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A quantitative evaluation of the microseismic activities in the far field

V. E. Asor[1] and E. O. Okeke[2]

[1] Subsurface Development Information Technology, Shell Petroleum, Warri, Nigeria
[2] Department of Mathematics, University of Benin, Benin City, Nigeria

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Abstract. This paper presents a derivation of the governing equations for microseisms in the far field in a visco-elastic half-space generated by ocean waves. The equations are for small amplitudes and for the range of microseismic frequencies associated with the first order bottom pressure effects of the generating shallow water swell. We apply the equations to a fairly hard rock geology and we calculate the frequency spectral amplitude components of microseisms as a function of depth below the Earth's surface in the far field. Further, by analysing the Rayleigh denominator (Burridge, 1972), usually derived from the governing equations, we verify that microseismic signals propagating away from the generating source are damped. In this consideration, we derived the limiting values of the damping coefficient to ensure effective damping of microseismic oscillations as function of the material shear wave speed.

1. Introduction

The phenomena of micro Earth tremors called microseisms had been observed since the early days of seismology. The efficiency with which these waves are transmitted from the generating source to the far field, their polarization, subsequent detection and recording are quite remarkable and fairly well understood. They are essentially surface waves propagating in the direction parallel to the Earth's surface and the associated energy trapped near the surface. Consequently, they could be detected at quite a distance from the generating source.

Analysis of the energy spectrum of the seismic records in the range of microseisms frequencies clearly indicate two main peaks. It has been established conclusively that the two peaks are largely associated with two distinct activities related to the ocean waves. The lower

Corresponding author: V. E. Asor, Subsurface Development Information Technology, Shell Petroleum, P.O. Box 230, Warri, Delta State, Nigeria; phone: +234 (0) 53255100; e-mail: Vincent.E.Asor@spdc.shell.com

peak corresponding to the primary frequency microseisms is associated with the first order effects of wave bottom pressure modulation as sea waves propagate through a sloping beach towards the shoreline (Darbyshire, 1950; Hasselmann, 1963; Hinde and Hartley, 1965; Okeke, 1972) and more recently (Goodman et. al., 1989; Trevorrow et al., 1989; Asor and Okeke, 1998; Okeke and Asor, 1998).

On the other hand, the upper frequency peak is associated with the double microseisms. This is so called for the microseisms frequencies in this band are double that of the generating sea waves. As determined by Longuet-Higgins (1950), the wave activities involved in this regard are the second order pressure effects. These are energized through the nonlinear interactions among progressive sea waves moving in opposite directions. The phenomena are not affected by the depth of the water layer. Consequently, they are effective generating mechanisms both in deep and shallow water areas.

In this study, we introduce a damping term in the governing equations. This will represent the effect of the material inelasticity which we shall assume to be slight. Newlands (1954), in particular, investigated a similar problem identified with Lamb (1904). In this case, the generating source is localised and of the type usually modelled with delta function and the damping coefficient is assumed to vary as the n^{th} root of the single frequency, ω of the induced seismic mode.

We shall however, adopt the Darbyshire and Okeke (1969) model in which the damping term in the equation of motion is merely assumed to be proportional to the time rate of change of material displacement components. In this model, the generating source is the high phase velocity pressure components of the shallow water gravity waves propagating towards the shoreline from a wide range of directions (Okeke and Asor, 2000). Thus, some of the Newlands' deductions are not readily applicable in our present study. Previous calculations based on this model were quite close to the measurements of seismic events in the far field.

Further, there are a number of interesting and innovative publications on the related problems of the evolution of the microseisms in the seafloor. Recent achievements in this area of geophysics owe a lot to the work of Yamamoto et al. (1977, 1978) and recently, Trevorrow et al. (1988, 1989). Information acquired therefrom had been effectively used in the study of such areas as the structural depth profile below the seabed. It is intended in this analysis to extend the identical calculations to the far field microseisms activities.

2. Boundary equations

In this model, x-axis is horizontal and directed normal to the shoreline; z-axis points vertically downwards with $z = 0$ as the seabed, t is time and $\varphi(x, z, t)$ and $\psi(x, z, t)$ are the scalar potentials associated with seismic events. ρ_ω and ρ_s are the densities of water and the underlying seabed respectively; λ and μ are elastic parameters. Thus, the compression and shear wave velocities are respectively defined by

$$\alpha^2 = \frac{\lambda + 2\mu}{\rho_s}, \quad \beta^2 = \frac{\mu}{\rho_s}$$

The governing equations as proposed by Darbyshire and Okeke (1969) are as follows

$$P(k)\exp ik(x-ct)=\lambda\left(\frac{\partial^2\phi}{\partial\ x^2}+\frac{\partial^2\phi}{\partial\ z^2}\right)+2\mu\left(\frac{\partial^2\phi}{\partial\ z^2}-\frac{\partial^2\psi}{\partial x\ \partial z}\right)+$$

$$+\gamma\rho_s\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial z}\right) \tag{1}$$

$$\mu\left(2\frac{\partial^2\phi}{\partial x\partial z}-\frac{\partial^2\psi}{\partial\ x^2}+\frac{\partial^2\psi}{\partial\ z^2}\right)+\gamma\rho_s\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x}\right)=0 \tag{2}$$

The displacement components of the medium assumed to be elastic, are given by U in x-direction and W in z-direction as follows

$$U=\frac{\partial\phi}{\partial x}+\frac{\partial\psi}{\partial z} \tag{3}$$

$$W=\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x} \tag{4}$$

γ is the damping coefficient. Generally, it is a function of the wave period. k and c are respectively, the low wave number and high phase velocity components in the wave number spectrum of the seawaves (Hasselmann, 1963). $P\ (k)$ is the seawave pressure amplitude. However, in the ensuing calculations, $P\ (k)$ will be expressed in terms of the wave periods.

The solutions of Eqs. (1) and (2) may be expressed in the form

$$\phi(x,z,t)=A\exp[ik(rz+x-ct)] \tag{5}$$

$$\psi(x,z,t)=B\exp[ik(sz+x-ct)] \tag{6}$$

where A and B are constants only with regards to space and time.

$$r^2=\left(\frac{c^2}{\alpha^2}-1\right),\ s^2=\left(\frac{c^2}{\beta^2}-1\right) \tag{7}$$

An effect of the damping term in Eqs. (1) and (2) is to make k and incidentally c complex number with a non-zero imaginary part. Thus, $k = k_0 + i\sigma k$, $c = c_o + i\sigma c$ with $\sigma k << k_o$, $\sigma c << c_o$.

3. The far field material dispersion

On the Earth's surface and in the far field, the wave forms are free, hence, Eqs. (1) to (7) give

$$A\left[\beta^2\left(s^2-1\right)+\gamma c\right]+B\left[2\beta^2 s-\gamma s c\right]=0 \tag{8}$$

$$A\left[2r\beta^2-\gamma c s\right]+B\left[\beta^2\left(1-s^2\right)-\gamma s\right]=0 \tag{9}$$

(8) and (9) are consistent if

$$f(c)=\left(2\beta^2-\gamma c\right)^2 rs+\left[\beta^2\left(1-s^2\right)-\gamma c\right]=0 \tag{10}$$

Using Eq. (7),

$$f(c)=\left(2\beta^2-\gamma c\right)^4\left[\left(\frac{\alpha_1 c}{\beta}\right)^2-1\right]-\left[\beta^2\left(2-\frac{c^2}{\beta^2}\right)-\gamma c\right]^4=0 \tag{11}$$

whereby introducing the following notation in Eq. (11), with v as the Poisson's ratio, we obtain

$$\alpha_1=\frac{\beta}{\alpha}=\frac{1-2v}{2-2v},\quad v=\frac{\lambda}{2(\lambda+\mu)}\quad \text{and}\quad k_1=\frac{c}{\beta},$$

Thus, from Eq. (11),

$$(2\beta-\gamma k_1)^4(\alpha_1^2 k_1^2-1)-\left[\beta(2-k_1^2)-\gamma k_1\right]^4=0 \tag{12}$$

Rearranging in powers of $k_1, f(k_1)$ is now

$$\begin{aligned}&k_1^6(\beta^4-\alpha_1^2\gamma^4)+4\gamma\beta k_1^5(\beta^2+2\gamma^2\alpha_1^2)-k_1^4[8\beta^4-\gamma^2\{6\beta^2(4\alpha_1^2-1)\\ &-\gamma^2(\alpha_1^2+1)\}]+4\beta\gamma k_1^3[\gamma^2(2\alpha_1^2+1)+2\beta^2(4\alpha_1^2-3)]\\ &+k_1^2[8\beta^4(3-2\alpha_1^2)+24\beta^2\gamma^2\alpha_1^2]+16\beta^3\gamma k_1(1-2\alpha_1)-16\beta^4(1-\alpha_1^2)=0.\end{aligned} \tag{13}$$

If $\gamma = 0$ in Eq. (13), it reduces to the usual relation for the non-dispersive Rayleigh waves in elastic solid. In this case, the equation will then be cubic in k_1^2 which has been thoroughly analysed (Bullen and Bolt, 1985) to obtain the propagational properties of the surface wawes for a range of values of ν. Eq. (13) is thus the case of the material dispersion associated with the inelastic term introduced in the model; for $k_1=c/\beta$ depends on the material parameters μ, λ and γ.

4. Analysis of Eq. (13)

$f(0)=-16\,\beta^2\,(1-\alpha_1^2)<0$, since $\alpha_1<1$ for surface waves.
$f(1)>0$ because

$$\gamma^3[\alpha_1^2(2\gamma-4\beta)+(\gamma-4\beta)]-16\beta^2\gamma^2(1-\alpha_1^2)+\beta^3(4\gamma-\beta)<0$$

for each term in the bracket is negative since $\alpha_1<1$ and $\gamma<\beta/4$. The reason for the last inequality follows from the realistic value of γ. Thus, there is a root of (13) between $k_1 = 0$ and $k_1 = 1$.

For $f(-1)$, we have

$$\beta^4+4\gamma\beta^3+6\beta^2\gamma^2(8\alpha_1^2-1)-4\beta\gamma^3(4\alpha_1^2+1)-\gamma^4(2\alpha_1^2+1)$$

$$>\frac{10}{16}\beta^4+59\frac{7}{8}\beta^2\alpha_1^2\gamma^2+3\frac{31}{32}\beta^3\gamma>0, \quad \text{for} \quad \gamma<\frac{\beta}{4}.$$

There is, at least, another root of Eq. (13) between $k_1 = 0$ and $k_1 = -1$.

Further, Eq. (13) contains terms involving γ, even and odd powers of k_1; thus, does not reduce to a cubic in k_1^2 that can be easily analysed.

Hence, to examine the nature of zeros of Eq. (13) in the unit circle $|k_1| < 1$, sequence $\{f_m(k_1)\}$, $m = 1, 2, ..., 5$ of Sturm's function (Kurosh, 1980) are computed from Eq. (13). We now let $f_o(k_1) = f(k_1)$ being Eq. (13). $f_n(k_1)$ is the first derivative of $f_{n-1}(k_1)$, n = 1, 2, …, 5. Let $c(0)$ be the number assigned to the changes of sign in these sequences when $k_1 = 0$. Attach an identical meaning to $c(1)$ when $k_1 = 1$. We find that the difference $c(0) - c(1)$ in $0 < k_1 < 1$ depends on the values assigned to γ. From symmetry, a similar conclusion applies to the range $-1 < k_1 < 0$.

Consequently, if $0 \leq\gamma\leq \alpha_1^2\, \beta/40$, then $c(0) - c(1) = 1$ and there is only one real root in each of the intervals $-1<k_1<0$ and $0<k_1<1$. In this case, the effect of damping is negligible in the propagation of elastic waves. With regards to the Sturm's sequence, all the leading coefficients are positive. If $\alpha_1^2\, \beta/40 < \gamma< \beta/4$, then the leading coefficients for $m = 2$ and $m = 3$ in the Sturm's sequence are negative and $c(0)- c(1) = 2$. Thus, in $|k_1|<1$, there are four complex conjugate roots for Eq. (13), one in each of the four quadrants of the k_1-plane. This analysis therefore seems to suggest that seismic waves in an elastic solid are effectively damped if the attenuation coefficient γ in the solid exceeds the value $\alpha_1^2\, \beta/40$; and the complex roots for which $Re(k_1) > 0$, $Im(k_1) > 0$ corresponds, to the observed damped vibrations.

We now apply this result to the microseisms signals recorded on geology made of fairly hard rock. The phase speeded, of the signals usually ranges from 1.1 km/s to 1.8 km/s and using the value $c_o = 0.8\beta_1$ the corresponding value of γ is between 0.021 km/s and 0.04 km/s. This range of values of

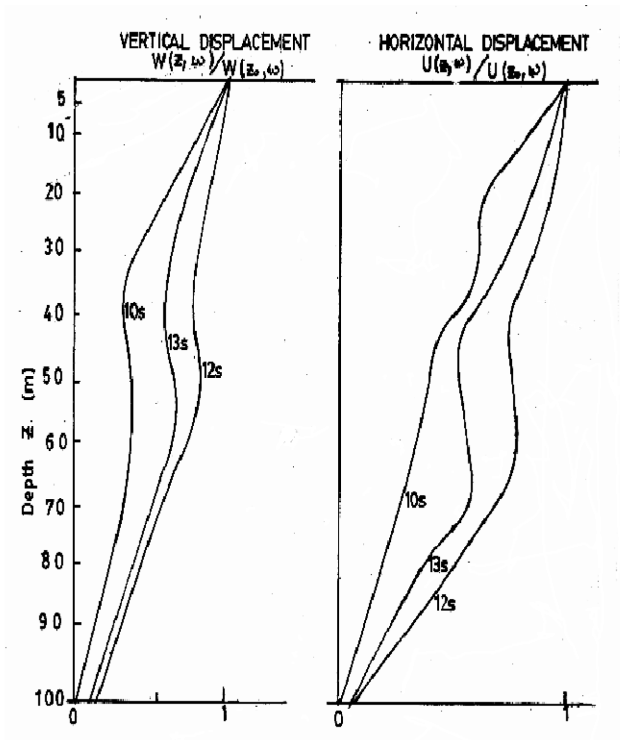

Fig. 1 - The variation of the vertical and horizontal ground movements with depth below the Earth's surface.

γ is between $\alpha_1^2\ \beta/40$ and $\beta/4$, suggesting that the microseisms signals propagating from the source to the recording station in the far field are damped to some extent.

The variation of this range of values of γ with depth is displayed in Table 1. The calculation in this model covers the case of the horizontally stratified Earth for which the elastic parameters and density are functions of the z-coordinates only. Further, the values of z in the computation are confined to the depth above which the microseisms signals are likely to be detectable (Okeke and Asor, 2000). From Table 1, the uniformity of this range with depth seems apparent.

Furthermore, calculations made by extrapolating from data in Table 1 suggests that the value of γ ranges from 0.0135 km/s to more than three times this value at a depth of 100 km below the Earth's surface. The implication of this state of affairs is illustrated in Fig. 1. Therefrom, it is apparent that the microseismic amplitude components are damped to a near extinction state at about the depth of 100 km.

5. The far field wave form

In this section, we calculate the frequency spectral amplitude components of microseisms signals as functions of depth variation below the Earth's surface. Identical calculations which have given rise to a number of useful results and publications describing these could be found in Trevorrow et al. (1991). However, the researchers seemed to have focused their interests on the seabed microseisms.

Table 1. The range of the possible values of γ as a function of depth below the Earth's surface.

Z (m)	β(z) km/s	α(z) km/s	ρ(z) g/cm^3	$\alpha_1^2 \frac{\beta}{40}$	β/10
5	1.60	2.73	2.05	0.0137	0.160
10	1.58	2.710	2.072	0.0134	0.158
20	1.71	2.952	2.091	0.0143	0.171
30	1.76	3.012	2.098	0.0152	0.176
40	1.94	3.340	2.10	0.0164	0.194
50	2.05	3.520	2.125	0.0218	0.205
60	1.97	3.401	2.132	0.0165	0.197
70	1.92	3.322	2.141	0.0160	0.192
80	1.90	3.124	2.143	0.0176	0.190
90	1.878	3.118	2.145	0.0168	0.188

Instead, our calculations will concern the microseisms as observed in the far field and in the range of primary frequency. Extrapolating from the data for the seabed vertical profile (Trevorrow et. al., 1988, 1991) of elastic shear modulus and related parameters, we have determined the corresponding density, compressional and shear wave speeds in the far field. Our results, as displayed in Table 1, are found to be in reasonable agreement with those observed locally by our field geologists.

In this consideration, Eqs. (1) and (2) are to be expressed in terms of the displacement component rather than scalar potentials. That is, we shall use the following representations

$$U(x,z,t) = \overline{U}(z)e^{ik(x-ct)} \tag{14}$$

$$W(x,z,t) = \overline{W}(z)e^{ik(x-ct)} \tag{15}$$

Introducing Eqs. (14) and (15) through (3) and (4) into (1) and (2), then rearranging, we obtain the following set of differential equations

Table 2. Calculated data of the microseismic activities in the far field. The last column is the amplitude distribution of the exciting gravity (water) waves, $\omega = 2\pi/T$, $\rho_w = 1.015$g/cm^3.

Period (s)	U(ω, z_0)	W (ω, z_0)	a(ω)
9	4.2μ	4.32μ	0.975m
10	4.5μ	4.5μ	0.992m
11	5.35μ	5.5μ	1.025m
12	6.15μ	6.4μ	1.130m
13	6.10μ	6.40μ	1.28m

$$\frac{d\overline{U}}{dz} = ik\left(\frac{c}{\beta^2}\right)\left(\gamma - \frac{\mu}{\rho_s c}\right)\overline{W} \tag{16}$$

$$\frac{d\overline{W}}{dz} = ik\left(\frac{c}{\alpha^2}\right)\left(\gamma - \frac{\lambda}{\rho_s c}\right)\overline{U} + \frac{P(k)}{\lambda + 2\mu}. \tag{17}$$

Eqs. (16) and (17) combine to give the usual matrix form (Bullen and Bolt, 1985)

$$\frac{df}{dz} = \mathbf{A}f + g_0 \tag{18}$$

where

$$\mathbf{f} = \begin{bmatrix} \mathbf{U} \\ \mathbf{W} \end{bmatrix}, \qquad \mathbf{g}_0 = \begin{bmatrix} 0 \\ \dfrac{P(k)}{\lambda + 2\mu} \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} 0 & \dfrac{\mathbf{1}}{\beta^2}\left(\gamma - \dfrac{\mu}{\rho_s \mathbf{c}}\right) \\ \dfrac{\mathbf{1}}{\alpha^2}\left(\gamma - \dfrac{\lambda}{\rho_s \mathbf{c}}\right) & 0 \end{bmatrix}$$

$$P(k) = \rho_w \; ga(k) \; \sec h(kd). \tag{19}$$

In Eq. (19), d is the depth of water layer measured from its undisturbed level. In shallow water, $kd \to 0$ and sech$(kd) \to 1$; thus $P(k) = \rho_w ga(k)$ and the pressure is now hydrostatic being unaffected by the depth of the water layer. In this case $a(k)$ is the amplitude of the water wave which is here referred to as the exciting source. In subsequent calculations, $a(k)$ will be expressed in terms of the observed wave periods, T, rather than wave number components, k.

In a perfectly damped elastic medium where the elastic parameters and density are assumed to be constants, Eq. (18) can be easily integrated to obtain

$$f(z) = (\mathbf{f}_0 + g_0 B)e^{-Az} - Bg_0 \tag{20}$$

where the column vector f_0 represents the observed components of the microseisms amplitude in the far field and $z = 0$.

$$B = A^{-1} = \frac{i}{kc}\begin{bmatrix} 0 & \alpha^2\left(\gamma - \dfrac{\lambda}{\rho_s \mathbf{c}}\right)^{-1} \\ \beta^2\left(\gamma - \dfrac{\mu}{\rho_s \mathbf{c}}\right)^{-1} & 0 \end{bmatrix} \tag{21}$$

$$e^{Az} = I - Az + \frac{A^2 z^2}{2!} + \ldots, \quad I = Identity \ \ matrix.$$

However, in the case of the horizontally layered Earth, Eq. (18) has a solution of the form

$$\mathbf{f(z)} = \mathbf{P}(z, z_0)\mathbf{f_0} + \mathbf{g_0}(k)\int_{z_0}^{z} \mathbf{P}(z, y)dy \tag{22}$$

where $\mathbf{P}(z, z_0)$ is related to the matrix **A** through

$$\frac{dP(z, z_0)}{dz} = P(z, z_0)A(z). \tag{23}$$

With $P(z_0, z_0) = I$, the relationship is now

$$P(z, z_0) = e\int_{z_O}^{z} A(z')d'z. \tag{24}$$

In the discussions to follow, $z_0 = 0$, is the Earth's surface.

6. Discussion

The depth dependent far field wavefield is calculated from Eq. (22). With layer elastic parameters extrapolated from Table 1, the matrix $\mathbf{P}(z, z_0)$ is easily determined at any given depth. The data input $a(\omega)$ in our calculations are displayed in Table 2; they are the amplitude distribution of the shallow water generating swell as functions of wave periods. In order to give a convergent and realistic result in the deeper part of the Earth's layers, the input data employed in the calculations are slightly less in magnitude than one would expect.

Specifically, the integral part of Eq. (22) is however less easily calculated as a function of depth. Nevertheless, Simpson's and Gaussian algorithms employed at each layer of thickness 5m give very smooth and identical results. Further, the damping coefficient γ is determined as a linear function of wave period using the relationship

$$\gamma = 2.96c^2 T\bar{\gamma}$$

where $\bar{\gamma}$ is the distance decrement given by $\bar{\gamma} = 7\times10^{-4}\ km^{-1}$, c=0.92 β.

The calculated variations of the components of the ground displacements in response to microseismic events are displayed in Fig. 1. Therefrom, the suggested apparent peak period of 12 s applies at all depths. Again, the usual behaviour of the guided mood of vibrations seems to have been well depicted by Fig. 1. However, the amplitude decay from the Earth's surface is not exponential as would have been expected by using the model of homogeneous elastic Earth's medium. Thus, this development suggests the layer effects on the amplitude of the vibrations.

Finally, the surface seismic amplitude components obtained from this model are quite close to the locally observed microseisms amplitude spectrum, though on the lesser side (Table 2). Thus, quantitatively, the computed amplitude components are almost half of the observed data (Okeke, 1972). The discrepancy is, as expected, owing to some other seismological constraints that are not included in this study.

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