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Statistical analysis of extreme waves on the Italian coasts from 1989 to 1999

R. INGHILESI, S. CORSINI, F. GUIDUCCI and A. ARSENI

Servizio Idrografico e Mareografico Nazionale,
Dipartimento per i Servizi Tecnici Nazionali, Rome, Italy

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Abstract - The statistical analysis in this paper concerns the evaluation of the extreme wave return periods on the Italian coasts and the calculation of the average duration of sea storms based on data collected by Italian marine measurement network Rete Ondametrica Nazionale. Ten years of wave data from 8 buoys were analysed following the standard statistical methods recommended by the Working Group on Extreme Wave Statistics - IAHR. Evaluations are based on the estimate of statistical distributions for the extreme wave episodes. Hypotheses of data independence and stationarity were discussed; independent episodes were extracted from the 10-year time series taking into account the directional characteristics of the Italian wave climate. Statistical models were applied in order to obtain return values of significant wave heights corresponding to return periods ranging from 5 to 50 years. The results were analysed and, in order to test the robustness of the analysis, return values for the 5-year return period were compared with observations. Sensitivity to the total number of selected episodes was explored and possible effects resulting from missing storms shown and discussed. An estimate of the mean wave duration was made, based on the evaluation of the statistical model for the probability of exceeding a threshold. A simple power-law formula was found to summarise the results.

1. The RON network

The Rete Ondametrica Nazionale (RON, National Sea Wave Measurement Network), run by the Servizio Idrografico e Mareografico Nazionale (SIMN, National Hydrological and Marine Survey), has been in operation since July 1989. Until the end of 1998 the network was

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Corresponding author: R. Inghilesi; Servizio Idrografico e Mareografico Nazionale, Via Curtatone 3, 00100 Roma, Italy; phone: +39 06 4442825; e-mail: roberto.inghilesi@dstn.it

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made up of eight directional pitch-roll wave buoys (Earle, 1985) located offshore at La Spezia, Alghero, Ortona, Ponza, Monopoli, Crotone, Catania and Mazara del Vallo (Fig. 1). The wave buoys were, and still are, equipped with the ARGOS satellite positioning system to check the buoys positions and, if necessary, quickly rescue the station after an unmooring event. The elaboration of the measurements is published quarterly by the SIMN in the Quarterly Wave Measurement Bulletin (*Bollettino Ondametrico Trimestrale*), and supplied directly to public institutions and companies with particular attention to specific needs. RON data are widely used in maritime structure design, climate studies, wave-forecast analyses and validation. Currently the available RON data (Corsini et al., 1998) are three-hourly time observations of significant wave height (H_{m0}), peak period (*TP*), mean period (*TM*), wave direction (*Dir*) and sea surface temperature (*SST*). In 1999 two new translatory wave buoys, installed offshore Ancona and Cetraro entered the RON, and the national data retrieval centre in Rome was improved by means of a real time data acquisition system. The RON will be made of 14, all real time, new buoys connected to the centre before August 2002.

Fig. 1 - The RON network.

The efficiency of the system can be evaluated by the following formula:

$$\eta = \frac{n_{obs}}{T_{obs}} = 1 - \frac{m_{obs}}{T_{obs}}$$

where η is the efficiency of the station between 1/7/89 and 30/6/99; n_{obs} is the number of three-hourly observations collected during the period; m_{obs} is the number of missing data; T_{obs} is the number of expected data in the ten year period. As Table 1 shows, the network's efficiency is greater than 90% for all the stations with the exception of Catania and Mazara del Vallo. Transmission problems, radio interference, sensor breakage, maintenance works and occasional unmoorings of the buoys are the main reasons for the lack of data.

Table 1 - Number of H_{m0} missing data and efficiency of the RON stations between 1989 and 1999.

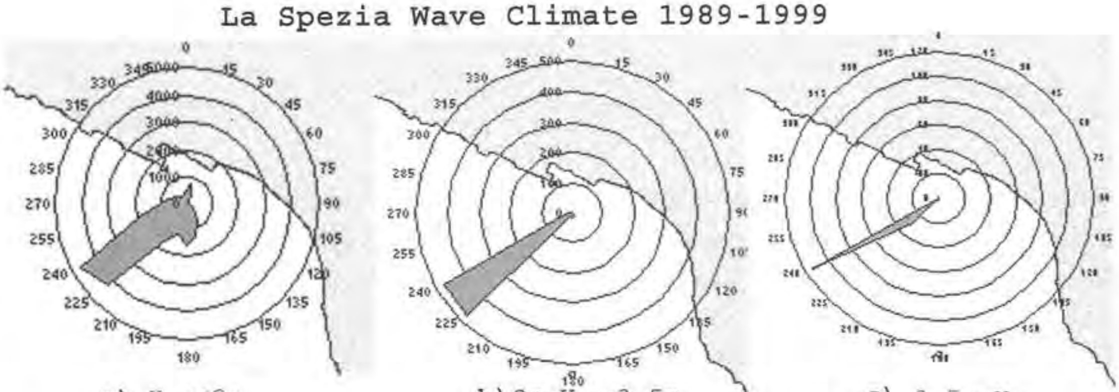
Location	m_{obs}	m_{obs} (%)
La Spezia	1744	94.0
Alghero	1155	96.0
Ortona	1577	94.6
Monopoli	1816	93.8
Ponza	2771	90.5
Crotone	1970	93.3
Catania	2984	89.8
Mazara	4315	85.2

2. Wave climate

The wave climate at the various buoy locations is evaluated by the National Hydrological and Marine Survey by means of tables reporting the number of measured data grouped into classes depending on H_{m0} and the incoming direction. The distributions obtained, divided by the number of valid measures, represent the joint frequency function of H_{m0} and *Dir*. The directional distributions of the observations, expressed in polar co-ordinates, are shown in figures 2-9 for all buoys. Three classes of H_{m0} were considered:

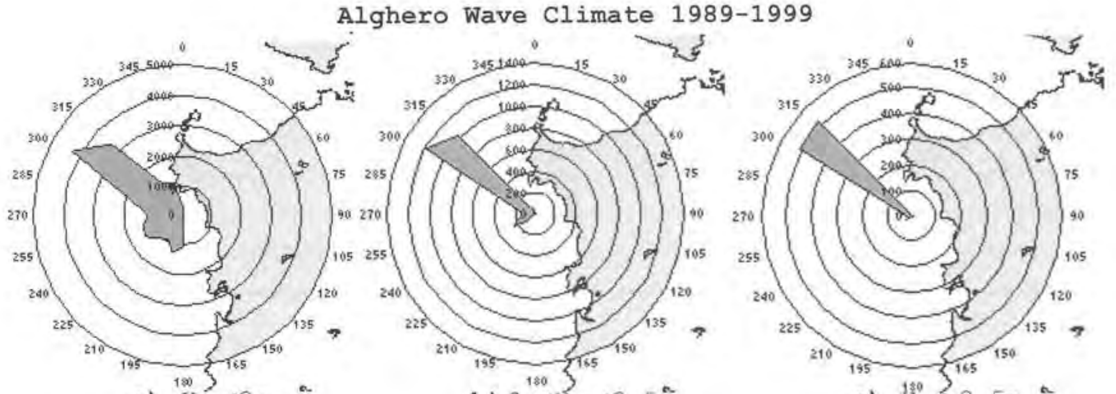
1st class (a) - small waves $0.5 \text{ m} < H_{m0} < 2.0 \text{ m}$;
2nd class (b) - moderate to high waves $2.0 \text{ m} < H_{m0} < 3.5 \text{ m}$
3rd class (c) - very high waves $3.5 \text{ m} < H_{m0}$

The highest waves are typical of the Tyrrhenian Sea, the most severe episodes are generally recorded west of Sardinia, in the Tyrrhenian Basin and on the Southern coasts of Sicily. Significant storms are observed in the Adriatic Sea, but here the storm activity is weaker and generally restricted to the winter period. Unimodal distributions of H_{m0} and wave direction are found in the very high wave range at La Spezia, Alghero, Ortona and Catania (Figs. 2a, 3a, 4a, 8a); bimodal distributions characterise the very high wave range of the other buoys. Moderate and high wave regimes have a bimodal distribution at Alghero (Fig. 3b), Ortona (Fig. 4b) and



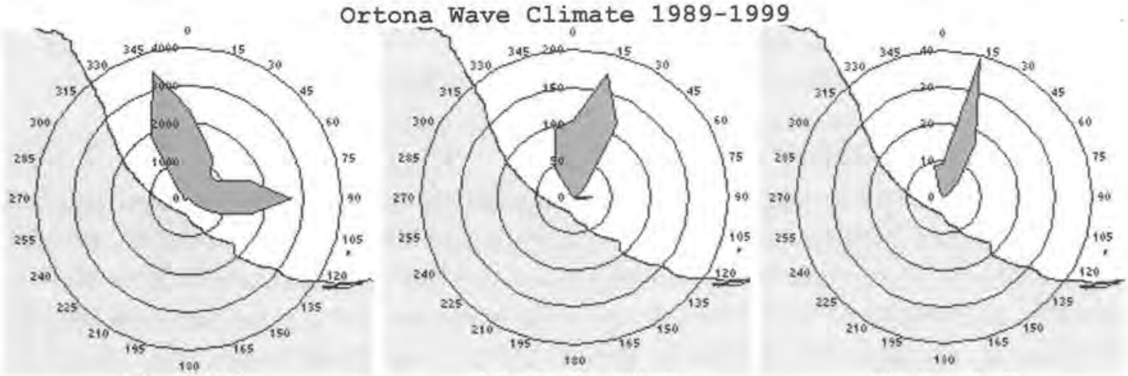
a) $H_{m0}<2m$ **b)** $2m<H_{m0}<3.5m$ **c)** $3.5m<H_{m0}$

Fig. 2 - Directional distribution of the 1989-1999 H_{m0} observations at the La Spezia Buoy.



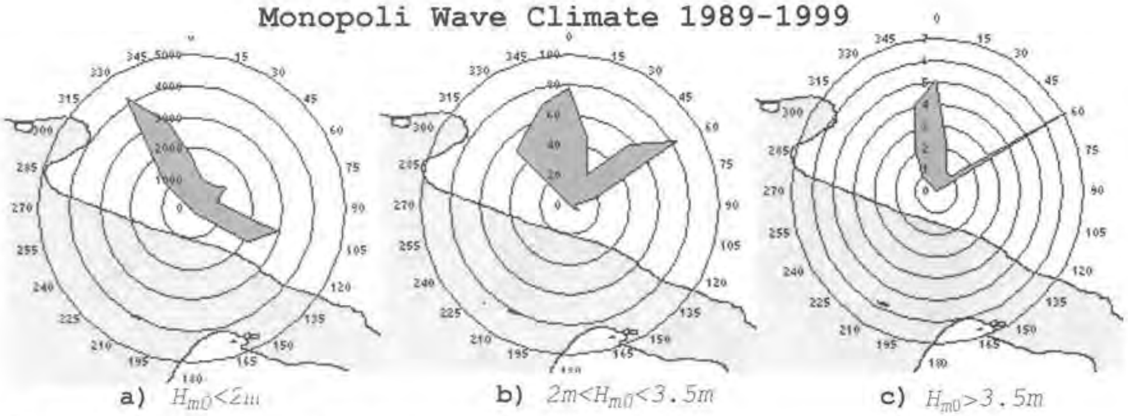
a) $H_{m0}<2m$ **b)** $2m<H_{m0}<3.5m$ **c)** $H_{m0}>3.5m$

Fig. 3 - Same as Fig.2 for the Alghero Buoy.



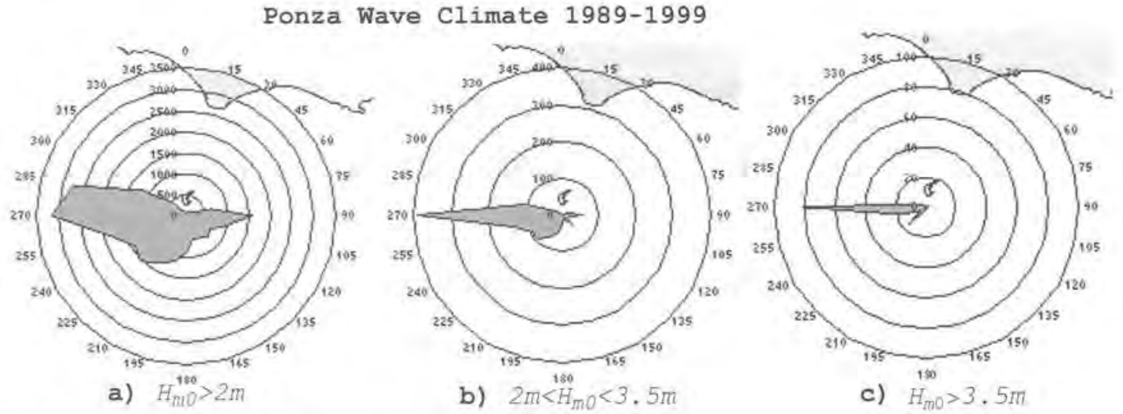
a) $Hm0>2m$ **b)** $2m<H_{m0}<3.5m$ **c)** $H_{m0}>3.5m$

Fig. 4 - Same as Fig.2 for the Ortona Buoy.



a) $H_{m0}<2m$ b) $2m<H_{m0}<3.5m$ c) $H_{m0}>3.5m$

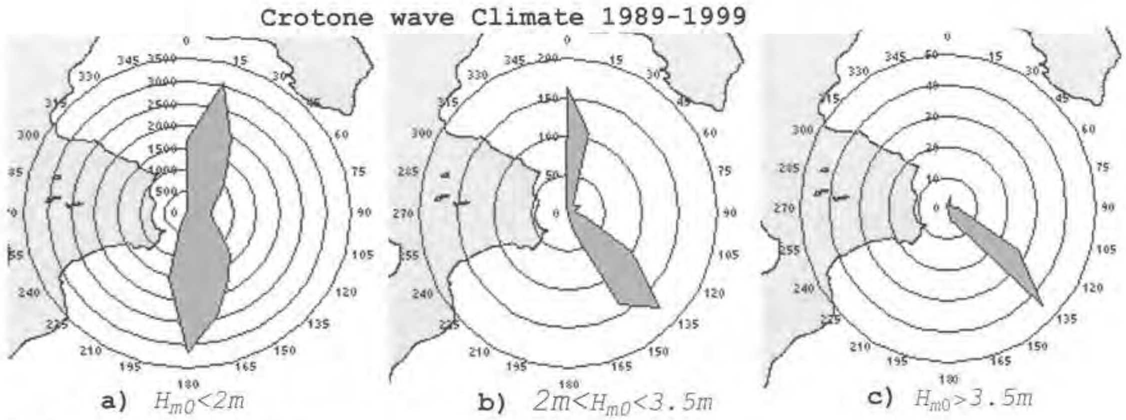
Fig. 5 - Same as Fig.2 for the Monopoli Buoy.



a) $H_{m0}>2m$ b) $2m<H_{m0}<3.5m$ c) $H_{m0}>3.5m$

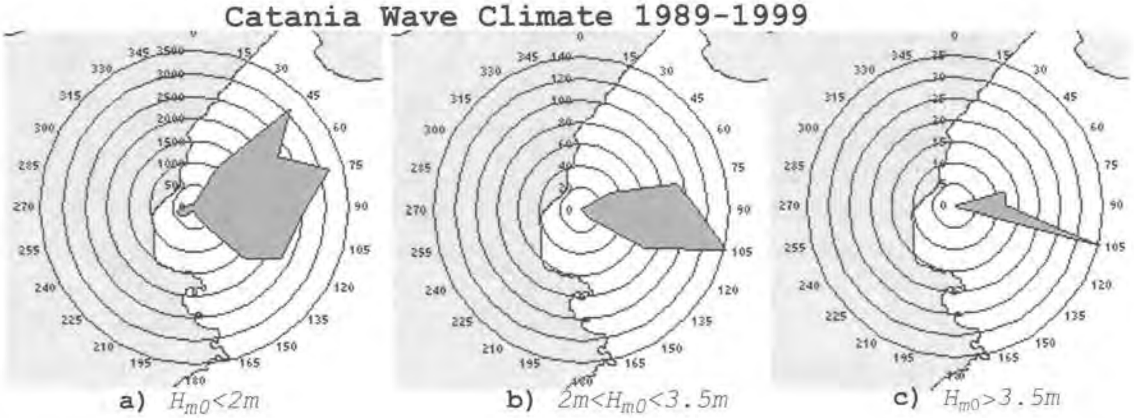
Fig. 6 - Same as Fig.2 for the Ponza Buoy.

Monopoli (Fig. 5b); a complex three-modal distribution is found at Ponza (Fig. 6b). The characteristics of these distributions are related to the synoptic scale meteorological climate. As an example, two different types of meteorological conditions contribute separately to the wave climate at Crotone (Fig. 7). The first contributors are the northern winds coming from the Balkans over the Adriatic Sea, and generating northerly waves over the Taranto Gulf. The second are due to low-pressure areas (moving from West to East), in the lower Mediterranean Sea and are responsible for south - eastward waves in the Ionian Sea. Conversely, the weak bimodal distribution in the high wave regime found at Alghero (Fig. 3) is related to a single phenomenon, the Mistral. When this wind occurs, often the north-westerly winds strengthen initially, then the storm may turn slightly anticlockwise and generate westerly waves. The resulting bimodal H_{m0} distribution will not be related to the separate occurrence of two different meteorological regimes. Events, that may be characterised by wave directions differing by up to 60 degrees, will not prove to be independent on a time scale of several days. The detailed investigation about the relationship between typical meteorological conditions and observed wave climate are far beyond the aim of the present work, but this issue, as viewed from a physical perspective, is being considered by SIMN by means of extensive numerical hindcasting



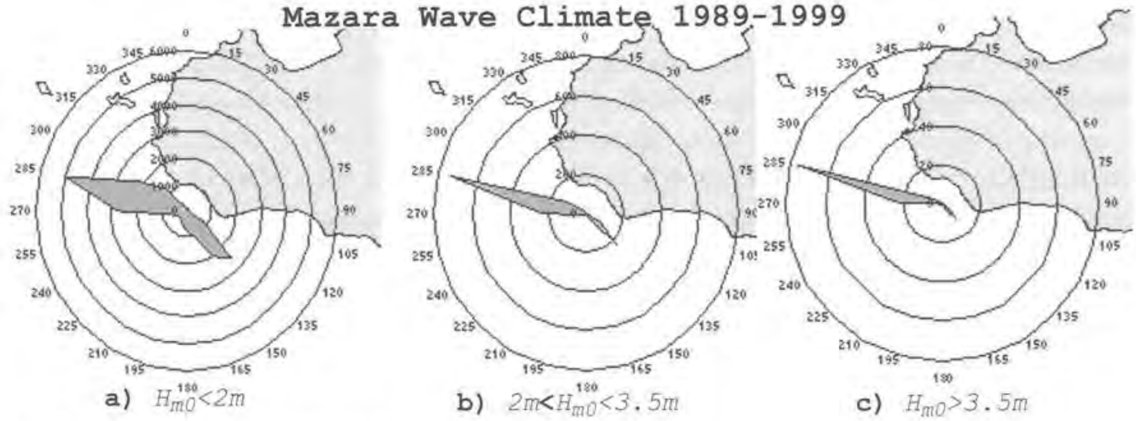
a) $H_{m0}<2m$ **b)** $2m<H_{m0}<3.5m$ **c)** $H_{m0}>3.5m$

Fig. 7 - Same as Fig.2 for the Crotone Buoy.



a) $H_{m0}<2m$ **b)** $2m<H_{m0}<3.5m$ **c)** $H_{m0}>3.5m$

Fig. 8 - Same as Fig.2 for the Catania Buoy.



a) $H_{m0}<2m$ **b)** $2m<H_{m0}<3.5m$ **c)** $H_{m0}>3.5m$

Fig. 9 - Same as Fig.2 for the Mazara Buoy.

techniques, including the use of very high resolution meteorological limited area and wave models.

3. Statistical analysis of extreme events

The statistical analysis of extreme waves was carried out using the Peak Over Threshold Method (POT), as described by Mathiesen (1994), Goda (1998) and Tomasicchio (1999), and following the indications of the IAHR Working Group on Extreme Wave Statistics (1990).

In a nutshell, the standard procedure followed is:

1 - selection of the extreme waves from the measured data and sorting;

2 - estimation of the distribution which best fits the extreme wave data in each sample;

3 - evaluation of the extreme values which corresponds to prefixed return periods from the distribution.

3.1. Directional analyses for extreme wave events

This method has been adopted in Italy by many Authors since 1990. The difference between one study and another arises mainly in the selection of the samples on which the standard procedure is applied. The first comprehensive study by Archetti and Franco (1995) was made on a set of RON observations up to 30.6.1995. In this study, the complete data set of measurements was divided into 12 sub-samples based on the direction of the waves. The sub-sample was defined as the ensemble of observations falling into one of the 12 equally spaced directional sectors, the first one starting from 0°N (sector I: all the observations from 0° to 30°, II sector: observations from 31° to 60°, and so on). The more recent study, carried out by Arena and Barbaro (1998), was based on a wider data set, taking into account the RON observations up to the 31st of July 1997. In this work 16 directional sectors (20° wide) were defined in a similar manner. This approach will be referred to in the present study as an a priori defined directional sectors framework. A problem with the a priori defined directional sectors is that more often than not the sub-set of data on which the statistics are applied is not homogeneous. In fact, even a simple set of over-threshold observations mainly characterised by straight north westerly waves (this is the case of observations collected at Alghero or La Spezia) would be artificially divided into two or more sub-samples, depending on the fixed directional framework (12, 16 or more sectors) introduced. Another problem is how to obtain a set of independent data from the 3-hourly observations. In the application of the POT method, attention is paid to extract only the observation of the maximum significant wave height for each distinct storm. Unfortunately it is not often easy to decide whether a series of waves, for instance seven or more days long, is made up of one or more different storm episodes. In the present study, to preserve the homogeneity of the data, directional sectors are defined with respect to the observed statistical distribution of the observations. To assure the independence of data a selection method based on the auto-correlation properties of the time series is presented in the following.

WAVE DIRECTIONAL SECTORS. - To select the independent events from each buoy data set, a sector, aligned according to the direction of the maximum measured value H_{m0}, was considered. The width of the sector, determining the subset of data to be processed, influences successive analyses. Therefore, in order to have a representative number of extreme events, this sector was defined wide enough to include both the direction related to the maximum observed H_{m0} and the direction corresponding to the most populated class of events (Fig. 10). Although the two directions often almost coincide, a more comprehensive analysis of the meteorological climate of the sector was then carried out. In particular, meteorological aspects were analysed in order to avoid not only splitting up a whole class of events of the same nature, but also to avoid merging different kinds of events into the same sector. The width of the sector cannot be easily determined when bimodal distributions show close (in terms of direction) peaks, as can be seen in the polar diagrams (Figs. 2, 4, 8, 9) referring to Alghero, Ponza, Pescara and Monopoli. In such conditions the sector width represents a critical parameter which must be subjected to further meteorological and climatic analyses. The difficulty in establishing the boundary between first, second and third sectors in a multimodal distribution can influence the reliability of the relative analyses. This is the reason why, in the following, separate results will be presented for principal (P) and secondary (S) sectors, taking into account the fact that the latter are often not uniquely definite as are the former.

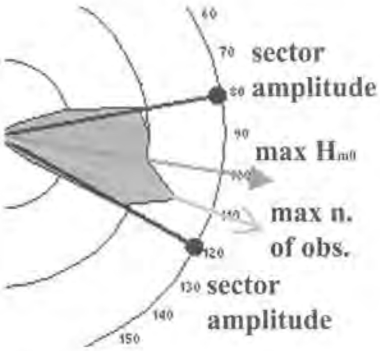


Fig. 10 - Representation of a directional sector based on wave climate.

SELECTION OF INDEPENDENT EPISODES. - For the analyses of the wave data to be statistically significant, the extreme wave events had to be selected in order to assure their mutual statistical independence. The procedure followed can be described in two steps:

1. build a time series of maxima of H_{m0} on such a time scale that the evaluation of maxima is statistically meaningful;
2. determine the independent extreme events from the time series of the maxima.

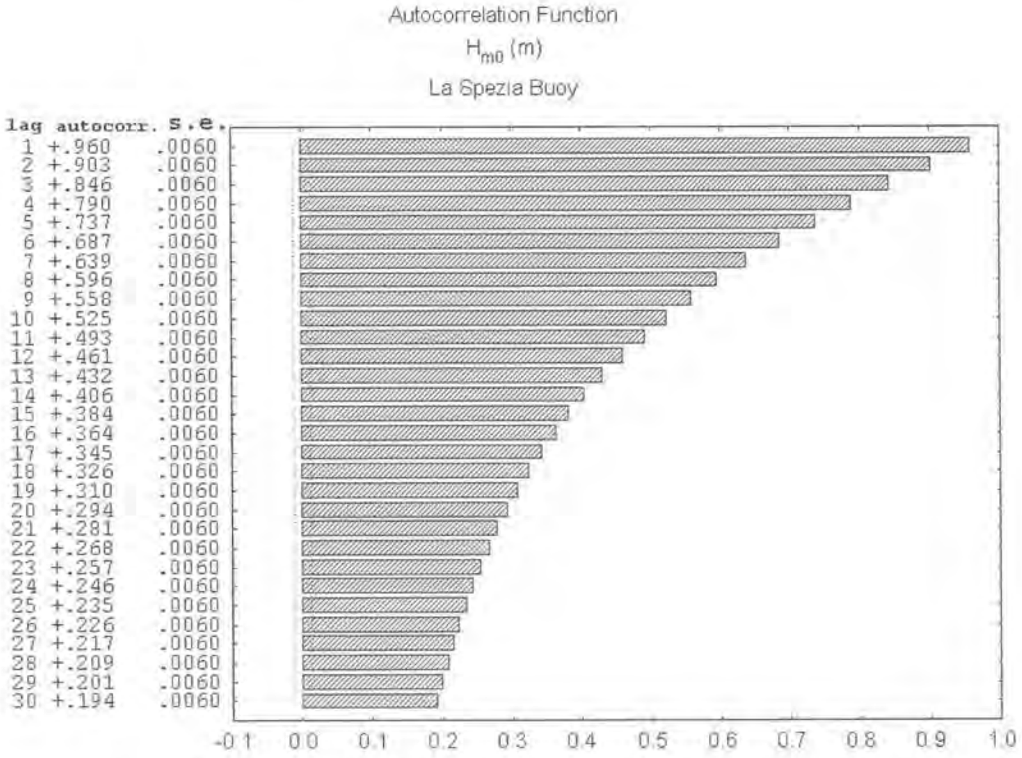
First step:

In order to analyse the influence between events at two subsequent instants of time, the autocorrelation function, defined as

$$c_{xx}(k) = \frac{1}{N}\sum_{t=1}^{N-k}(x_t - \bar{x})\,(x_{t+k} - \bar{x})$$

(Jenkins et al., 1969), was evaluated. In the above formula x_t is the observation collected at time t; N is the total number of observations; $k = 1, N\text{-}1$ is the gap between the observations, $\bar{x} = \frac{1}{N}\sum_{t=1}^{N} x_t$ is the arithmetic mean of the sample. The autocorrelation function $c_{HH}(k)$ was evaluated over the complete 10-year time series of H_{m0} for all buoys, the selected gaps ranging from $k = 1$ to $k = 30$ (i.e. from 3 hours to approx. 4 days). As an example, results for La Spezia are reported in Fig. 11, where the lag, the autocorrelation function value C_{HH}, and the standard deviation are shown. For all buoys the value of $c_{HH}(k)$ corresponding to $k = 16$ ($\Delta t = 48$ h) was found in the range 0.4-0.3, which is a value small enough for two successive events to be considered weakly correlated. Consequently, under the assumption that the 48 h-maximum value over the threshold h* is a statistically consistent estimate of the maximum, a temporal window of 48 h was used for the selection of the maximum values of H_{m0}. Besides, it can be shown that the evaluation of the total number of independent sea storms depends on the amplitude of the temporal window used for the evaluation of the maximum. When the window is increased from 3 h to 48 h the number of independent storms decreases and the average storm duration increases. But, as shown for the La Spezia buoy in Fig. 12 shows, when the length of the window exceeds the value $k = 15$ (45 h, value at which c_{HH} falls below 0.4) the number of storms gradually tends towards a stable value (near $N = 160$ in this example).

Second step: Independent episodes were selected from the time series of 48 h-maxima based on the following assumptions: two or more subsequent maxima over the threshold H* belong to



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lag autocorr. s.e.
 1 +.960   .0060
 2 +.903   .0060
 3 +.846   .0060
 4 +.790   .0060
 5 +.737   .0060
 6 +.687   .0060
 7 +.639   .0060
 8 +.596   .0060
 9 +.558   .0060
10 +.525   .0060
11 +.493   .0060
12 +.461   .0060
13 +.432   .0060
14 +.406   .0060
15 +.384   .0060
16 +.364   .0060
17 +.345   .0060
18 +.326   .0060
19 +.310   .0060
20 +.294   .0060
21 +.281   .0060
22 +.268   .0060
23 +.257   .0060
24 +.246   .0060
25 +.235   .0060
26 +.226   .0060
27 +.217   .0060
28 +.209   .0060
29 +.201   .0060
30 +.194   .0060
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Fig. 11 - Autocorrelation Function c_{HH} of H_{m0} between 01.07.1989-30.06.1999.

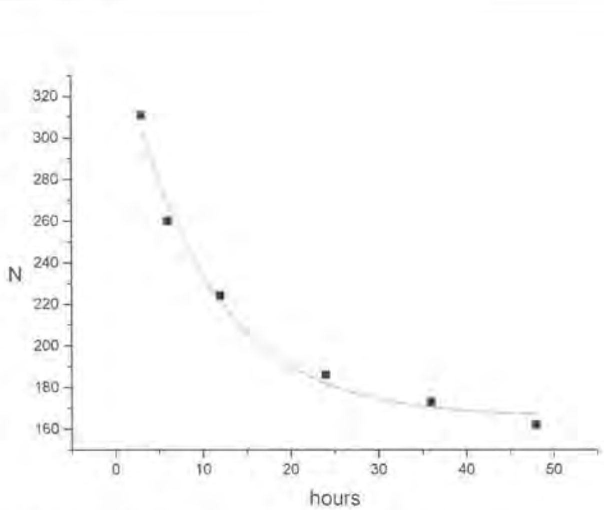


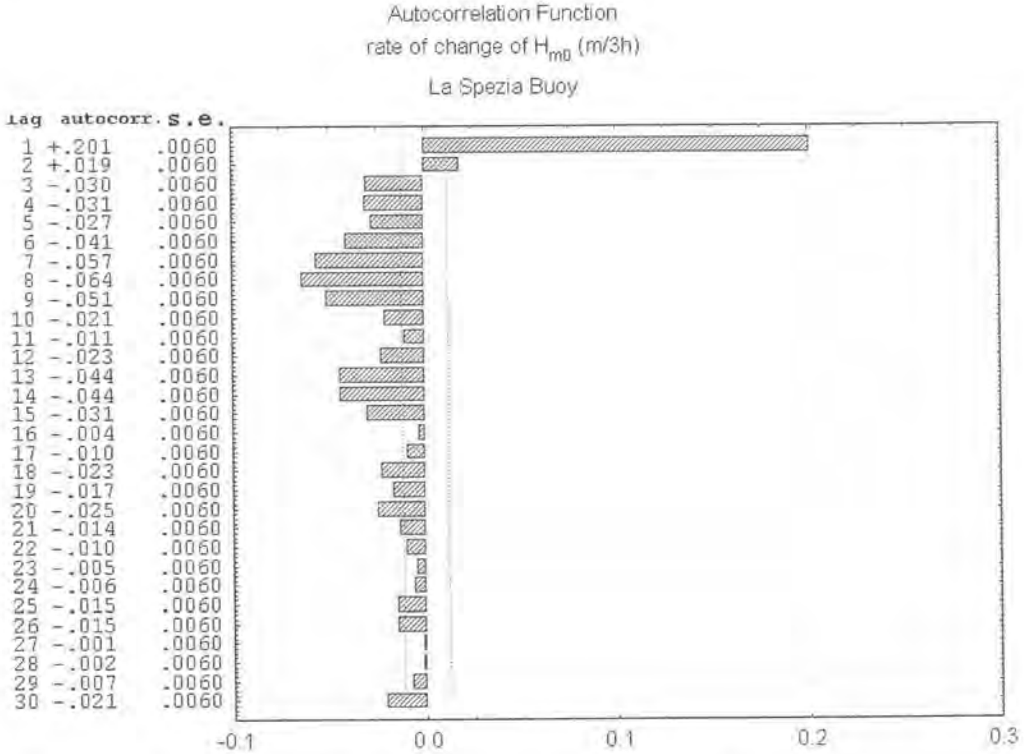
Fig. 12 - Number of estimated independent sea storms vs. the time gap at La Spezia.

the same storm; the storm is represented by the absolute maximum among them. The threshold value for the analyses of the 48 h-maxima was taken to be H* = H_{m0} = 2.0 m for all buoys. The same threshold value H* was considered in the analysis of the wave directions in the selection of the directional sectors. A second threshold value H** was assigned in order to select *n* cases out of the *N* independent 48 h maxima available. The number *n* is representative of the average number of significant storms in one year. Due to the intense storm activity during the year, and considering that the summer calm wave period in Italy is generally short, *n* was assumed to be as large as 50. Values of H** for all the sectors considered are shown in Table 2. The autocorrelation analyses carried out on the time series of the rate of change of H_{m0} were also evaluated. It

Table 2 - Directional sectors and threshold values H*- H**.

Buoy	**Dir(°N)**	**H*(m)**	**H** (m)**	**Sector Code**
La Spezia	240°	2.0	3.5	P1
Alghero	300	2.0	5.3	P2
Ortona	40	2.0	2.5	P3
Monopoli	60	2.0	2.2	P4
Ponza	280°	2.0	3.3	P5
Crotone	140	2.0	2.7	P6
Catania	100	2.0	2.3	P7
Mazara	290	2.0	3.4	P8
Alghero	70	2.0	2.8	S1
Ortona	280	2.0	2.0	S2
Monopoli	360	2.0	2.0	S3
Ponza	50	2.0	2.5	S4
Ponza	280	2.0	2.0	S5
Crotone	200	2.0	2.3	S6
Mazara	320	2.0	2.4	S7

was found that even in the 3-hour lag the series of the differences are not significantly correlated. An example of the autocorrelation function for the La Spezia time series is shown in Fig. 13.



lag	autocorr.	s.e.
1	+.201	.0060
2	+.019	.0060
3	-.030	.0060
4	-.031	.0060
5	-.027	.0060
6	-.041	.0060
7	-.057	.0060
8	-.064	.0060
9	-.051	.0060
10	-.021	.0060
11	-.011	.0060
12	-.023	.0060
13	-.044	.0060
14	-.044	.0060
15	-.031	.0060
16	-.004	.0060
17	-.010	.0060
18	-.023	.0060
19	-.017	.0060
20	-.025	.0060
21	-.014	.0060
22	-.010	.0060
23	-.005	.0060
24	-.006	.0060
25	-.015	.0060
26	-.015	.0060
27	-.001	.0060
28	-.002	.0060
29	-.007	.0060
30	-.021	.0060

Fig. 13 - Autocorrelation Function of the rate of change of H_{m0} between 01.07.1989-30.06.1999.

3.2. P(H) estimate

The *N* available independent over-threshold H* episodes were sorted into order to find the best fit of the data by means of a parametric probability distribution function (pdf). A rank index n_j was then associated to the ordered array (n_i ranging from 1 to *N*). Only the n events which were found to exceed the second threshold H** were taken into account in the following analysis; a tentative plotting position function (PPF) was associated to all the selected probability distributions. The cumulative probability distributions adopted in this work were the Gumbel distribution, $P(H) = e^{-e^{-\left(\frac{H-B}{A}\right)}}$ (where *A* is the location parameter and *B* the scale parameter); and the 3-parameter Weibull distribution $P(H) = e^{-\left(\frac{H-a}{b}\right)^{\gamma}}$ (where *a* is the location variable, *b* the scale variable and γ is the shape parameter). Here, and in the following, the distributions will be referred to as Gumbel (*A*,*B*) and Weibull (a,b,γ), respectively. To determine the set of parameters which gives the best fit of the distribution, the Least Square Method (Goda, 1988) was adopted. The method was used on 5 groups of distributions, i.e. Gumbel (*A*,*B*) and Weibull

Table 3 - Results from linear regression to data by sectors.

Sector	*r**2*	*s*	*A*	*sa*	***B***	*s b*	*g*
P1	0.98	0.09	2.32	0.04	1.15	0.02	1.4
P2	0.99	0.06	1.21	0.04	3.24	0.03	2.0
P3	0.99	0.07	0.85	0.04	2.13	0.03	2.0
P4	0.99	0.07	1.90	0.02	0.68	0.01	1.0
P5	0.97	0.12	0.86	0.08	2.31	0.06	2.0
P6	0.98	0.10	1.25	0.05	2.11	0.04	2.0
P7	0.99	0.10	0.84	0.04	2.23	0.04	2.0
P8	0.97	0.12	0.65	0.07	2.35	0.05	2.0
S1	0.99	0.06	1.07	0.04	2.0	0.03	2.0
S2	0.94	0.95	1.63	0.05	0.88	0.05	2.0
S3	0.98	0.09	0.88	0.04	1.69	0.04	2.0
S4	0.98	0.10	0.74	0.05	2.09	0.04	2.0
S5	0.92	0.11	1.78	0.06	0.96	0.03	1.4
S6	0.98	0.07	1.67	0.02	0.91	0.02	1.4
S7	0.99	0.07	1.89	0.02	1.06	0.01	1.4

(a,b,γ_i) with 4 possible values of i . The best fit from the 5 groups of parametrised distributions was evaluated on the basis of the total variance explained by the model. The distributions applied are the Gumbel (A,B) and the four Weibull (a,b,γ_i) (with: $\gamma_1 = 0.75$, $\gamma_2 = 1.0$, $\gamma_3 = 1.4$, $\gamma_4 = 2.0$). Results of the regressions are summarised in Table 3. For each sector in the table we are show: the fraction of explained variance (or the square of the correlation parameter, r^2), the standard deviation (SD) of the estimate of the regression; the values of the regression coefficients a,b with their variances; the shape factor of the Weibull distribution. It can be observed that the SD of the estimate σ is greater where the buoy exhibits a lower efficiency. In other words, the estimate of the best fit distribution is significantly influenced by the number of

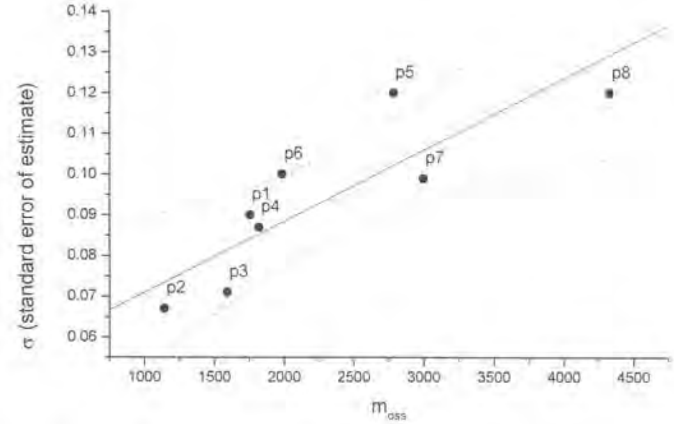


Fig. 14 - Estimate SD vs. total number of missing observations by principal sectors.

missing observations in the period. The scatter diagram of SD against the total number of missing observations is shown in Fig. 14. In this figure, both the regression line and the confidence bands with a 95% level of significance are shown. The two sectors that stray from the confidence bands are p5 (Ponza buoy) and p6 (Crotone buoy). For these two sectors the number of missing observations is likely to influence the estimates more than in the others sectors. The strong or weak dependence on the number of missing data is, in all probability, due to whether there is a significant number of extreme events in the missing data (for example, missing observations due to buoy unmoorings or to radio communication problems during extreme events). In the analyses carried out on the secondary sectors (Fig. 15) it can be observed that the high percentage of missing data found at the Mazara del Vallo station (s7 sector) did not result in a fit to data worse than in other stations. On the contrary, the Ortona sector (s2) shows a higher sensitivity to the number of missing data. When a sector is sensitive to the number of missing observations, to consider of all expected cases would result in a different shape of the distribution. Consequently, a substantial underestimation of the extreme values corresponding to long term return periods would ocens. On the other hand, in the case of a robust distribution, appropriate corrections could in principle be used in the estimation of $H_{m0}(T)$ in order to take into account missing time periods of data.

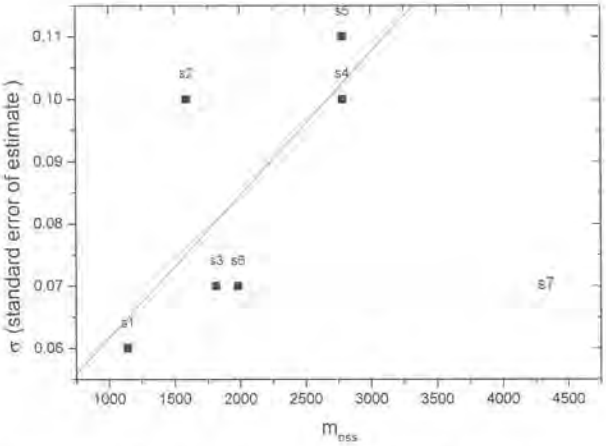


Fig. 15 - Estimate SD vs. total number of missing observations by secondary sectors.

3. 3. Evaluation of return values for extreme events

According to the definition (Tomasicchio, 1998) the return period T is the average value, expressed in years, of the number of observations which lie between two subsequent events with magnitude equal to or greater than a given threshold. The return value of the significant wave height, $H_{m0}(T)$, is the value of H_{m0} which corresponds to the return period T. From this definition, the following expressions are derived:

$$P(H) = 1 - \frac{1}{\bar{N} \cdot T}, \qquad (1)$$

$$\bar{N} = \frac{N}{\Delta T} \tag{2}$$

where T is the return period in years; N the total number of considered events; ΔT the considered time period in years and $\bar{N}$ the average number of events per year. The $P(H)$ in (7) is the three-parameter Weibull distribution.

By substituting the analytical expression for $P(H)$ in (1), the equations for the evaluation of $H_{m0}(T)$ are:

$$H_{m0}(T) = a + b\left\{\ln\left[\bar{N}T\right]\right\}^{\frac{1}{\gamma}} \tag{3}$$

The standard deviation σ associated to significant wave height was evaluated on the basis of the expressions (Tomasicchio, 1998):

$$\sigma = \sigma_r \, \sigma_H \tag{4}$$

$$\sigma_r = \sqrt{\frac{\left[1 + \alpha\left(\{\ln[\bar{N}T]\}^{\frac{1}{\gamma}} - c + \varepsilon \ln \nu\right)^2\right]}{N}} \tag{5}$$

$$\alpha = \alpha_1 \, e^{\alpha_2 N^{-1.3} + k\sqrt{-\ln \nu}} \tag{6}$$

with α_1, k, c, ε, as parameters which depend on the choice of γ, indicated in Table 4.

Table 4 - Values of the parameters for the evaluation of the standard deviation s of $H_{m0}(T)$.

g	a1	k	C	e
0.75	1.65	-0.63	0.0	1.15
1.0	1.92	0.0	0.3	0.90
1.4	2.05	0.69	0.4	0.72
2.0	2.24	1.34	0.5	0.54

Other parameters are: $\alpha_2 = 11.4$, $\nu = n/\mathrm{N}$ (censor parameter) and σ_H which is the standard deviation of H_{m0} in the wave sector. Return values $H_{m0}(T)$ corresponding to return periods $T = 50$, 25, 10, 5 years were obtained by using Eqs. (3) and (4) with the a, b, γ previously derived. Results relevant to the distributions obtained are shown in Table 5. In order to assess the reliability of the estimates, the second maximum (the value with rank 2, if data are sorted in descending order) measured in each wave sector was compared with the estimate of the corresponding $H_{m0}(T = 5$ years). The choice of $T = 5$ years follows from the definition of the return period. In fact, at least two values of H_{m0} equal to or greater than the given threshold level are necessary in order to define an interval between two events. Taking into account only the absolute maximum observed in the considered period of time (rank 1) would not be accurate, as

Table 5 - Return values $H_{m0}(T)$ in m corresponding to return periods of 50, 25, 10 and 5 years by sectors.

Sector (T)	H_{m0} **(50)**	σ **(50)**	H_{m0} **(25)**	σ **(25)**	H_{m0} **(10)**	σ **(10)**	H_{m0} **(5)**	σ **(5)**
P1	6.79	0.82	6.46	0.73	6.00	0.61	5.63	0.52
P2	9.9	0.68	9.47	0.61	8.87	0.52	8.38	0.44
P3	6.07	0.61	5.76	0.56	5.32	0.49	4.95	0.43
P4	5.72	1.07	5.25	0.93	4.62	0.75	4.15	0.61
P5	6.81	0.67	6.49	0.61	6.04	0.53	5.67	0.46
P6	6.43	0.61	6.12	0.56	5.68	0.49	5.32	0.43
P7	6.24	0.58	5.91	0.53	5.44	0.47	5.04	0.41
P8	6.79	0.68	6.47	0.62	6.02	0.53	5.65	0.46
S1	6.11	0.65	5.83	0.59	5.43	0.52	5.10	0.45
S2	3.55	0.61	3.4	0.56	3.19	0.48	3.01	0.42
S3	5.04	0.61	4.79	0.56	4.44	0.49	4.15	0.43
S4	5.92	0.63	5.62	0.58	5.19	0.5	4.84	0.44
S5	5.11	0.73	4.82	0.65	3.31	0.55	3.31	0.48
S6	4.95	0.77	4.68	0.69	4.3	0.59	3.99	0.50
S7	5.75	0.78	5.43	0.7	4.99	0.59	4.64	0.51

the single maximum could be associated in principle with an unknown (but larger than the length of the time series) return period. The comparison of the observed data with $H_{m0}(T = 5$ years) is shown in Fig. 16 for the principal sectors and in Fig. 17 for the secondary ones. Results agree quite well, even though it can be observed that they are not completely independent as the observed values were also used in the estimation of the $P(H)$.

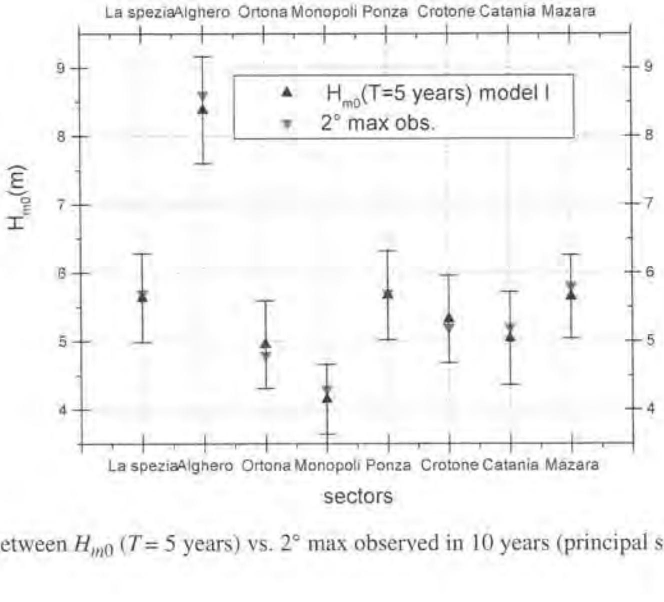


Fig. 16 - Comparison between H_{m0} ($T = 5$ years) vs. 2° max observed in 10 years (principal sectors).

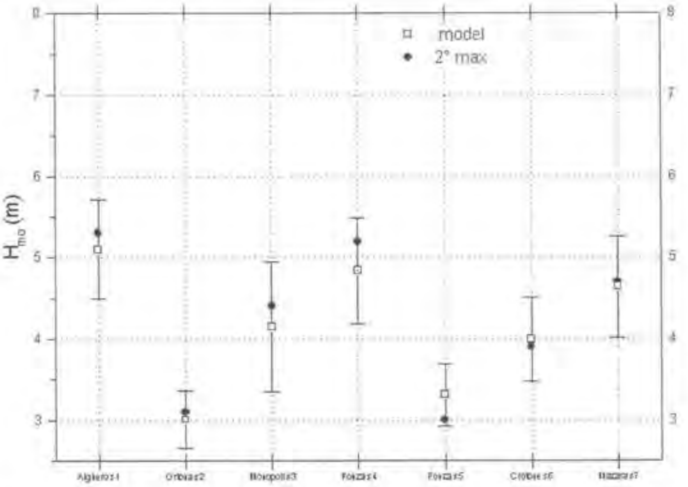


Fig. 17 - Comparison between H_{m0} ($T = 5$ years) vs. 2° max observed in 10 years (secondary sectors).

COMPARISON OF ESTIMATES OF RETURN VALUES FOR $T = 50$ YEARS. - Estimates of return values for $T = 50$ years were compared with the results obtained by other authors, i.e. the previously mentioned studies of Archetti and Franco (1995) and Arena and Barbaro (1998). It must be noted that most of the differences found arise in the selection criteria adopted and in the total amount of data considered rather than in the methodology of analysis. As different definitions

Table 6 - Average duration of exceedance of a threshold H in hours by principal sectors.

Hs' (m)	P1 (h)	P2 (h)	P3 (h)	P4 (h)	P5 (h)	P6 (h)	P7 (h)	P8 (h)
2.5	98	240	70	45	84	110	110	88
3.0	48	140	47	32	55	65	70	59
3.5	34	98	34	25	39	44	50	43
4.0	26	71	26	21	29	31	38	32
4.5	21	54	21	17	23	24	30	25
5.0	18	43	17	15	18	19	24	21
5.5	15	35	14	13	15	15	20	17
6.0	13	29	12	12	13	12	17	14
6.5	12	25	10	11	11	10	14	12
7.0	11	21	9	9.6	9.3	8.9	12	11
7.5	9.6	18	7.9	8.8	8.1	7.6	11	9.2
8.0	8.7	16	7	8.1	7.2	6.6	9.6	8.1
8.5	8	14	6.3	7.5	6.4	5.8	8.6	7.2
9.0	7.4	13	5.6	7	5.7	5.2	7.7	6.5
9.5	6.8	11	5.1	6.5	5.1	4.6	6.9	5.8
10.0	6.4	10	4.7	6.1	4.6	4.2	6.3	5.3

of the sub-samples (directional sectors) were used, statistics are hardly comparable except for the sectors which are more or less coincident. Nevertheless, where possible, something could be inferred by the comparison of the results. First of all, it was seen that both the return values H_{m0} ($T = 50$ years) of Archetti and Arena were found higher than the estimates presented here. Differences ranged from 10-12% to 50%, the greatest, were at Ortona, Alghero, Monopoli and Ponza. In particular, for Alghero p2 (the highest waves recorded in the Italian seas are observed in this sector) Arena gives an estimate which is approximately 2.0 m greater that the value ($H_{m0}(T = 50$ years$) = 9.8$ m) reported here. The main reason for these differences (at least for Alghero, La Spezia, and Catania, for which the statistical distribution of the observations with respect to the wave direction is unimodal) is largely due to the present definition of the directional sectors with respect to the local wave climate. As only the sectors aligned with the *P* and *S* defined here have been taken into account, the smaller the directional sectors, the higher the return value estimates obtained. It can also be noted that, for fixed, a priori determined sectors, the sectors adjacent to the sharp sector (more or less) aligned with the *P* or *S* directions give high estimates due to the tails of the data distribution. For Ponza, Monopoli, and Ortona, the effect of the non-homogeneity is even more evident, due to the complexity of the local wave climate.

SENSITIVITY TO MISSING SEA-STORM DATA. - The accuracy and the degree of reliability of the statistics are directly related to the completeness of the analysed time series. That's not often the case when dealing with a network of instruments mostly placed in severe climatic conditions. Many authors (Earle and Bishop, 1985; Mathiesen, 1994; Tomasicchio, 1998; for example) introduce empirical methods to overcome some lack of data. Two easy and straightforward ways to overcome missing storms are the direct substitution of the data taken from neighbouring buoys (when possible) and the empirical correction of the expression of ΔT in Eq. 2). The latter method assumes a substantial robustness of the estimated $P(H)$, while the former can be applied only when the measurement network gives spatially well-correlated data. Even though RON was not originally designed to give globally redundant information (each station was taken to be representative of a particular wave climate), stations in the Tyrrhenian Sea and Adriatic Sea often record data relative to the same storms as seen from different locations. Unfortunately this does not always occur for the buoys in the Ionian Sea and off the coasts of Sicily. Moreover, the morphology of Italian coasts and the local climate are limiting factors to be taken into consideration when comparing data from neighbouring stations. Nevertheless, as a very rough approximation, it is possible to infer sea-storms' missing data values from the series of observations recorded at the nearby locations showing high cross-correlation values. The analyses, repeated after the addition of 'external' storm episodes, did not provide significant changes in the results previously shown. Where a significant number of episodes could be added (La Spezia p1 and Crotone p6, Ponza s4 and Mazara s7, for which at least 5 events were introduced), this can be seen as an indicator of the robustness of the estimates of the $P(H)$. Unfortunately, the method failed to provide reliable estimates of missing storms in the Middle and Southern Tyrrhenian Sea, where the statistics were known to be less robust. Other methods have been in use at the present time, in particular Neural Networks methods (Puca, 2001), which will hopefully provide some reliable refinements for the current results in the near future.

Sensitivity to the mean annual number of significant sea-storms (n). - The height of the second threshold level H** is commonly related to the annual average of meteorological storms in the area (Galeati, 1997). However, it is quite difficult to get an estimate of this number independently of the sea-storm H_{m0} records. In fact, the counting of the storms should be based on several meteorological parameters in areas eventually far from the buoy locations. The underestimation of n generally leads to the underestimation of $H_{m0}(T)$. On the other hand, the sensitivity of the estimates to the overestimation of n is very small. This is because $H_{m0}(T)$, found by means of the least square method, is little sensitive to the addition of episodes that are smaller in magnitude than all the others considered (Corsini et al., 1999). All the analyses were repeated after having raised the second threshold level, so as to fall to $n = 40$, which corresponds to a mean frequency of occurrence of meteorologically significant events of approx. 1 out of 9 days. Estimates of return values were not found to differ significantly from those with $n = 50$.

4. Average duration

A theoretically founded parametric model (Mathiesen, 1993) was adopted for the estimate of the average duration of sea states over a given threshold. The empirical method assumes that the characteristics of the distribution of H_{m0} and of the average rate of change of H_{m0} determine the mean duration of exceedance of the threshold. The hypothesis behind the application of the model, as seen in the previous analyses, is the stationarity of the time series. The procedures previously exposed have been adopted to ensure the validity of the assumptions. As seen in Fig.13, the autocorrelation function of the rate of change of H_{m0} falls immediately below the value for which data are considered as weakly dependent. Therefore, all the observed data in the time series were taken into account. The expression for the average duration of exceedance of a threshold H is

$$\tau(H) = \frac{2P(H)}{f(H)\,S(H)} \tag{7}$$

where $P(H)$ is the three parameter Weibull distribution. $f(H)$ is the three-parameter probability density function, i.e.

$$f(H) = \frac{\gamma}{b}\left(\frac{(H-a)}{b}\right)^{\gamma-1} e^{-\left(\frac{(H-a)}{b}\right)^{\gamma}}. \tag{8}$$

$S(H)$ is the rate of change of H_{m0}, known empirically to be proportional to the r power of H:

$$S(H) = qH^{r} \tag{9}$$

q and r are determined from the time series, according to the expression for the rate of change of H:

$$S_i\,(H_i) = \frac{\left[\dfrac{H\,((i{+}1)\Delta t)}{\Delta t} - \dfrac{H(i\Delta t)}{\Delta t}\right]}{\Delta t} \tag{10}$$

and, finally,

$$H_i = \frac{H\,((i+1)\Delta t) + H\,(i\Delta t)}{2} \tag{11}$$

where i varies from 1 to the number of data available in the 3-h time series. The average absolute rate of change was calculated grouping the $S_i(H_i)$ data into intervals of 0.2 m and evaluating the expression for $S(H)$. The result of the estimate for Alghero is shown as an example in Fig. 18.

Results of the duration analysis are summarised in Table 16 for main sectors and in Table 17 for secondary sectors. Fig. 19 and Fig. 20 show that, when the threshold levels for H_{mo} are conveniently made non dimensional by the use of a length scale H_o, and the time duration is made non dimensional by means of a convenient time scale $\tau^* = 1$ hour, the following relation holds for all sectors:

$$\tau_* = H_*^{-\beta}, \tag{12}$$

where

Table 7 - Average duration of exceedance of a threshold *H* in hours by secondary sectors.

Hs' (m)	S1	S2	S3	S4	S5	S6	S7
2.0	100	30	43	70	29	41	58
2.5	62	16	38	46	22	32	40
3.0	43	11	34	32	18	26	30
3.5	31	7.6	31	24	15	22	24
4.0	24	5.8	28	19	13	20	20
4.5	19	4.5	26	15	11	18	17
5.0	15	3.7	25	12	10	16	14
5.5	13	3	23	10	9.4	15	13
6.0	11	2.6	22	8.6	8.7	13	11
6.5	9.2	2.2	21	7.4	8	13	10
7.0	8	1.9	20	6.4	7.5	12	9.1
8.0	7	1.7	19	5.6	7	11	8.3

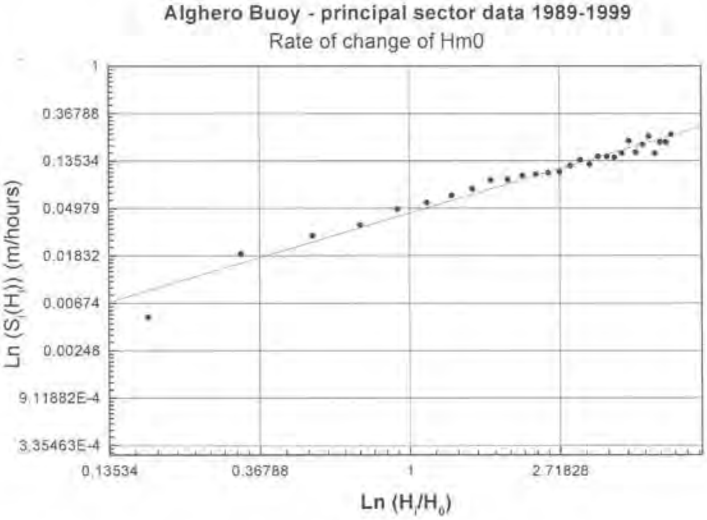


Fig. 18 - Estimate of the average absolute rate of change of H_{m0} for the Alghero Buoy.

$$H_* = \frac{H_{m0}}{H_0}, \tag{13}$$

and

$$H_0 = e^{\frac{\alpha}{\beta}}. \tag{14}$$

with β, α parameters depending on the sector considered.

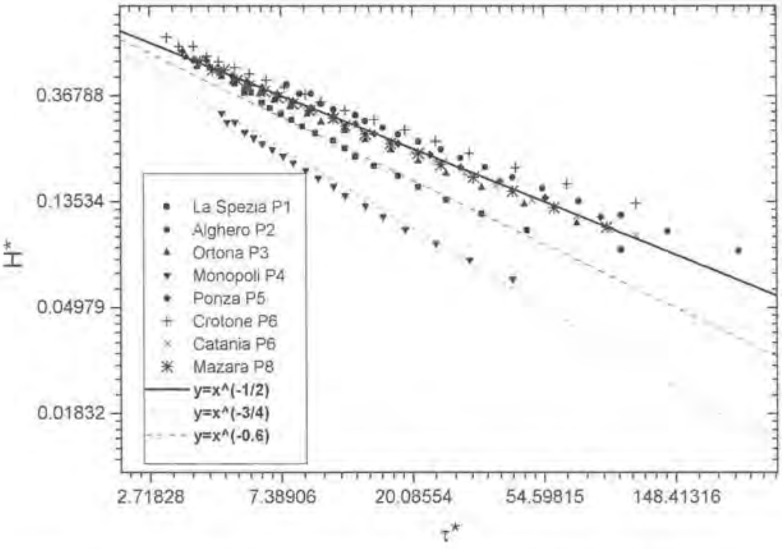


Fig. 19 - A dimensional average duration of exceedance of a threshold H^* by principal sectors.

Values for β and α for each sector were determined by means of power-law regressions of duration against H_{m0}; results are shown in Table 8. The exponent β was found to be close to a value of 2 on the average, being in the range 1.51 (La Spezia) - 2.31 (Alghero). Taking into consideration that secondary sectors are weaker and have fewer episodes, analysis of these sectors provided similar results, in particular the value of β was found to be close to 2 for the secondary sectors at Alghero s1, Ortona s2, Ponza s4 and Mazara s7. Lower values were found at Crotone s6, Ponza s5, and Monopoli s3. The α parameter was seen to be dependent to the absolute maximum of H_{m0} observed at the buoy (i.e., by definition, to the maximum in the principal sector) for all principal and secondary sectors considered. It must be finally observed that even if different power laws seem to apply in different sectors, a general power law seen as

$$\tau_* = H_*^{-1/2} \tag{15}$$

cannot be excluded, as, due to the dependence of the length scale H_0 on β, small uncertainties in the evaluation of β can be responsible for the separation of the lines in Figs. 19 and 20

Table 8 - Estimates of α and β by directional sectors.

Sector	β	α
P1	-1.51	4.53
P2	-2.31	10.07
P3	-2.06	7.85
P4	-1.98	7.24
P5	-2.12	8.33
P6	-2.21	9.12
P7	-1.93	6.89
P8	-2.06	7.85
S1	-2.30	9.97
S2	-2.59	13.33
S3	-0.77	2.16
S4	-2.23	9.30
S5	-1.52	4.57
S6	-1.27	3.56
S7	-1.74	5.70

5. Conclusions

A classical methodology extensively adopted in hydrology has been applied to the analysis of the extreme waves in the first decade of RON observations. A great deal of attention has been given to the accuracy of the application of the procedure to Italian sea waves. The methods proposed to preserve the homogeneity and stationarity of the samples led to the definition of

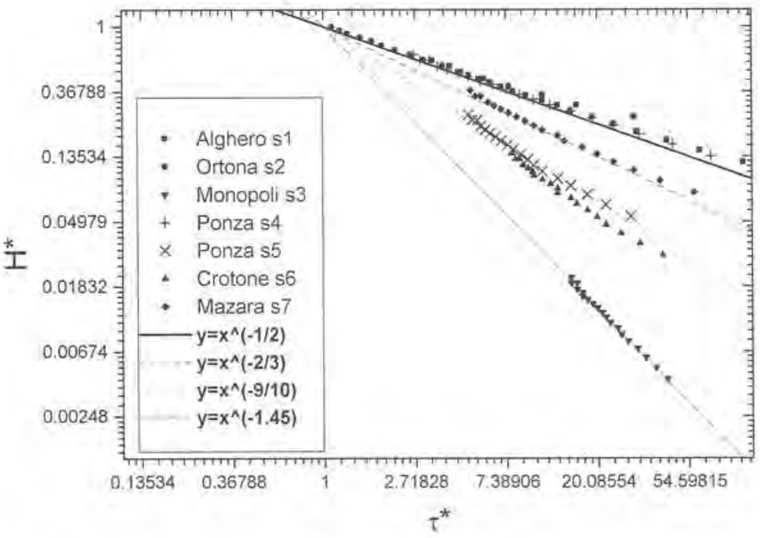


Fig. 20 - A dimensional average duration of exceedance of a threshold H^* by secondary sectors.

directional sectors bounded on the basis of the observed local wave climate. Results for return values $H_{m0}(T = 5$ years), were found to agree fairly well with the second maxima observed in the 10-year period for all sectors considered within the standard error limits. Return values for $T = 5$ to 50 years were not found to be sensitive to changes in the number of events considered. The introduction, where possible, of a limited number of extrapolated data hardly changed the estimates. The overall quantity of missing data, in spite of the ambiguity due to the unknown percentage of missing storms, was found to be useful in assessing the robustness of the estimated distribution. On the whole, the adopted methodology was found to be little influenced by data discontinuities (i.e. small gaps), providing reliable estimates even with 10-13 % missing data, as for Crotone and Mazara. Yet a degree of uncertainty on the reliability of the estimates for Ponza and Crotone cannot be ignored: a significant number of missing data, particularly if systematically related to sea-storms, can lad to an underestimation of the results. In the near future, more accurate analyses and measurements will be carried out in the area of the Lower Tyrrhenian Sea (where a new buoy has already been positioned at Cetraro, in the Tyrrhenian Basin) in order to improve the reliability of the long term estimates where significant storms usually take place. Comparison with previous analyses showed that $H_{m0}(T)$ might have been overestimated in the past, differences arising mainly in the different definition of the directional sectors and in the different criteria adopted for the selection of independent events introduced in the statistical analyses. Following the procedure of Mathiesen (1994) for the evaluation of the average duration above a threshold level H, the analyses were carried out by means of the results previously obtained. The results can be summarised for all the sectors with a parametric formula relating the average duration and the significant wave height threshold, suggesting the possible existence of a general law.

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