BOLLETTINO DI GEOFISICA TEORICA ED APPLICATA VOL. 40, N. 3-4, pp. 219-226; SEP.-DEC. 1999

Marine gravity field recovery by combining satellite altimetry and shipborne gravimetry

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(Received October 4, 1998; accepted August 5, 1999)

Abstract. The approximation of the gravity field in the Newfoundland Sea area off the east coast of Canada is carried out. Geoid heights, geoid gradients, gravity anomalies and deflections of the vertical are predicted by combining altimeter data from the Geosat and ERS-1 Geodetic Missions (GMs) and shipborne gravity data, using both spectral and collocation techniques. The computed geoid heights are compared with corresponding TOPEX altimeter Sea Surface Heights (SSHs) and the differences show an accuracy close to 3 cm in terms of standard deviation (sd). The predicted gravity anomalies are compared with gravity anomalies from global marine gravity data banks and the differences are at the level of 4 mGal (1 sigma). Deflections of the vertical are computed by inverting gravity anomalies and SSHs. The results are intercompared and found to agree at the level of 0.6"-1" (1 sigma). Finally, conclusions are drawn on the usefulness of each data type and the effectiveness of each processing method.

1. Introduction

In the present study, shipborne gravity anomalies are combined with the GM altimeter data of Geosat and ERS-1 using the spectral Input/Output System Theory (IOST) method (see, e.g., Sideris, 1996; Sansò and Sideris, 1997). The computed by the IOST method geoid heights are compared with corresponding heights derived only from shipborne gravity data using the well-known 1-D FFT technique. Then, the geoid heights from the combined solution and the gravimetric one are compared with TOPEX SSHs in order to assess the importance of satellite altimetry in the approximation of the gravity field over oceanic areas.

Another objective of this study is the computation of gravity anomalies by (a) the IOST method

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Table 1 - Statistics of the 20 000 gridded free-air gravity anomalies. Unit: [mGal].

	mean	min.	max.	sd	rms
original Δg	12.61	- 53.47	132.40	± 24.33	27.40
Δg reduced to EGM96	- 2.15	- 54.97	88.12	± 10.97	11.18

combining GM satellite altimetry and shipborne gravimetry data and (b) the FFT method directly inverting the GM satellite SSHs. The results derived by the two methods are intercompared and also compared with corresponding gravity anomalies from the global marine data base of Sandwell and Smith (1997). The last objective of the present paper is the efficient recovery of deflections of the vertical by the direct inversion of (a) sea gravity anomalies and (b) SSHs using FFT.

2. Data processing

2.1. Marine gravity data

The original shipborne free-air gravity data covering the off Newfoundland Sea test area in the east coast of Canada belong to the Gravity Data Centre of Canada. The data processing as well as the data adjustment were carried out by the Geological Survey of Canada. 84 169 point free-air gravity anomalies were originally selected in the region. A gap existing in the north-east part of the area was filled by 3'×3' gravity data extracted from the KMS worldwide data bank (Knudsen and Andersen, 1996). From this gravity data set, a 3'×6' (~ 5.56 km × 7.15 km) grid was produced in the area $45° \leq \phi \leq 55°$, $-55° \leq \lambda \leq -45°$ (200×100 grid). The 20 000 gridded free-air anomalies are on the GRS80 reference system and are referenced to the EGM96 geopotential model (Lemoine et al., 1996). The statistics of the gravity anomalies are given in Table 1. The atmospheric correction, equal to 0.87 mGal, has been also applied to these anomalies. The residual gravity anomalies are still biased by -2.15 mGal.

2.2. Altimeter data

ERS-1 and Geosat radar altimeter data of the corresponding GMs were used in our test area. From the ERS-1 GM, 39 335 sub-satellite points belonging to 366 tracks were selected in our test area. To these altimeter data, a simple bias crossover adjustment was applied after the removal of a number of erroneous observations. The 11 742 crossovers had, before adjustment, a mean value equal to -0.10 m and an rms value equal to 0.21 m. After the adjustment, these values dropped to 0.00 m and 0.08 m, respectively. Additional numerical tests carried out in a larger area by applying a 3-parameter (bias and two tilts) transformation did not improve the results in terms of mean and rms values. From the adjusted altimeter SSHs, corresponding heights were computed on the same 3'×6' gravity grid (see previous section). For these gridded altimetric heights, the EGM96

Table 2 - ERS-1 and Geosat GM altimetry statistics before and after crossover adjustment (200x100 grid). Unit: (m).

	mean	min.	max.	sd	rms
observed ERS1 SSHs	20.34	- 1.39	38.11	± 8.48	22.03
observed Geosat SSHs	21.83	- 2.07	38.43	± 7.85	23.19
ERS-1 GM SSHs to EGM96 (before)	- 0.87	- 2.80	0.55	± 0.29	0.92
ERS-1 GM SSHs to EGM96 (after)	- 0.01	- 0.62	1.27	± 0.25	0.25
Geosat GM SSHs to EGM96 (before)	- 0.71	- 5.71	1.46	± 0.45	0.84
Geosat GM SSHs to EGM96 (after)	- 0.00	-3.71	2.09	± 0.27	0.27
ERS-1/GEOSAT GM SSHs reduced to EGM96 (before)	0.02	-2.08	1.95	± 0.29	0.29
ERS-1 and GEOSAT SSHs reduced to EGM96 (after)	0.00	-0.60	1.29	± 0.25	0.25

geopotential model was used as a reference surface as well. The statistical results of the 20 000 gridded altimeter heights are listed in Table 2. It is worth mentioning that all the along track SSHs and the derived gridded heights are considered in this study as geoid heights, since no attempt was made to recover sea surface topography and no oceanographic corrections were applied. However, the effects of the errors introduced by this assumption should be further investigated.

From the Geosat GM, 53 225 sub-satellite points belonging to 632 tracks were selected in our test area. To these altimeter data, a simple bias crossover adjustment was applied after the removal of a number of erroneous observations. 25 398 crossovers had, before adjustment, a mean value equal to -0.15 m and an rms value equal to 0.23 m. After the bias adjustment, these values dropped to 0.01 m and 0.15 m, respectively. At this processing stage, a common adjustment of the ERS-1 and Geosat altimeter heights was also carried out. 37 140 crossovers from both missions had, before adjustment, a mean value equal to -0.14 m and an rms value equal to 0.22 m. After the bias adjustment procedure, these values dropped to 0.07 m and 0.14 m, respectively. These values were interpolated again onto the same 3'×6' grid. The statistical results of the 20 000 gridded altimeter heights derived from the Geosat GM, as well as those derived after merging the heights from both GMs, are listed in Table 2.

We also used 817 TOPEX altimeter heights in the inner part of our test area. These values were used only as control values in order to assess the accuracy of the predicted geoid heights. The TOPEX data have been used without ocean tidal or other corrections similarily to the GM-altimetry data. The TOPEX heights used in the study have a much better orbital error (less than 5 cm) than that of the two GMs, but present poor resolution between neighbouring tracks (~ 260 km).

3. Results and discussion

3.1. Prediction of geoid heights

The first geoid approximation in our test area is a pure gravimetric geoid solution derived by the 20 000 gridded free-air gravity anomalies using the 1-D FFT procedure (see, e.g., Tziavos et

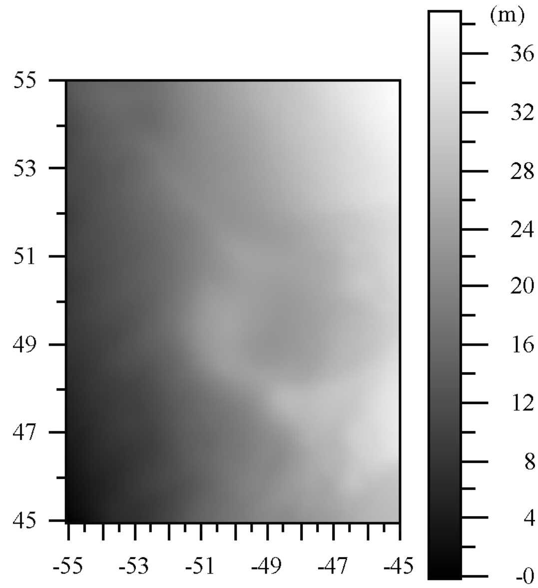

Fig. 1 - Complete geoid height solution by IOST.

al., 1998). The geoid heights calculated by this procedure refer to the same 3'×6' grid as the gravity anomalies. The statistical results of this residual solution, after the subtraction of the reference field (EGM96 geopotential model), are listed in Table 3, together with the statistics from the comparison of this solution with the combination one and with the 817 TOPEX geoid heights of the central part of the test area.

In order to assess the importance of satellite altimetry in combination geoid solutions,

Table 3 - Residual geoid height predictions and geoid height differences between different methods. Unit: [m]

(1) Gravimetric solution, (2) Combined solution (gravity+ERS-1 GM), (3) Combined solution (gravity+Geosat GM), (4) TOPEX					
	mean	**min**	**max**	**sd**	**rms**
(1)	- 1.07	- 2.26	0.45	± 0.34	1.12
(2)	0.00	- 0.61	1.23	± 0.25	0.25
(3)	- 0.01	- 0.59	1.31	± 0.28	0.28
(4)	- 0.63	- 1.16	0.10	± 0.22	0.67
(1) - (2)	- 1.07	- 2.45	- 0.05	± 0.39	1.07
(1) - (3)	- 1.10	- 2.88	0.02	± 0.45	1.10
(2) - (3)	0.01 (0.00)	- 0.11 (- 0.08)	0.44 (0.14)	± 0.03 (± 0.03)	0.03 (0.03)
(1) - (4)	- 0.55 (0.00)	- 0.17 (- 0.26)	- 1.03 (0.25)	± 0.24 (± 0.08)	0.27 (0.08)
(2) - (4)	- 0.58 (0.00)	- 0.72 (- 0.12)	- 0.46 (0.14)	± 0.04 (± 0.04)	0.58 (0.04)
(3) - (4)	- 0.60 (0.00)	- 0.75 (- 0.14)	- 0.49 (0.19)	± 0.05 (± 0.05)	0.60 (0.06)

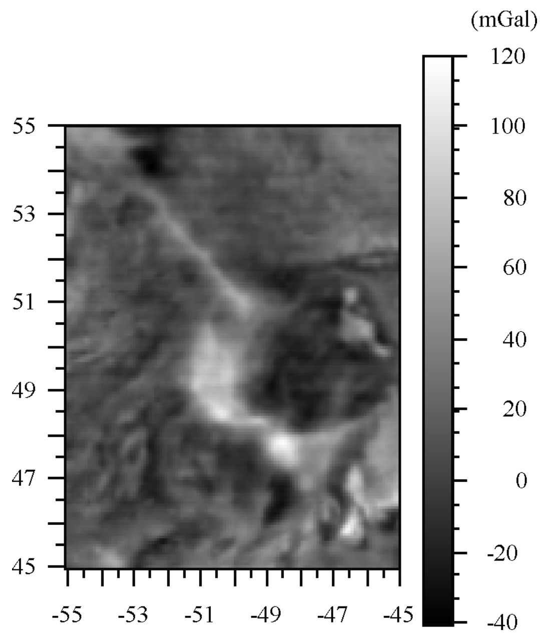

Fig. 2 - Complete gravity anomaly solution by IOST.

additional geoid computations were carried out by combining the 3'×6' free-air gravity anomalies with (a) the 3'×6' ERS-1 satellite altimeter heights and (b) the 3'×6' ERS-1 and Geosat satellite altimeter heights that were derived from the common adjustment mentioned in the previous section. The geoid heights derived by the IOST method are intercompared and compared also with the gravimetric solution heights and the 817 TOPEX heights. The statistics of the differences are tabulated in Table 3. In Fig. 1, a 2-D map of the complete geoid heights derived from gravity, ERS-1 GM altimetry and the EGM96 model is shown. In the IOST method, the gravity data were used with an error estimate equal to 3 mGal and the altimeter heights with an error equal to 10 cm (Li, 1996). In both spectral techniques, 100% zero-padding was used around the data to eliminate circular convolution effects. In order to absorb long-wavelength errors between the different geoid comparisons, a 4-parameter datum shift was applied to the computed geoid height differences. The numbers in parentheses in Table 3 refer to the results after removing these datum differences.

From the results of Table 3, it is obvious that the two combined solutions (1 and 2) derived by IOST give very close results at the level of 3 cm in terms of sd. The gravimetric geoid heights (solution 1) is considerably biased, as compared with the TOPEX heights, due to the bias of the input gravity data (see Table 1). It is important to note the improvement of the gravimetric solution after removing the systematic differences and comparing with the TOPEX altimeter heights. The excellent agreement of the combined solutions (2, 3) with the TOPEX data at the level of 4 and 5 cm, respectively, in terms of sd before the removal of systematic differences, indicates that the contribution of satellite altimetry is very critical in these combined gravity field

Table 4 - Statistical results of (a) “observed” Δg (b) gravity/geoid derived Δg (IOST), (c) ERS-1 GM derived Δg, (d) ERS-1/Geosat derived Δg and their differences. Unit: [mGal]

(1) Δg_{obs}, (2) Δg_{IOST}, (3) Δg_{ERS-1}, (4) $\Delta g_{ERS-1/GEOSAT}$					
	mean	**min**	**max**	**sd**	**rms**
(1)	- 2.44	- 31.18	42.17	± 8.82	9.21
(2)	- 0.37	- 25.66	40.56	± 9.04	9.05
(3)	- 0.36	- 26.11	32.07	± 7.96	7.97
(4)	- 0.46	- 23.35	30.89	± 8.30	8.31
(1) - (3)	- 2.05	- 24.28	17.88	± 3.58	4.12
(1) - (4)	- 2.12	- 21.30	15.18	± 5.06	5.49

models. In this study, the sd value drops by 83% (the sd of differences from 24 cm in the gravimetric solution decreases to 4 cm in the combined ones). From the two combination solutions, slightly better results are obtained by the combination of marine gravity with ERS-1 GM data. No improvement was observed when merging ERS-1 and Geosat heights and combining them with marine gravity data.

3.2. Prediction of gravity anomalies

Different sets of gravity anomalies are used and intercompared in this section. The first set is the 3'×6' (200×100) grid of the “observed” gravity values derived from 84 169 point free-air gravity anomalies selected in the test area and using a collocation procedure. The “observed” gravity data grid was compared with the corresponding gravity values extracted from the global marine gravity data base of Sandwell and Smith (1997). The differences of the two gravity data sets gave a sd close to 3.86 mGal and a mean value equal to -1.98 mGal. The grid size of the global data set is 2' and we finally interpolated onto the 3'×6' grid to form the differences.

In a second numerical test, gravity anomalies were predicted by the IOST method using the same altimetry and marine gravity data as in the case of the geoid height prediction before. The internal estimation error was found to be close to 6 mGal. The residuals of the gravity anomaly prediction by IOST are given in Table 4. The complete gravity anomaly solution derived by sea gravimetry, altimetry and the EGM96 geopotential model is shown in Fig. 2. All gravity values in Table 4 are referenced to EGM96 and refer to the central part of the test area. In another numerical test, gravity anomalies were predicted by the inversion of ERS-1 GM altimeter SSHs and combined ERS-1/Geosat by FFT.

The statistical results of the latter two numerical tests are tabulated in Table 4. The gravity anomalies derived by the inversion of ERS-1 and ERS-1/Geosat geoid height grids are intercompared and also compared with the “observed” free-air anomaly data set. The geoid derived gravity anomalies show an agreement with the “observed” values at the level of 3.5 - 5.1

Table 5 - Deflection of the vertical predictions and their differences. Unit: [arcsec].

Deflections of the vertical from: (1) Δg, (2) SSHs(ERS1-GM)										
	mean		**min**		**max**		**sd**		**rms**	
	ξ	η	ξ	η	ξ	η	ξ	η	ξ	η
(1)	- 0.18	0.07	- 5.21	- 5.54	4.89	6.11	± 1.42	± 1.33	1.43	1.33
(2)	1.25	- 1.10	- 6.63	- 7.19	7.59	6.75	± 3.18	± 3.14	3.41	3.33

mGal in terms of sd of the differences. Better results are always obtained with the ERS-1 GM grid of geoid heights, with about an 1.5 mGal improvement in terms of sd of the differences.

3.3. Prediction of deflections of the vertical

Two methods are used in this study to compute deflections of the vertical ξ and η. Both methods are based on the inversion of the 3'×6' free-air gravity anomalies and altimeter SSHs into deflection components using the planar FFT procedure (Tziavos et al., 1998). In Table 6, the statistics of the deflection components by the two methods are given in the inner zone of the test area. Statistics of their differences are given as well. From the results of Table 5, a very good agreement between the gravity and geoid derived deflections can be seen.

4. Conclusions

Marine gravity field modeling can be performed in an efficient and accurate way using the IOST spectral procedure. Geoid heights and gravity anomalies were produced over an oceanic area with high accuracy and resolution by merging marine gravity data and dense satellite altimetry data from the GMs of ERS-1 and Geosat. Comparing the predicted geoid heights with TOPEX data, the sd of the differences was found equal to 4 cm. From the results of this study, the importance of satellite altimetry in combined solutions becomes evident. The improvement over a pure gravimetric geoid solution reaches in our case the level of 20 cm (drops from 24 cm for the gravimetric solution to 4 cm for the combined solution). When the predicted gravity anomalies derived by different methodologies are intercompared or compared with marine gravity data from global bases, the gravity anomaly prediction results vary between 1 and 4 mGal in terms of sd of the differences. The predicted deflections of the vertical derived from the inversion of gravity anomalies and altimeter heights of the GMs of ERS-1 and Geosat by FFT agree at the level of 0.6" to 1.0" in terms of sd of the differences.

Acknowledgments. The financial support provided by grant number CRG 951041 to the authors under the NATO Collaborative Research Grants Program is gratefully acknowledged. Many thanks are due to Marc Veronneau of the Geodetic Survey of Canada, who provided the gravity data.

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