly in the Global Navigation Satellite Systems (GNSS) during the last few decades have brought out a new tool called GNSS heighting to determine the physical heights far more cheaply and quickly than the traditional methods (Meyer *et al.*, 2006). GNSS heighting requires a precise geoid or quasi-geoid model consistent with the national vertical datum and height system adopted. Nowadays, most countries are seeking to acquire 1 cm or better geoid/quasi-geoid models covering their territories (Smith *et al.*, 2013; Farahani *et al.*, 2017; Oršulić *et al.*, 2019; Ellmann *et al.*, 2020).

A five-year-long collaborative project entitled "Turkish Height System Modernization and Gravity Recovery (2015-2020)" has been conducted in Turkey to estimate a few centimetres national geoid model at 1-3 km spatial resolution. The project has four major sub-objectives that must be accomplished in turn: i) densification of Turkish gravity standardization network (TR-GravNet), ii) determination of the best fitting Gravity field and Ocean Circulation Explorer (GOCE)-based satellite-only Global Gravity field Model (GGM), iii) removing the discrepancies in historical gravity data using TR-GravNet reference data set along with the best fitting satellite-only GGM, and

iv) computation of a Turkish geoid model with a precision of a few centimetres at 1-3 km spatial resolution.

Simav and Yildiz (2019) focused on the first and second objectives of the project. They described the TR-GravNet densification procedure in south-western Turkey and assessed the Earth Gravitational Model 2008 (EGM2008) (Pavlis *et al.*, 2012) along with release 5 of GOCE-based satellite-only GGMs. They concluded that the GOCE-based models perform better than the EGM2008. In this study, we focus on the determination of Turkish Geoid Model-2020 (TG-20) and explore how close we are to the final project objective of high precision geoid modelling. Subsequently, we quantify the amount of improvement gained by the new geoid with respect to the previous official geoid model of Turkey, Turkish Geoid Model-2003 (TG-03) (Kilicoglu *et al.*, 2006).

Second and third parts of the manuscript are devoted to the description of the data and the methodology used. The quality assessment of historical gravity data compiled from different sources is, then, presented in the fourth part. The fifth part is related to the determination of the gravimetric geoid model, and the sixth part describes transforming of the gravimetric geoid to the current Turkish vertical datum. Comparison of the new and existing geoid models, with each other and with the GPS/levelling geoid heights, are presented in the seventh part. Finally, the results are discussed and the conclusions are presented.

2. Data

2.1. TR-GravNet data

Simav and Yildiz (2019) described the strategy of measurement and adjustment of the Turkish gravity standardisation network, shortened as TR-GravNet, in detail. In this study, we use 12,905 TR-GravNet points depicted with red plus signs in Fig. 1. The average spacing between the TR-GravNet points is about 5 arcmin corresponding nearly 10 km in mid-latitudes. The mean formal error of these points, obtained from the constrained least-squares network adjustment, is about 20 μ Gal. The permanent Earth tides are treated according to the zero-tidal gravity concept (Ekman, 1989) during the data reduction.

The 3D positions of the TR-GravNet points are determined via the network-based Real-Time Kinematic GNSS service called CORS-TR. The overall precision of the site positions given in the ITRF96 coordinate reference frame at epoch 2005.0 is of few centimetres in the horizontal components and better than 1 dm in the vertical component. The TG-03 is used to convert the ellipsoidal heights of TR-GravNet points to Helmert orthometric heights. Subsequently, free-air gravity anomalies at the TR-GravNet points are computed with respect to the GRS80 reference ellipsoid (Moritz, 2000).

2.2. Historical gravity data

There are almost 267,000 historical gravity data in Turkey compiled from the General Directorate of Mineral Research and Exploration (MTA) and from the Turkish Petroleum Corporation (TPAO). This type of data is regarded as historical data as there is almost no metadata available for them. The data were generally collected by old kind of gravimeters. We have no information about the reductions applied to the gravity readings, little knowledge

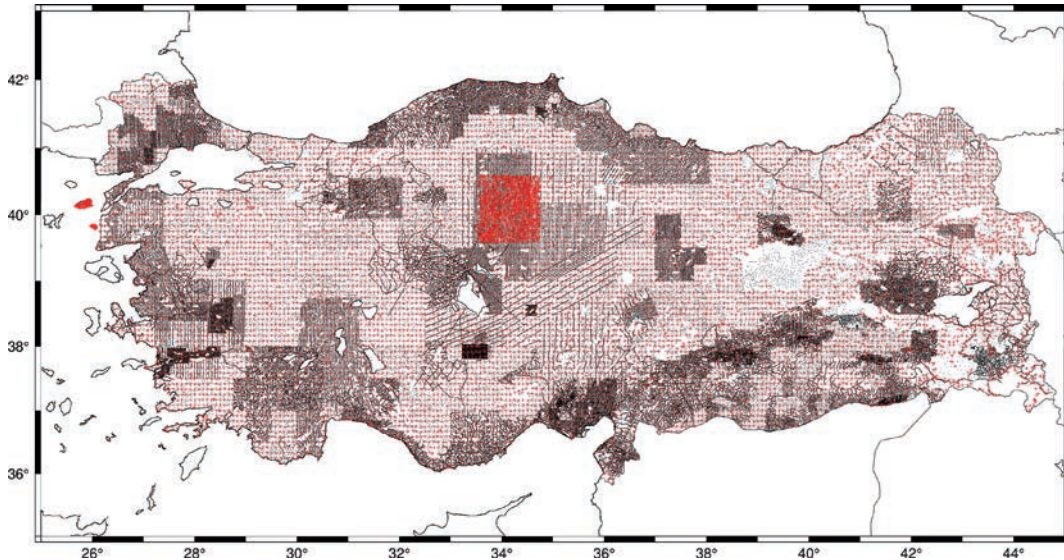


Fig. 1 - Terrestrial gravity data in Turkey. TR-GravNet (red plus sign) and quality-controlled historical gravity data (black dots).

about the gravity data, and no reports about any adjustment results. However, the horizontal and vertical coordinates of the MTA data were digitised from the 1:25,000 topographic maps published in the European Datum-1950 (ED-50) whereas the coordinates of TPAO data were generally obtained by classical triangulation measurements in the ED-50 datum. The transformation of the horizontal coordinates from ED-50 to the Turkish National Reference Frame (TUREF), defined based on ITRF96 (epoch 2005.0), is carried out by a three-dimensional modified Helmert model in which the effect of height component is minimised. The precision of this transformation is about 1.1 m (Aktug *et al.*, 2011). Thus, during the transformation of the coordinates to TUREF, the orthometric heights of the historical gravity points referenced to the Turkish vertical datum (Demir and Cingoz, 2004) are left unchanged. Similar to the TR-GravNet data, free-air gravity anomalies at historical gravity points are calculated with respect to the GRS80 ellipsoid (Moritz, 2000).

2.3. Global gravity field model

Satellite-only GGMs that are independent of any errors in the terrestrial gravity observations are very rewarding data set for the quality assessment of historical gravity data in the absence of reliable terrestrial data (Bonfim *et al.*, 2013), and they are crucial for the modelling of the long-wavelength component in geoid modelling. Simav and Yildiz (2019) demonstrated that the GOCE-based satellite-only models show improvements over EGM2008 to spherical harmonic degree ~ 220 . The zero-tide version of the GOCO06S satellite-only GGM (Kvas *et al.*, 2021) is used in agreement with tide system adopted for the TR-GravNet gravity data.

The free-air gravity anomalies over the marine areas and neighbouring countries at $1' \times 1'$ resolution are computed using the experimental gravity field model XGM2019e (Zingerle *et al.*, 2020) up to degree and order 5399 corresponding to a spatial resolution of 2 arcmin. The XGM2019e is a recently released combined GGM based on the GOCO06S (Kvas *et al.*, 2021) in the low-frequency

band in combination with the same the terrestrial data used for the computation of XGM2016 (Pail *et al.*, 2018) at 15'×15' resolution and spectrally enhanced with the Earth2014 topographic model (Hirt and Rexer, 2015) over the land areas and Denmark Technical University-2013 (DTU13) (Andersen *et al.*, 2014) satellite altimetric gravity anomalies over the oceans.

2.4. Digital elevation model

Elevation models are used to evaluate the terrain effects in gravity field modelling. In this study, Digital Elevation Model (DEM) of 7.2 arcsecond resolution over the land areas, that was constructed by Hirt *et al.* (2014) based on the SRTM V4.1 250 m (Jarvis *et al.*, 2008) are utilised. The data are downloaded from Western Australian Centre for Geodesy, Curtin University Perth at <http://ddfe.curtin.edu.au/gravitymodels/ERTM2160/data/dem/>.

2.5. GPS/levelling data

We have utilised two types of GPS/levelling data during the TG-20 computations. The first type is the 192 historical data homogeneously distributed throughout the country, which has already been utilised in the productions of the previous geoid models. GPS and levelling measurements for these 192 co-located sites were acquired at completely different time periods. While the GPS measurements were collected within a few years in the 1990s, the levelling survey period has lasted for at least 60 years. The vertical displacements, particularly caused by the earthquakes and groundwater changes during the long survey period, might cause some changes on the point heights. On account of its homogeneous distribution, the historical GPS/levelling data have been used with some consideration in the gravimetric-hybrid geoid model transformation.

The non-concurrently data acquisition problem in the historical GPS/levelling data has led us to make the new precise levelling and GPS measurements simultaneously to avoid possible vertical displacements. This second type of GPS/levelling data was collected during the summers of 2017, 2018, and 2019. Labour-intensive precise levelling measurements were performed at seven different profiles countrywide (Fig. 2). These profiles have been particularly selected in areas where the geoid slopes are the highest (Fig. 2b). A total length of 1705 km was surveyed using well-calibrated Topcon DL-500 digital levels and rods by two motorised field teams. Two hundred and seventy-eight new GPS/levelling co-located benchmarks were constructed along the levelling profiles for the validation of the geoid model.

GPS and first-order double run precise levelling measurements were performed simultaneously at the co-located benchmarks. GPS data were collected in static mode for at least 8 hr with 30 s sampling rate by Topcon GR-5 integrated receiver and antennas. A specially designed aluminum mast, that has a fixed antenna height (1.8 m), was used to avoid antenna height measurement errors.

GPS processing has been done via GAMIT/GLOBK 10.70 software (Herring *et al.*, 2018) to derive precise geodetic coordinates in the International Terrestrial Reference Frame-2014 (ITRF2014) (Altamimi *et al.*, 2016) and subsequently transformed to the ITRF1996. The standard deviations of the ellipsoidal heights vary between 0.33 and 3.10 cm with a mean of 1.10 cm.

We also observed gravity at both the GPS and intermediate levelling benchmarks by a Scintrex CG5 relative gravimeter to compute the geopotential numbers. Relative gravity measurements at each independent profile were tied to at least two absolute gravity points of the TR-GravNet and adjusted in the same way as described in Simav and Yildiz (2019).

Each levelling profile was adjusted by fixing geopotential number of any single secure benchmark in the vicinity in order not to introduce any possible vertical displacement errors into

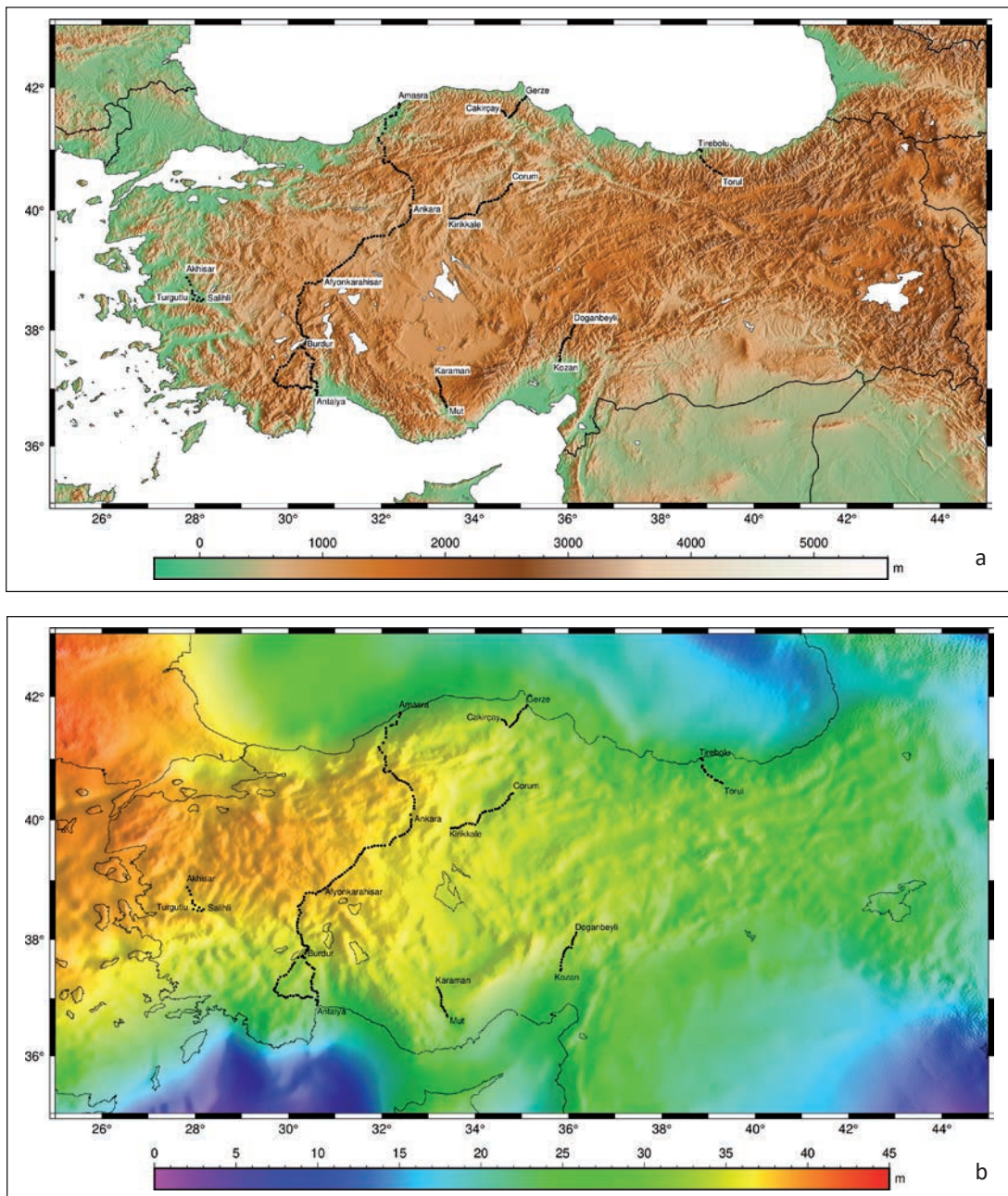


Fig. 2 - Simultaneously measured seven GPS/levelling profiles: a) over the 7.2 arcsec SRTM DEM, b) over the TG-20 geoid model.

the new GPS/levelling geoid undulations. Helmert orthometric heights were computed after the adjustment of the geopotential numbers. The longest profile running from the Mediterranean to Black Sea includes a closed loop between Antalya and Burdur cities (Fig. 2a). The closure error is $0.01516 \text{ kGal} \times \text{m}$ (g.p.u) in terms of geopotential numbers corresponding approximately to 1.5 cm. Based upon this information, the error propagation of the GPS and levelling errors yields a 1.8 cm error for the GPS/levelling geoid undulations.

In order to ensure the tide system consistency for whole data sets, both the ellipsoidal heights and Helmert orthometric heights have been transformed to zero tide systems using the formula given by Ekman (1989). The correction terms for these transformations vary between -3.2 and -0.7 cm.

2.6. TG-03 hybrid geoid model

The TG-03 is the latest official hybrid geoid model of Turkey (Kilicoglu *et al.*, 2006), which was constructed in two successive steps. First a gravimetric quasi-geoid was computed by the Least-Squares Collocation (LSC) method using EGM96 up to degree and order 360, MTA historical gravity data, marine gravity anomalies derived from ERS1, ERS2 and TOPEX/POSEIDON altimetry data (Kilicoglu, 2005), and a national DEM of 450 m spatial resolution. Subsequently, the gravimetric quasi-geoid was combined with 192 historical GPS/levelling quasi-geoid heights by a 5-parameter bilinear model. Finally, the hybrid quasi-geoid model was converted to a geoid model at 3'×3' resolution as the Helmert orthometric heights are officially adopted in Turkey.

3. Methodology

The new gravimetric geoid model is computed based on the Least-Squares Modification of Stokes' Formula with Additive Corrections (LSMSA) method (Sjöberg, 2003) where the geoid undulation N is attained as the sum of geoid height from stochastic least-squares kernel modification and four additive corrections for the topography, downward continuation, atmospheric, and ellipsoidal corrections (Ågren *et al.*, 2009):

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma_0} S^M(\psi) \Delta g d\sigma + \frac{R}{2\gamma} \sum_{n=2}^M (s_n + Q_n^M) \Delta g_n^{GGM} + \delta N_{DWC} + \delta N_{COMB}^{TOP} + \delta N_{ELL} + \delta N_{COMB}^{ATM} \quad (1)$$

where R is the mean Earth radius, σ_0 is the spherical cap, γ is the mean normal gravity, Δg is the unreduced gridded Molodensky-type surface free-air gravity anomaly, $S^M(\psi)$ is the modified Stokes' function, s_n is the modification parameters, M is the maximum degree of the GGM, Q_n^M are the Molodensky truncation coefficients, Δg_n^{GGM} is the Laplace surface harmonic of the gravity anomaly computed from the selected GGM up to degree n , δN_{DWC} is the downward continuation correction (Sjöberg, 2003), δN_{COMB}^{TOP} is the combined topographic correction (Sjöberg, 2007), δN_{ELL} is the ellipsoidal correction to the modified Stokes' formula (Sjöberg, 2004), and δN_{COMB}^{ATM} is the combined atmospheric correction (Sjöberg and Nahavandchi, 2000).

The detailed formulas of four additive corrections are described in Ågren *et al.* (2009). Scientific Software for Precise Geoid Determination Based on the Least-Squares Modification of Stokes' Formula (LSMS-GEOLAB) (Kiamehr and Sjöberg, 2010) is used for the practical implementation of the LSMSA method.

4. Quality assessment of historical gravity data by the LSC method

The quality assessment of the historical gravity data is carried out by the LSC method with the remove-compute-restore procedure. The LSC is a very efficient and flexible method that enables

the use of different Earth gravity field functionals (Arabelos and Tscherning, 1998). The theory of the method is well described by Krarup (1969), Moritz (1980), Sansò and Sideris (2013), and Tscherning (2015), and there exist numerous publications including the application of LSC method for gravity field modelling (Forsberg and Tscherning, 1981; Barzaghi *et al.*, 1996, 2002, 2007; Arabelos and Tscherning, 1998; Tscherning *et al.*, 2001; Tscherning, 2013).

LSC considers the data located at different altitudes by a spatial covariance function. The required auto- and cross-covariance matrices are computed using the analytically modelled covariance function of the disturbing potential by covariance propagation (Sadiq *et al.*, 2010):

$$cov(T_p, T_q) = \alpha \sum_{i=2}^N \left[\frac{R_E^2}{r_p r_q} \right]^{i+1} \sigma_i^2 P_i(\cos \psi) + \sum_{i=N+1}^{\infty} \left[\frac{R_B^2}{r_p r_q} \right]^{i+1} \frac{A}{(i-1)(i-2)(i+4)} P_i(\cos \psi) \quad (2)$$

where ψ is the spherical distance between p and q , r_p, r_q are the radial distances from the origin, N is the degree of GGM, σ_i^2 is the error degree-variance related to the GOCO06S GGM up to degree 220, R_E is the mean Earth radius and R_B is the radius of Bjerhammar sphere. An iterative adjustment (Knudsen, 1987) is used to determine the covariance parameters α (scale parameter), A [constant parameter with units of (m/s)⁴], and the Bjerhammar radius R_B . In the case of using LSC for free-air gravity anomaly (Δg) prediction from Δg , the equation modified from Moritz (1980) can be written as:

$$\Delta g(p) = C_{pq}^{\Delta g \Delta g} (C_{qq}^{\Delta g \Delta g} + C_{\varepsilon\varepsilon}^{\Delta g \Delta g})^{-1} \Delta g_q \quad (3)$$

where $C_{pq}^{\Delta g \Delta g}$ represents signal cross-covariance matrix of variables Δg_p and Δg_q between the computation point p and the observation point q , $C_{qq}^{\Delta g \Delta g}$ is the auto-covariance matrix of the observations (Δg_q) at observation point q and $C_{\varepsilon\varepsilon}^{\Delta g \Delta g}$ is the covariance matrix of the observational errors. For the evaluation of the above equations, the latest version of GEOCOL (i.e. GEOCOL19) that can implement multi-processing LSC (Kaas *et al.*, 2013) is used.

The free-air gravity anomalies at the gravity points are separated into three components: namely long-wavelength part, short-wavelength part, and residual part. While the long-wavelength part is computed from GOCO06S ($N = 2$ to 220), the short-wavelength part is compensated by Residual Terrain Modelling (RTM) effects taking into account the deviations of topography from a mean elevation surface (Forsberg, 1984). A mean elevation surface with 15 arcmin resolution is constructed by low-pass filtering the 7.2 arcsec high-resolution DEM by applying a Gaussian filter. Setting an integration radius of 167 km by GRAVSOF TC program (Forsberg, 1984; Forsberg and Tscherning, 2008), the RTM effects in terms of the free-air gravity anomalies are computed.

The empirical covariance function is determined from the TR-GravNet residual gravity anomaly data employing the GRAVSOF EMPCOV program and fitted to the analytical Tscherning/Rapp model (Tscherning and Rapp, 1974) using the program COVFIT (Knudsen, 1987) of GRAVSOF, considered as the standard method on the sphere that can generally provide a good fitting only up to the first zero of the empirical function (Barzaghi *et al.*, 2001). Fig. 3 shows the empirical and modelled covariance values indicating a good fit that reflects the capability of the used fitting procedure. The estimated covariance parameters are the depth to the Bjerhammar-sphere

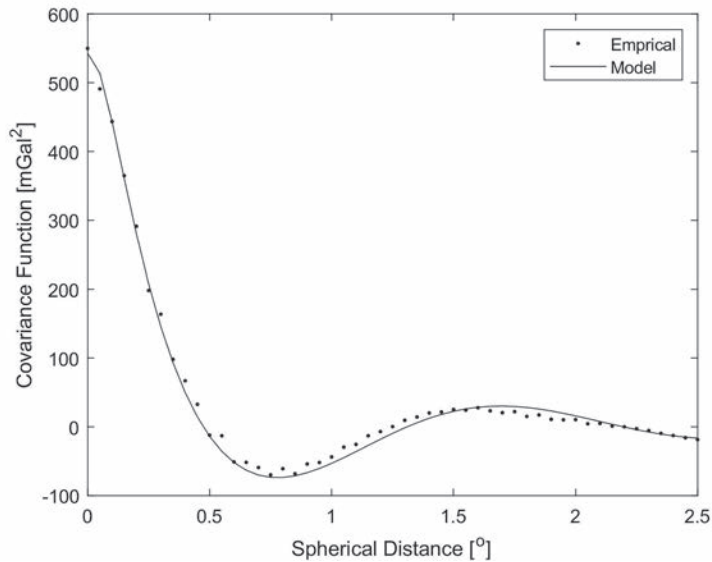


Fig. 3 - Empirical (dotted line) covariance values derived from TR-GravNet residual gravity anomalies and fitted analytical values of Tscherning and Rapp (1974) covariance model.

(-5.55 km), the gravity variance at zero altitude (635.09 mGal²), and the error degree variance scale factor (77.54).

Subsequently, using the estimated covariance parameters, LSC is implemented to the residual gravity anomalies to predict the same quantities at historical gravity data points. Fig. 4 shows the differences between the observed and predicted gravity anomalies at MTA historical points in a region where the historical data have considerably large systematic errors. It is evident from Fig. 4 that a local bias exists in the area that lies between the latitudes 38.5° to 39.0°N and the longitudes 30.0° to 30.5°E. The mean of the estimated bias is about 14 mGal, which suggests a gravity datum inconsistency. Most probably, the gravity value of the tie or reference gravity point in that sub-region was in Potsdam datum, which used to be the adopted gravity datum in Turkey until the release of IGSN-71 (Morelli *et al.*, 1973). While the initial mean and standard deviations of the differences between the observed and predicted gravity anomalies at historical gravity data points are 1.6 and 6.3 mGal, respectively, the standard deviation drops down to 5.3 mGal after the correction of the 14 mGal local datum bias is applied. Following the elimination of the datum inconsistency, the well-known 3-sigma criteria are applied iteratively for the removal of

Table 1 - Statistics of the differences between the predicted and observed free-air gravity anomalies at MTA historical gravity data points. Units are in mGal (N: number of data, N^o: number of outliers).

Historical Gravity Data	Max	Min	Mean	Std	N	N ^o
Without any quality control	200.9	-114.6	1.6	6.3	64908	-
After 14 mGal is subtracted in a sub-region	200.9	-114.6	1.6	5.3	64908	-
After 14 mGal is subtracted in a sub-region and the outliers are removed by 3 σ criteria after 8 iterations	9.4	-9.4	1.4	3.1	61512	3396

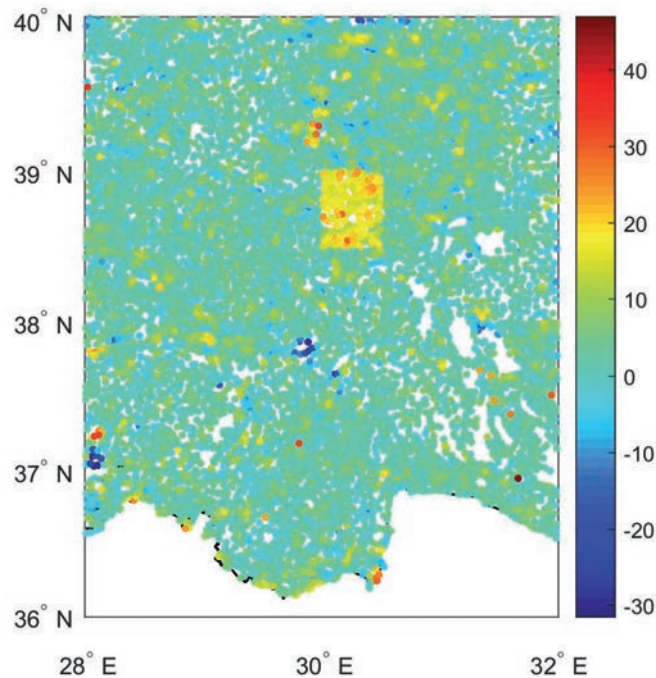


Fig. 4 - Differences between the observed and predicted free-air gravity anomalies at MTA historical points in a region where the historical data have considerably large systematic errors. Unit: mGal.

the possible outliers in the historical data set. After 7 iterations, the standard deviation and mean of the differences are found to be 3.1 and 1.4 mGal respectively in the range of ± 9.4 mGal (Table 1).

Similar procedure is applied for the TPAO historical gravity data. No systematic error is detected such as datum inconsistency in this data set. The standard deviation and mean of the differences after 10 iterations result in 2.7 and 1.4 mGal, respectively, with a range of ± 8.0 mGal (Table 2). Following these analyses, 5.2% of the MTA and TPAO historical gravity points are flagged as outliers and removed from the data set.

Fig. 1 shows the spatial distributions of the quality-controlled MTA and TPAO historical point-wise gravity data and TR-GravNet point-wise gravity data used for the gravimetric geoid determination.

5. Gravimetric geoid determination by the LSMSA method

This section describes how the new gravimetric geoid model is computed by the LSMSA method requiring gridded surface Molodensky-type free-air gravity anomaly data as input (see Eq. 1). To reduce the discretisation errors for the gridding of point-wise free-air gravity anomalies, a remove-grid-restore strategy is applied (Ågren *et al.*, 2009). Initially, the free-air gravity anomalies from GOCO06S ($N = 2$ to 220) GGM and RTM free-air gravity anomaly effects are removed from all point-wise free-air gravity anomalies in Fig. 1, then the residual gravity anomalies at 265923 points, that are constructed merging TR-GravNet and quality-controlled historical MTA and TPAO point-wise residual gravity anomalies (Table 3), have been gridded on a regular $1' \times 1'$ geographical grid by the 2-D LSC method implemented in the GEOGRID

Table 2 - Statistics of the differences between the predicted and observed free-air gravity anomalies at TPAO historical gravity data points. Units are in mGal (N: number of data, N^o: number of outliers).

Historical Gravity Data	Max	Min	Mean	Std	N	N ^o
Without any quality control	88.4	-86.0	1.6	3.7	202086	-
After the outliers are removed by 3 σ criteria after 10 iterations	8.0	-8.0	1.4	2.7	191506	10580

Table 3 - Statistics of the TR-GravNet, quality-controlled MTA and TPAO point-wise residual gravity anomalies. Units are in mGal (N: number of data).

Gravity Data	N	Max	Min	Mean	Std
TR-GravNet	12905	108.1	-97.6	-0.0048	23.19
Quality-controlled MTA	61512	107.4	-94.4	-0.1500	23.41
Quality-controlled TPAO	191506	113.6	-79.2	0.6500	23.39

program of the GRAVSOFIT package (Forsberg and Tscherning, 2008) with correlation length of 21.2 km. The correlation length is determined using the model covariance function fitted by a logarithmic function (Forsberg, 1987) to the empirical covariance function estimated from the residual gravity anomalies. This logarithmic function is defined by three parameters: C_0 as the variance of the residual gravity anomalies, D and T as the parameters representing the degree of damping the high and low frequencies of the gravity signal, respectively (Forsberg, 1987). These parameters are determined using the GPFIT program of the GRAVSOFIT package (Forsberg and Tscherning, 2008) whereas the correlation length is determined as the distance where the modelled covariance function reaches the value of $C_0/2$.

The LSMSA method is implemented using GOCO06S up to degree 300, setting the capsize radius $\psi_0 = 1^\circ$. The optimal modification parameters in Eq. 1 is determined using the modification approach of Ågren *et al.* (2009) based on the determination of the crossing point of the error degree variances of the GGM and terrestrial gravity data that are computed from its formal errors of the spherical harmonic coefficients and the combination of white noise and the reciprocal distance model (Ågren *et al.*, 2009), respectively. The degree variance models described in Table 4 gives the best fitting gravimetric geoid model in comparison with GPS-levelling geoid undulations over the seven levelling profiles shown in Fig. 2a. The optimal modification parameters suggest the degree 150 as the crossing point of the error degree variances of the GOCO06S GGM and the terrestrial gravity data.

Table 4 - Degree variance models used in new gravimetric geoid computation by the LSMS method using the modification approach of Ågren *et al.* (2009).

Type of degree variance	Explanation
Signal	Tscherning and Rapp (1974). Degree variances rescaled using the factor 0.5 ² .
GGM error	Computed from the formal error of the spherical harmonic coefficients of GOCO06S (Kvas <i>et al.</i> , 2021). GGM error degree variances are rescaled using the factor 0.4 ² .
Terrestrial gravity error	Combination of the reciprocal distance and white noise models (Ågren <i>et al.</i> , 2009). The reciprocal distance part is specified by the standard deviation of 0.2 mGal and the correlation length. The white noise part is specified by the standard deviation of 0.2 mGal and the Nyquist degree 10800.

Fig. 5 shows four additive corrections for the final gravimetric geoid model. The downward continuation (DWC) correction shown in Fig. 5a reaches its maximum value of approximately 2 m in the mountainous areas in eastern parts of Turkey and stays around a few cm levels in the flat areas. The combined topographic correction shown in Fig. 5b reaches approximately -2 m in the mountainous areas. The combined atmospheric correction is computed using the global worldwide DEM at 15'×15' resolution derived from SRTM30plus spherical harmonic coefficients to the maximum degree 720 (Ågren *et al.*, 2009). The combined atmospheric correction shown in Fig. 5c is in the range from -1.0 to 0.2 cm and reaches its maximum in the mountainous areas, which should not be neglected. The ellipsoidal correction shown in Fig. 5d is in the order of a few millimetres.

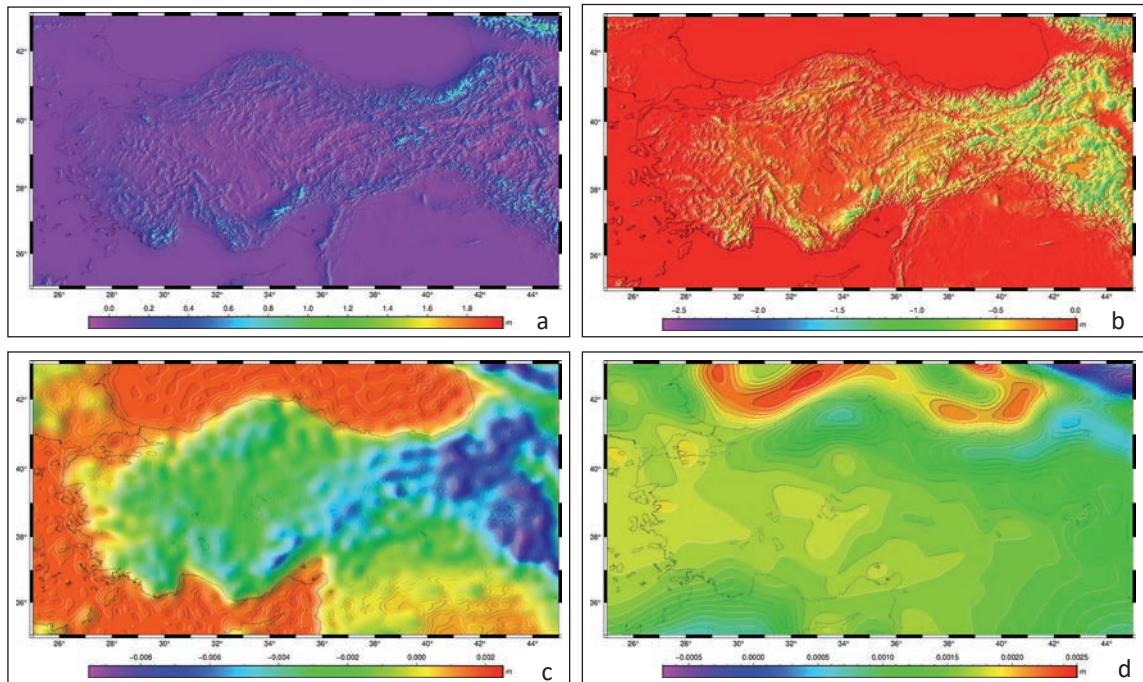


Fig. 5 - Four additive corrections for the new gravimetric geoid computation by the LSMSA method: a) Downward continuation, b) Combined topographic effect, c) Combined atmospheric correction, d) Ellipsoidal correction. Unit: m.

Table 5 - Statistics of the residuals of new gravimetric geoid model after a 1- and a 4-parameter fit using 182 historical GPS/levelling benchmarks. Units are in m.

# Parameter	Statistics (GPS/levelling geoid- Geoid model)			
	Mean	Min	Max	Std
1	-0.857	-1.112	-0.638	0.097
4	0.000	-0.245	0.211	0.086

6. Shifting the gravimetric geoid to the current Turkish vertical datum

Considering the sustainability of practical surveying works in the country, the gravimetric model should be fitted to the current vertical datum via GPS/levelling data. However, this is one of the most challenging parts of our TG-20 geoid computation scheme. On the one hand, we have

historical and non-concurrently measured but well-distributed GPS/levelling data convenient for classical 4-parameter transformation (Heiskanen and Moritz, 1967), but on the other hand we have profile-based, recently acquired, and simultaneously measured GPS/levelling data but not suitable for tilt estimation (see section 2.5). For that reason, we decided to apply both 4-parameter and 1-parameter transformations to the same GPS/levelling data set, compare the post-fit residual statistics, and assess their performances at the seven new GPS/levelling profiles not used in the transformation.

The new gravimetric geoid is transformed into the current vertical datum by 4-parameter and 1-parameter transformations using the well-distributed 192 historical GPS/levelling geoid undulations that have already been used in the TG-03 hybrid geoid model production. After excluding the 10 outlying GPS/levelling data, the final statistics of the post-fit residuals between new gravimetric geoid and 182 historical GPS/levelling undulations are presented in Table 5. The estimated mean value of -0.857 ± 0.097 m from 1-parameter transformation represents the zero degree-term (Smith, 1998) neglected in gravimetric geoid determination and the deviation of Turkish vertical datum from the global datum. The remaining post-fit residual statistics in Table 5 show that 4-parameter transformation performs better than 1-parameter case. Table 6 gives the statistics of the comparisons at the seven different new GPS/levelling profiles which indicates the superiority of 4-parameter transformation over 1-parameter. Therefore, the eventual transformation of gravimetric geoid to vertical datum is carried out by 4-parameter transformation using 182 historical GPS/levelling geoid undulations and final hybrid geoid model is obtained in the current Turkish vertical datum.

Table 6 - Comparison of the 1- and 4-parameter fitted geoid model using 182 historical GPS/levelling with independent GPS/levelling geoid undulations over seven different profiles shown in Fig. 2a. The bracketed values represent the 4-parameter transformation. Units are in m (N: number of GPS/levelling benchmarks).

Profile name	Statistics (GPS/levelling geoid - Geoid model)				
	N	Mean	Min	Max	Std
Antalya - Burdur - Amasra	176	0.071 (0.078)	-0.040 (0.020)	0.015 (0.160)	0.038 (0.028)
Doğانبeyli - Kozan	18	-0.020 (-0.078)	-0.073 (-0.110)	0.027 (-0.050)	0.032 (0.018)
Çakırçay - Gerze	17	0.041 (0.088)	-0.011 (0.040)	0.072 (0.110)	0.024 (0.019)
Kırıkkale - Çorum	30	0.021 (0.004)	-0.020 (-0.020)	0.049 (0.020)	0.019 (0.013)
Akhisar - Salihli	11	-0.113 (-0.139)	-0.130 (-0.150)	-0.083 (-0.110)	0.012 (0.013)
Tirebolu - Torul	13	-0.026 (0.046)	-0.110 (-0.080)	0.019 (0.100)	0.040 (0.063)
Karaman - Mut	13	-0.080 (-0.086)	-0.091 (-0.100)	-0.061 (-0.060)	0.010 (0.012)

7. Assessment of the TG-20 and TG-03 geoid models

The TG-20 hybrid geoid model shown in Fig. 2b is evaluated using recent GPS/levelling measurements over seven levelling profiles and compared with the previous official Turkish geoid model TG-03.

The statistics of the differences between the TG-20 and geoid heights at simultaneously measured GPS/levelling co-located benchmarks are presented in Table 7 and the de-measured differences for each profile are shown in Fig. 6a. GPS/levelling geoid undulations successfully validate the TG-20 with standard deviations of 1.2 to 6.3 cm. The statistics show that the standard deviations of the differences are below 3.0 cm in most of the levelling profiles crossing through relatively rugged terrains where the geoid slopes are highest. The results are very promising on reaching the project objective of 1-3 cm at 1-3 km geoid model. The maximum differences are observed in Tirebolu-Torul profiles near the Black Sea coasts, which could be attributed to the marine gravity data used. The relatively large bias of -13.9 cm in Akhisar-Salihli profile indicates a subsidence of the historical levelling benchmark used as a tie point. This low-lying wetland area has been used for agricultural purposes for many years. Possible groundwater depletion might cause local subsidence and lower the vertical position of the tie point physically. In this situation, the published Helmert orthometric height (H) of the tie point does not reflect its true value at the recent measurement epoch and would be higher. If we compute the GPS/levelling geoid undulations ($N^{GPS/Lev}$) using the ellipsoidal heights of the new GPS/levelling benchmarks at the recent epoch ($h^{GPS_recent_epoch}$) by the formula:

$$N^{GPS/Lev} = h^{GPS_recent_epoch} - H \quad (4)$$

then, a negative bias is introduced into the $N^{GPS/Lev}$ values.

The TG-03 model has also been compared with the new 278 GPS/levelling geoid undulations and with the TG-20 to assess the advancement. The statistics of the comparison with GPS/levelling are given in Table 7 and the de-measured differences for each profile are shown in Fig. 6b. It is clear that the standard deviations are at least 50% higher than the TG-20 model and reach up to 11.2 cm. The extreme values in the differences ranging from -39.0 to 17.8 cm over the Antalya-Burdur-Amasra profile are reduced by 75% with the new model. Fig. 6b indicates the areas having significant errors in the TG-03. It also points out a significant tilt in the TG-03 increasing from NE to SW direction in the Antalya-Burdur closed loop.

The differences between TG-20 and TG-03 are shown in Fig. 7. Remarkable variations in the coastal areas particularly in the eastern Black Sea coasts, reaching up to 80 cm, are evident. This is probably due to GOCE based GGM improvement over the EGM96 model. The quite large differences in some inland areas may potentially be caused by the gross errors in some of 192 historical GPS/levelling data, shown in the background of Fig. 7, used in the production of TG-03 but eliminated as outliers in TG-20 computation. As explained in section 6, 10 outlying historical GPS/levelling data are eliminated, and 182 historical GPS/levelling data are used in the computation of TG-20. Also emphasised in section 2.5, one possible explanation for the gross errors in the historical GPS/levelling data may possibly be the non-coincidence in time between the levelling and GPS measurement acquisitions.

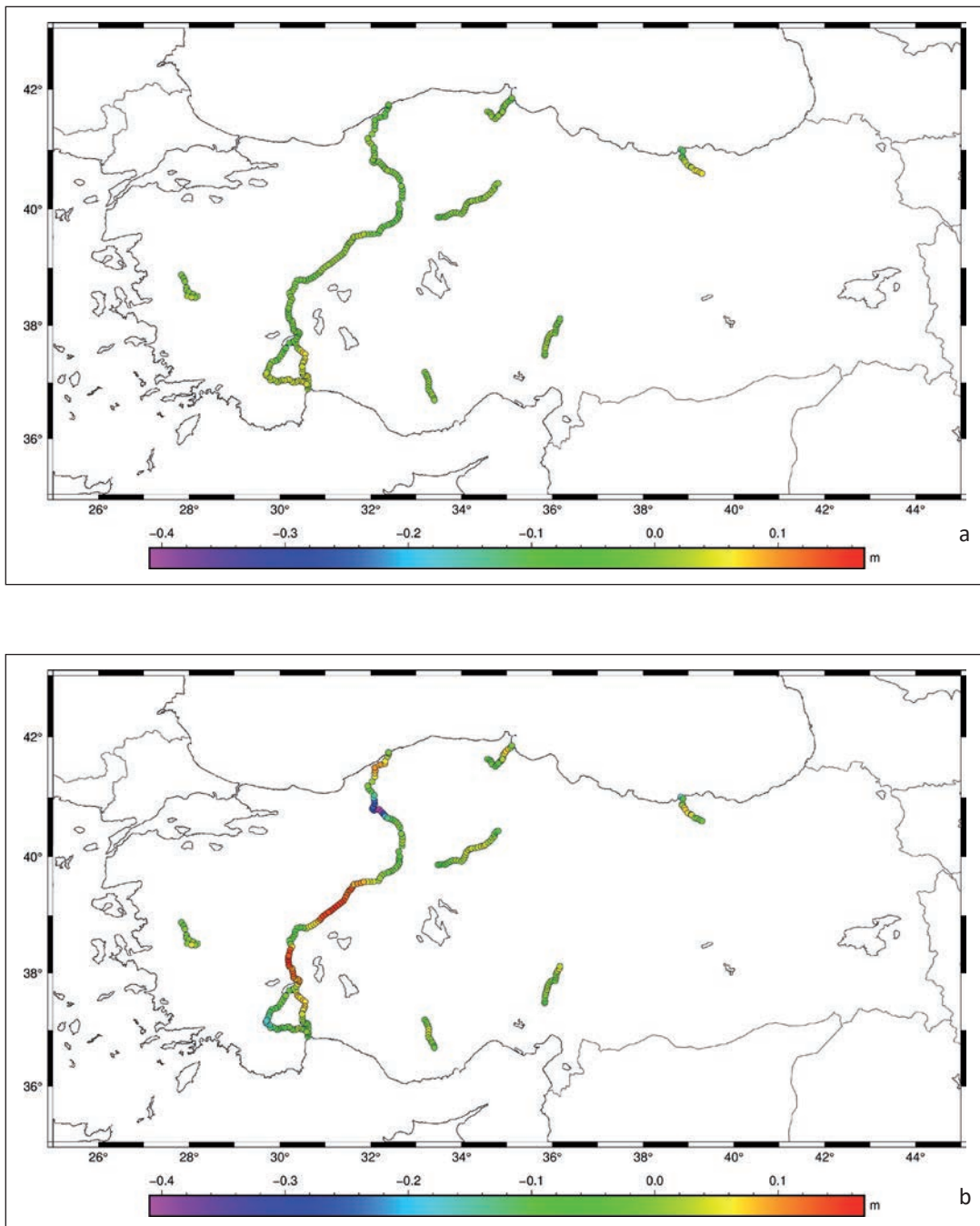


Fig. 6 - Differences between GPS/levelling geoid undulations and geoid models: a) TG-20 model, b) TG-03 model. The same colour bar is used in both figures. Unit: m.

Table 7 - Comparison of the TG-20 and the TG-03 geoid models with GPS/levelling geoid undulations over seven different profiles shown in Fig. 2a. The bracketed values represent the TG-03 geoid model. Units are in m (N: number of GPS/levelling benchmarks).

Profile name	Statistics (GPS/levelling geoid - Geoid model)				
	N	Mean	Min	Max	Std
Antalya - Burdur - Amasra	176	0.078 (0.010)	0.020 (-0.390)	0.160 (0.180)	0.028 (0.111)
Doğ anbeyli - Kozan	18	-0.078 (-0.004)	-0.110 (-0.050)	-0.050 (0.060)	0.018 (0.029)
Çakırçay - Gerze	17	0.088 (0.047)	0.040 (-0.010)	0.110 (0.120)	0.019 (0.041)
Kırıkkale - Çorum	30	0.004 (0.044)	-0.020 (-0.030)	0.020 (0.090)	0.013 (0.039)
Akhisar - Salihli	11	-0.139 (-0.129)	-0.150 (-0.170)	-0.110 (-0.080)	0.013 (0.022)
Tirebolu - Torul	13	0.047 (-0.024)	-0.080 (-0.210)	0.110 (0.050)	0.063 (0.071)
Karaman - Mut	13	-0.086 (-0.092)	-0.100 (-0.150)	-0.060 (-0.040)	0.012 (0.030)

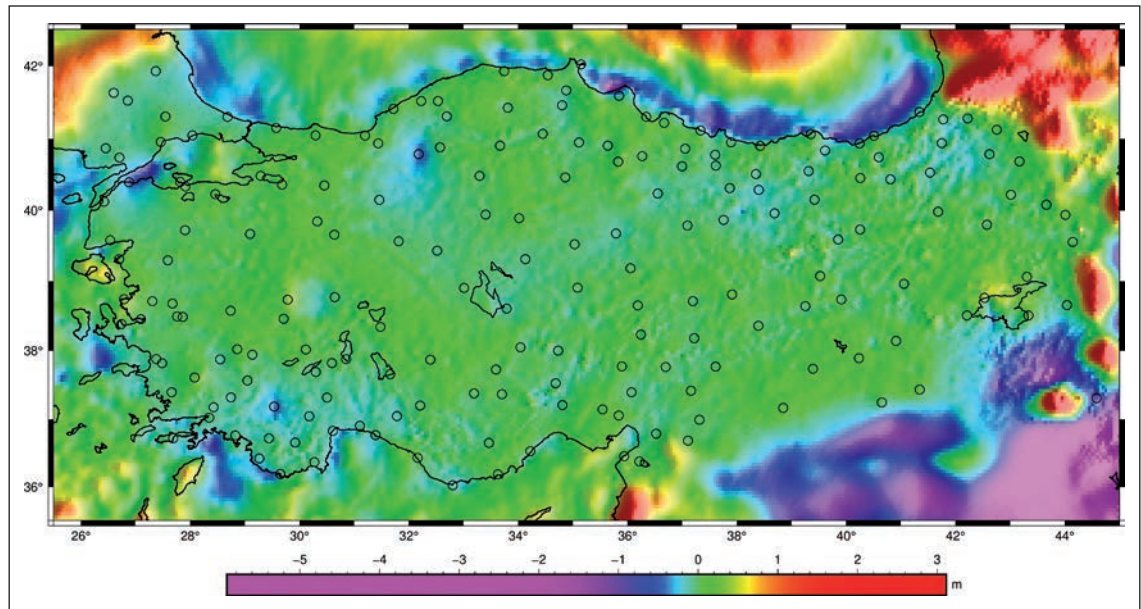


Fig. 7 - Differences obtained subtracting the TG-03 model values from the TG-20 model values at 3'x3' grid nodes. The scatter of 192 historical GPS/levelling benchmarks is shown with black circles. Unit: m.

8. Discussion and conclusions

The new Turkish hybrid geoid model entitled TG-20 has been computed within the Turkish Height System Modernization and Gravity Recovery (2015-2020) project. Different from the previous model, TG-20 incorporates state-of-the-art GGM, DEM, and recently collected terrestrial gravity and GPS/levelling data as well as the quality-controlled historical gravity and GPS/levelling data.

The use of TR-GravNet data successfully detects a 14 mGal bias in the historical gravity data in a 55×55 km² region located between the latitudes 38.5° to 39.0° N and the longitudes 30.0° to 30.5° E which could be attributed to the gravity datum inconsistency.

Recently measured GPS/levelling data validates TG-20 with standard deviations ranging between 1.2 and 6.3 cm, which outperforms the previous geoid model. Except for Tirebolu-Torul profile, the accuracy of the TG-20 geoid model is less than 3 cm demonstrating the project objective of 1-3 cm geoid model is achieved. The largest differences between TG-20 and GPS/levelling geoid heights are observed particularly in coastal areas. Thus, a further improvement can be expected in the accuracy of the TG-20 model when the coastal areas are properly measured by airborne gravimetry that we plan in the coming years.

Comparison of the TG-03 geoid model and TG-20 reveals significant errors in the TG-03 reaching up to -82 cm caused by the gross errors in some of the historical GPS/levelling geoid heights used in the combination of the TG-03 model. Because the historical GPS and levelling data were measured at quite different periods, some of the benchmarks were probably prone to vertical deformations due to effects such as groundwater variations during the time until the GPS measurements were performed. Consequently, this study also highlights the benefit of simultaneous measurements of levelling and GPS to precisely validate and demonstrate the accuracy of a national geoid model to the surveying community.

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