Geodetic measurements in the Himalayas and new determination of the height of Mount K2

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Abstract - In July-August 1996 GPS measurements along the profile from Skardu to Askole and to the K2 Base Camp were performed. Along the route gravity and deflections of the vertical were measured. In the K2 Base Camp Area, across the Godwin Austen glacier a base geodetic network was established with points on bedrock and on the moving glacier surface. A tripod with a sight target and two reflecting prisms was installed at the top of K2 in order to measure angles and distances from the points of the base network to the summit. Meteorological observations were performed to determine the atmospheric refraction coefficient. Taking into account the movements of the glacier it was possible to link the summit of Mt. K2 to the trigonometric network of Pakistan and to the IGS network improving the accuracy of the height of the mountain compared with the previous measurements.

1. The Karakorum Mountains

When in 1816 the Prussian geographer Alexander von Humboldt reported in the famous magazine "Annales de Chemie et de Physique" the results of the first measurements of the Himalayan chain, performed by British topographers, he already asked if behind those mountains there might be other mountain chains with even higher summits. At that time it was believed that the highest peak was that of Mt. Dhawlagiri, but in order to obtain reliable numerical values of the height of those mountains reference baselines were still missing (i.e. points in the valley where the height is already known, from which further measurements can be made). Being hundreds of km from the sea the baseline elevations were measured with heavy and fragile barometers, subject to meteorological variations.

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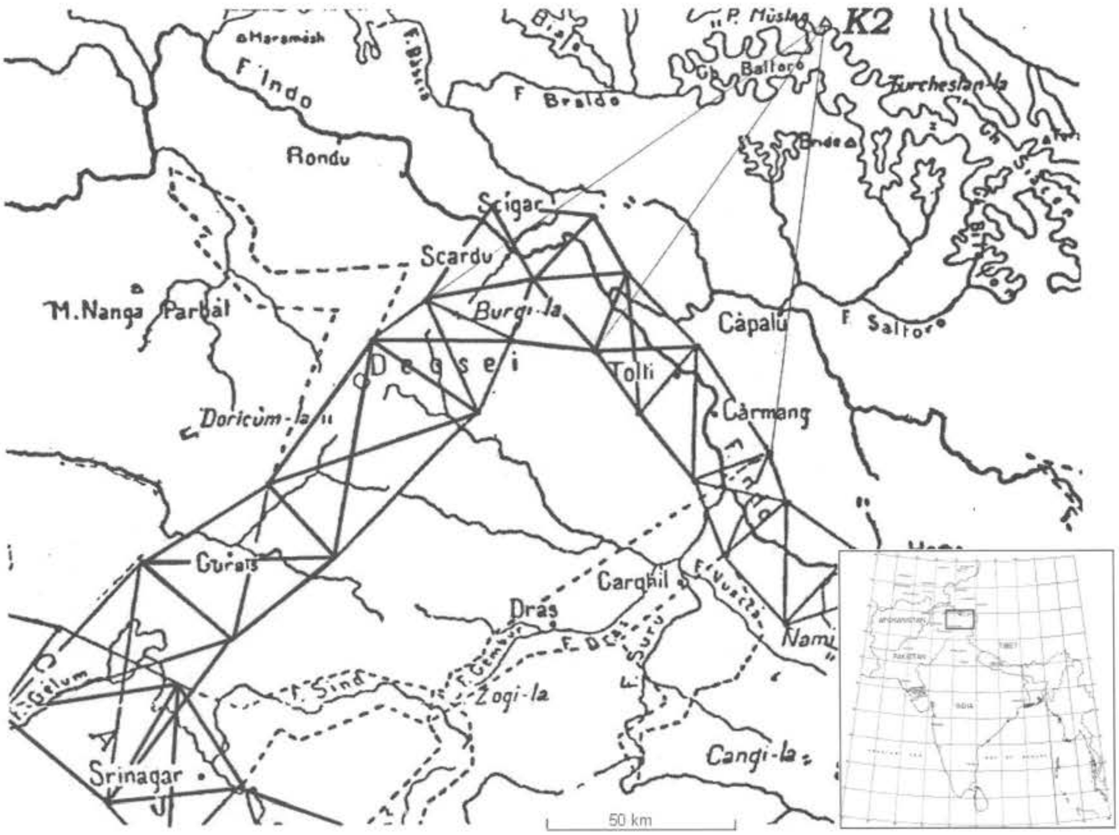

Fig. 1 - The British trigonometric network near K2 established in 1859.

Trigonometric baselines were only established 20 years later when William Lambton and George Everest arrived at the foothills of the Himalayas with the measurements of the Great Trigonometric Arc of India. At that time Kanchenjunga and Peak XV (later called Mount Everest) were pointed out, but the researchers of the Survey of India, the Indian agency for cartography and topography founded by Lambton, continued their measurements westwards where the chain bends forming an arch and assumes the name of Karakorum.

The British topographers, from Srinagar pointed north-eastwards through the Nanga Parbat massive and after crossing the Deosai Plains went down into the valley of Skardu, at the junction between the Indus river and the Shigar which comes down from the large glaciers that extend on one side towards the Hunza valley and on the other to the Karakorum pass (Fig. 1).

2. The height of K2

During the winters of the years 1857-59 angular measurements were performed for several unknown mountains of the Karakorum range from 9 points located at a height of 5000 metres and at distances of between 95 and 221 km. The peaks were identified with the symbol K (for Karakorum) followed by a number: K1, K2, K3, … The highest one, labelled K2, turned out to reach 8611 metres and took the second place among the highest mountains of the world.

During the processing of the data, the curvature of the earth was taken into account, but a rather empirical refraction coefficient was used. For the measurements from the valley to the peaks, at first a constant value was assumed that was then modified in order to minimise the range of variation of the measurement from every station. For K2, Colonel Montgomerie chose values of between 0.04 and 0.05 (Burrard and Hayden,1933).

TABLE XII.—Height of K^2.

Station of observation.	Year of observation.	Height of station of observation.	Distance from K^2.	Values of height, if no correction for refraction be applied.	Resulting height as determined by Montgomerie with co-efficients of refraction varying from 0·04 to 0·05.
		Feet.	Miles.	Feet.	Feet.
Shangruti	1859	17531	78·9	28640	28246·6
Bīāchūthūsa	1859	16746	99·0	28846	28218·7
Marshāla	1858	16906	58·6	28472	28240·0
Kāstor	1858	15983	66·0	28560	28261·4
Thurigo	1858	17246	61·8	28515	28254·1
Harāmukh	1856	16001	136·5	29300	28293·9
Kanūri-Nār	1857	15437	114·3	28920	28218·4
Bārwāi	1857	16304	88·0	28666	28258·5
Thalanka	1857	16830	74·7	28613	28322·7
Mean	..	..	..	..	28253
Range of variation in values *	..	..	..	828	104

Fig. 2 - The first measurements of K2 (Burrard & Hayden 1933).

Further studies on the refraction coefficient were carried out 40 years later by Colonel S. G. Burrard and Dr. De Graaf Hunter who in 1931 also computed the heights of Everest, Kanchenjunga and Dhawlagiri with respect to the geoid, the physically defined reference surface of the Earth.

However, none of these studies were carried out for K2, the height of which remained over the years that calculated by Col. Montgomerie i. e. 8611.36 metres with an estimated error of 10.2 metres (see Fig. 2).

3. Subsequent measurement

After the initial measurements of the Himalayan peaks carried out by the Survey of India, in the middle of the 19th Century, as mentioned above, other measurements followed for Mt. Everest (1904, 1954, 1975, 1992), Kanchenjunga and Dhawlagiri, but no other determination was performed for K2 until 1986 when George Wallerstein and then Alessandro Caporali, with

Professor Ardito Desio, remeasured the two highest mountains of the Earth with the purpose of comparing their heights.

On these occasions, Global Positioning System (GPS) receivers and laser distance meters (EDM) for the measurement of the base lines were employed for the first time. The theodolites still pointed at the snow covering of the summit of the mountains for the angular data.

The measurement by Caporali was performed from Concordia at a distance of about 15 km where the ellipsoidal height was measured as a Single Point Position (SPP) of a Leica WM101 single frequency GPS receiver. The summit turned out to be 8579 metres above the WGS84 ellipsoid. The geoid-ellipsoid separation deduced from the global geoid (Caporali et al., 1990) was of -37 metres and, therefore, the geoidal height of K2 turned out to be 8616 metres (Desio, 1988).

4. New topographic instruments

With respect to the past, a remarkable improvement in topographic measurement methods was made during the '80's with the introduction of laser distance meters along the base lines. But considerably better results could only be obtained with the installation of instruments on the summit. This implied the collaboration of a mountaineering expedition and climbers skilled also on the technical side and capable of transporting, installing and operating scientific instruments.

The refinement of GPS technology produced lighter, more precise and reliable instruments and consequently in September 1992 a Leica System 200 GPS receiver recorded on the summit of Mount Everest for one hour (Poretti et al., 1994).

The installation of a tripod with two triplets of reflecting prisms and a red silk optical target permitted the measurement of the distance from the summit with millimetre accuracy, by means of the Leica ME5000 and DI3000 distance meters. As far as the angular measurements are concerned the Leica T3000 and T2002 electronic theodolites were employed.

The launch of sounding balloons and the installation of a meteorological station at the South Col (8000 m), transmitting constantly to the Base Camp, permitted a calculation of the refraction coefficients with the highest accuracy, by means of the values of temperature, pressure and humidity recorded along the atmosphere between the station in the valley and the summit.

The new height of Mount Everest: 8846.10 metres was calculated averaging over the values obtained by satellite and terrestrial instruments both on the Nepali and Tibetan side.

5. The triangulation to the summit of K2

In July-August 1996 GPS measurements along the profile from Skardu to Askole and to the K2 Base Camp were performed. The base network for the measurement of K2 was formed by four points in view of the summit, indicated by the letters C, E, G and F in Figs. 3 and 4.

Fig. 3 - Bird's eye view of the K2 Base Camp area.

Two of the points (C and E) were materialised on the bedrock, at the sides of the Godwin Austen glacier, while the other two (G and F) were located on the glacier's moraine. The central point (G) was linked to a point (SK) located at the "K2 Motel" in Skardu at a distance of about 98 km. Point F was used only as a temporary support because C and E were not in sight of each other.

Point SK was later linked to a fundamental trigonometric point of the triangulation network of Pakistan (also point of the Survey of India) on the rocky hill above the Fort of Skardu and indicated as TR (see Fig. 6). This last point was also linked to Askole, a village located half way between Skardu and K2 Base Camp.

6. Linking the trigonometric point of the Survey of India to the IGS network

The coordinates of the trigonometric point (TR) were provided by the Geological Survey of Pakistan. They are referred to the Everest ellipsoid and were measured for the first time by the Survey of India between 1857-63 (see also Dainelli, 1922). The most recent measurements refer the geoidal elevations to the Karachi mareograph.

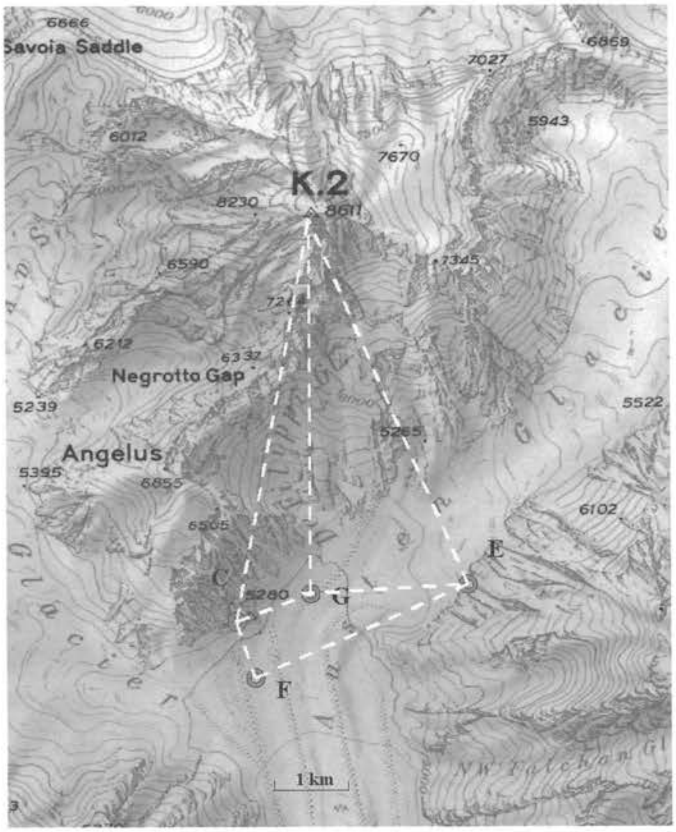

Fig. 4 - The triangulation network.

Table 1 - The coordinates of the trigonomeric point (TR) above Skardu.

Ellipsoid	Everest	WGS84
Latitude	35° 18' 40".11 N	35° 18' 38".11 N
Longitude	75° 37' 40".63 E	75° 37' 36".40 E
Geoidal Height	2698.35 m	
Ellips. Height		2671.40 m

They were transformed into the WGS84 ellipsoid and the geoid-ellipsoid separation N = -26.95m was computed from the EGM96 model giving an ellipsoidal height of 2671.40 metres.

Table 2 - The basic reference points linked by GPS observations.

GPS Point	Kitab	Poligan	Skardu K2Motel	Skardu TR
Latitude	39° 08' 05".16 N	42° 40' 47".17 N	35° 17' 45".81 N	35°18' 37".83 N
Longitude	66° 53' 07".59 E	74° 41' 39".34 E	75° 38' 52".05 E	75°37' 37".31 E
Ellips. Height	622.51 m	1714.19 m	2218.00 m	2669.16 m

New coordinates for the TR point were computed with reference to IGS (International GPS Service) stations Kit3 located in Uzbekistan, at a distance of 885 km and Pol2 located at Poligan in Kyrghyzstan at a distance of 824 km.

The difference of elevation at the trigonometric point between the two determinations is of the order of 2.24 m.

The relative position between the TR and the K2 Motel points was measured with a 2-hour GPS session that showed a height difference of 451.01 m.

Table 3 - Geoidal and ellipsoidal elevations (in metres).

Point	H ellipsoid	N	H geoid
Skardu TR Pt.	2669.16	-26.95	2696.11
Skardu K2 Motel	2218.00	-26.88	2244.88
K2 Base Camp	4929.75	-22.51	4952.26

A 30-hour GPS session linked the K2 Motel to the point G at Base Camp and a height difference of 2711.76 metres was calculated between them.

7. The movement of the Godwin Austen Glacier

Point G was located on the glacier in the Base Camp area and was therefore mobile with respect to C, E and SK. This fact was also pointed out by the GPS measurements. Dividing the recording time (30 continuous hours between July 24 and 25) into four parts, one can notice that the distance between the two stations slowly decreased.

Processing with precise Ephemeris gave the following results:

Table 4 - Distances from Skardu.

Day	Hour	Distance
24.7.97	15-23	98195.13 m
	23-07	98195.06 m
25.7.97	7-14	98194.96 m
	14-21	98194.92 m

Considering the movement of the glacier as constant during the day and night, a shift of 22.6 cm/day can be seen in the Base Camp-Skardu direction.

The movement of point G and of the whole glacier was controled by triangulation and photogrammetry measurements during the entire period that the researchers were at Base Camp.

The network for the calculation of the movement of the glacier included a baseline AB on the glacier linked to point G and to two points C and D on the bedrock. Seven more points (1-7)

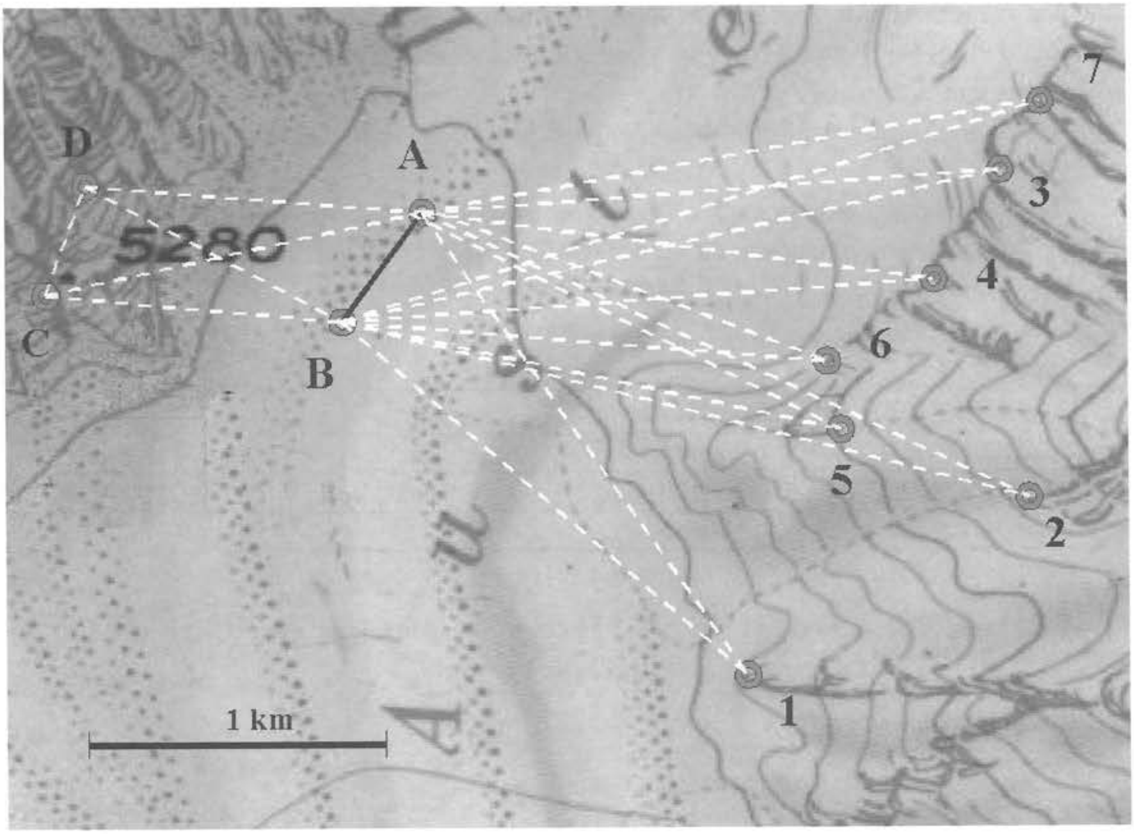

Fig. 5 - The triangulation points for measuring the glaciers displacement.

were located on the left side of the glacier (Fig. 5). The remeasurements were performed at the same time of the day in order to minimise the effects of refraction. Angular measurements were performed with Leica T3000 and T2002 theodolites providing an accuracy of 0.15 mgon, while distances were measured by Leica's DI3000 distance meter with an error of 3 mm ± 1 ppm.

For the data processing the measurements were referred to a local datum.

It is possible to calculate the approximate amount and direction of the glacier's movements by comparing the coordinates from July 28 to August 1 (see Table 5).

Table 5 - Horizontal movement of baseline AB between 28.07 and 1.08.96 (in centimetres).

Point	date	coord. x	coord. y
A	28.07	1000.00	10000.00
A	01.08	1000.17	10001.37
B	28.07	1000.00	10283.70
B	01.08	999.78	10285.02

The differences between the coordinates show alterations in the positions of 1.38 m for point A and 1.34 for point B during the time of observation. This means, supposing a steady movement, a glacier surface velocity of 35 cm/day at A and 33 cm/day at B.

There is another determination of velocity for the period July 25 to 28 which is based on measurements from base AB to the fixed points C and D. The results are shown in Table 6.

Table 6 - Horizontal movement of baseline AB between 25.07 and 28.7.96 (in centimetres).

Point	date	coord. x	coord. y
A	28.07	1000.00	10000.00
A	25.07	999.92	9999.15
B	28.07	1000.00	10283.70
B	25.07	1000.11	10282.95

This means a movement of 85 cm of point A and 76 cm of B. i.e. daily velocities of 28 and 25 cm. The uniform tendency of both results is obvious.

These differences deserve further discussion.

The results provided in Table 5 have been found on the basis of angle measurements from A and B to all 1-7 and C, D points and distance measurements to C and D and reciprocal distance measurements between A and B. The results of Table 6 are only based on measurements of angles and distances to points C and D and reciprocal distance measurements between A and B (Purukherr et al., 1997).

Because of the numerous multiple determinations and better distribution of the target points, the results of Table 5 are statistically more meaningful. The following table supports this conclusion:

Table 7 - Statistics of the results.

Point	date	sigma(x)	sigma(y)	sigma(z)
A	01.08	1.1	1.0	1.5
A	25.07	17.0	16.0	23.3
B	01.08	1.3	1.0	1.6
B	25.07	5.8	14.3	15.4

changes in height of the final points A and B during the survey cannot be proved significantly from the material of the measurements and can be neglected with reference to the K2 summit height determination.

8. The triangulations to the summit

Point G was linked to points C and E by triangulation. Theodolites and distance metres were aimed at the summit from points C, E and G.

Table 8 - Results of the triangulations to the summit (in metres).

	ΔH	**ΔH: G-K2**
G-K2	3659.143	3659.143
E-K2	3598.291	3659.186
C-K2	3634.588	3659.180
G-K2 Average		3659.170

At the time of the observations, temperature, pressure and humidity were recorded at Base Camp. On arrival of the alpinists the temperature at the summit was -25°C degrees and the pressure 357 hP.

These data allowed the calculation of the refraction coefficient of the atmosphere K with a good approximation though this is not as important as it was when the measurements were performed from a distance of 100 km.

K was computed according to the method of Bock (Bock, 1950):

$$K = 5.03 \frac{p}{T^2} (3.42 - \tau) \left[1 - \frac{\tau}{T} \delta H\right]^{\frac{3.42}{\tau} - 2}$$

where p is the pressure (in hP), T is the temperature (in °K) of the theodolite's station point, τ is the mean vertical temperature gradient ($-\delta T / \delta H$ in °C/hm) and δH is the height difference in hm. For the period of observation the refraction coefficient K used in the reduction of the angular data varied between 0.071 and 0.074.

Taking into account that the elevation of point G is of 4931.02 m one can obtain the elevation of the reflecting prisms on the summit (Table 9).

Subtracting the height of the prisms on the snow and the depth of the snow one obtains the value of the elevation of K2 with reference to the rocky surface of the summit.

Table 9 - Geoidal and ellipsoidal elevations of K2 (in metres).

	Geoid	**Ellipsoid**	**Error**
K2 prisms	8611.59	8589.08	
Prisms-snow	2.25	2.25	± 0.03
K2 snow	8610.46	8586.83	
Depth of snow	2.22	2.22	± 0.05
K2 rock (ref. IGS Stations)	**8606.97**	8584.61	± 1.09
K2 rock (ref. Karachi)	**8609.36**	8586.85	± 1.05
K2 average	**8608.17**	8585.73	± 1.25

The error of the measurement is formed by several components:
coordinates of Kitab, Poligan and Skardu TR (0.60), global geoid (0.67), GPS baselines

Kitab-Skardu (0.85) and Skardu-K2 Base Camp (0.12), vertical triangulation (0.025), prisms' height above the snow (0.03) and depth of the snow (0.05), for a total amount of 1.25 m.

9. The single point position of the GPS measurements

Between July 14 and 25 more than 30 hours of GPS recording was performed at the point at the K2 Motel of Skardu.

The processing in "single point position" with the Leica SKI programme showed an elevation of 2211.36 metres i.e. 7.79 m less than the trigonometric official value, providing an ellipsoidal elevation of K2 of 8585.73 metres.

Though this value does not differ much from the 8579 metres observed by Alessandro Caporali in 1987, while remembering that it was an error in the calculation of the single point which in 1986 led to believe that K2 was higher than Everest, it has been preferred to consider this result as a mere indication. The new height was obtained as an average between the values resulting from the official trigonometric elevation at point TR and the one determined by connection with the Kit3 and Pol2 IGS permanent stations.

The processing of the GPS data for the TR-SK-G profile (from the trigonometric point to Base Camp) was performed assuming as initial values those obtained from the processing of the Kit3-Skardu baseline for which more data was available.

The results obtained can be also resumed in the scheme indicating the differences in height between the various points involved in the measurement (Fig. 6).

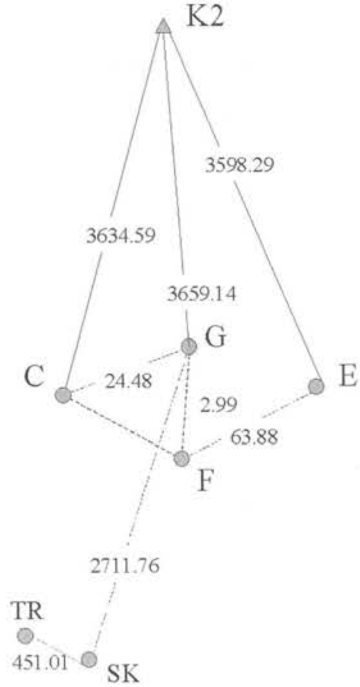

Fig. 6 - The vertical triangulation results.

10. Conclusions

After 170 years the problems involved in the determination of the elevation of the mountains of India remained much the same, though there were some obvious changes such as the elevation of the base stations, the refraction coefficient and the geoid-ellipsoid separation, with the problem of the deflection of the vertical that at that time had not been taken into account.

The amount of error introduced by these unknown factors has been greatly reduced due to the accuracy of modern instruments and to the closeness between the baselines and the summits. The greatest surprise lies in the fact that despite the large amount of the errors involved in the first measurement (Burrard and Heiden, 1908) its numerical value turns out to be very close to the new one.

The remeasurement of Mount K2 induced the proposal for a new value for the geoidal elevation of the mountain as 8608.17 m. For a comparison with the measurements of the past one must take into account the values calculated with respect to the snow surface.

Table 10 - Historical scheme of the measurements of Mount K2 (in metres).

	Rock	Snow	Ellipsoid	Geoid-Ell.
Montgomerie (1859)		8611.		
Caporali-Desio (1987)		8616.	8579.	-37.00
Poretti et al. (1996)	8608.17	8610.46	8587.95	-22.51

The new elevation of K2 is 50 cm lower than the one determined by the Survey of India in 1867 and 5.54 m lower than that calculated by Caporali and Desio in 1987. The closeness of the two geoidal heights is mainly due to the error introduced by the GPS single point position computation. In fact, the difference in results is much more substantial if the elevations on the ellipsoid are compared and is compensated by the different value of the geoid-ellipsoid separation that amounts to almost 15 metres.

The probable error of this measurement of K2 unfortunately turns out to be much higher than that obtained for Everest (Poretti et al., 1994) in 1992.

This is mainly due to the lack of certain data on the error of the base point in Skardu that could be even higher than the internal error of the measurement (estimated in at least 0.56 m), and to the great distance of the nearest IGS permanent station.

In accordance with the method used for Mt. Everest, the height of the mountain was calculated with reference to the rocky basement of the summit.

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