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Gravimetric geoid computations in Hungary and the surrounding area

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Abstract. Different local gravimetric geoid solutions were carried out in Hungary and the surrounding continental area. These solutions were based on terrestrial gravity data and height data using as a reference surface the EGM96 geopotential model. The gravity data used in the area $45.5° < \phi < 49°$, $16° < \lambda < 23°$ were finally gridded on a 1.5' × 2.5' geographical grid. These included more dense gravity data with respect to a previous gravimetric solution for Romania and Yugoslavia. The height data were available on a 1km × 1km grid. The methods used were the spherical 1D Fast Fourier Transform (FFT) method and the Fast Collocation (FCOL) procedure. In order to assess the accuracy of the computed geoid heights we compared them with 43 Hungarian GPS/leveling stations belonging to the EUREF89 network. The statistical results of the derived differences give an accuracy close to 12 cm in terms of standard deviation, which decreases to 8 cm after subtracting a linear trend and bias model. Excluding 14 GPS stations located at the borders of Hungary the aforementioned accuracies reached the level of 7 cm and 6 cm, respectively. Moreover, we have compared our geoid solution in the entire test area with the European geoid EGG97 and found a standard deviation of differences close to 41 cm and 20 cm before and after subtracting a linear trend and bias model. In a last numerical experiment we computed geoid heights by 1D FFT using different cap sizes for the gravity data. The geoid height results were similar and no significant improvement has been achieved.

1. Introduction

In the last decade different geoid solutions were carried out for the Hungarian territory using heterogeneous data and different methodologies. The main goal of this paper is to improve our

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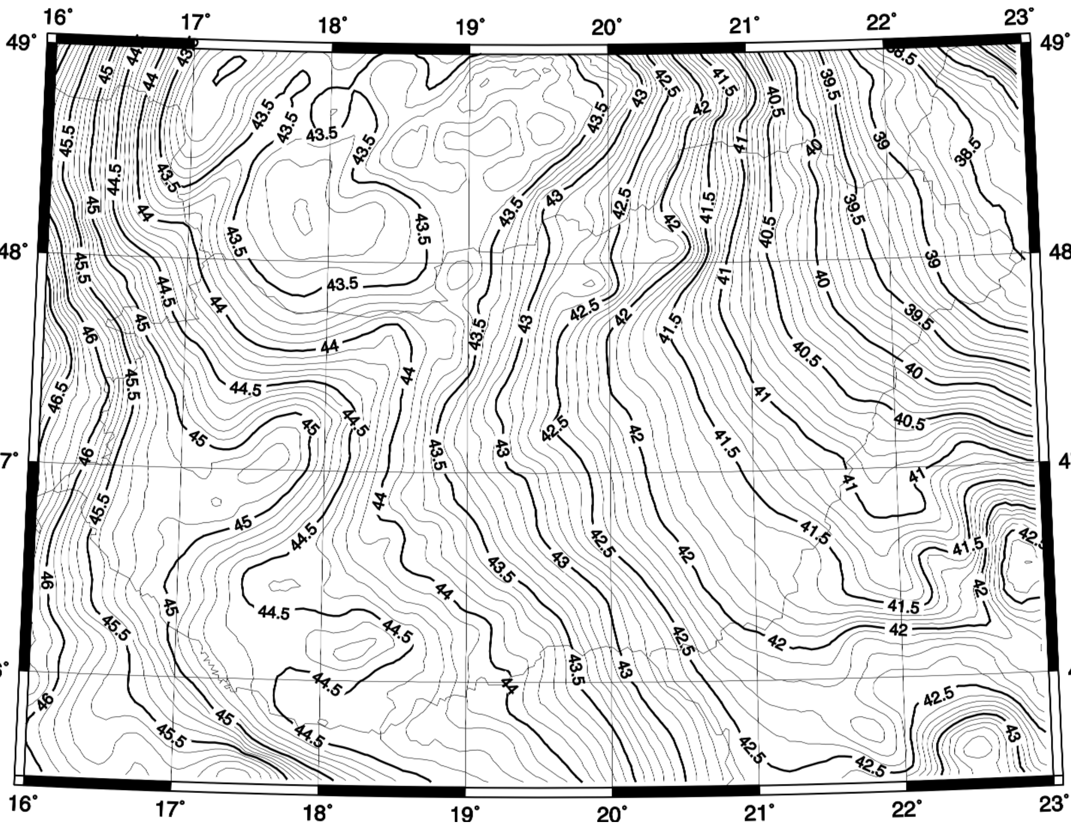

Fig. 1 - Geoid heights of the FFT solution in Hungary (meter).

previous geoid computation for the area of Hungary and the surrounding regions taking advantage of a new updated gravity database, with particular emphasia on several neighbouring countries. From the numerical tests accomplished so far and the results already summarized in the abstract, important conclusions and recommendations are drawn relevant to the computation of large-scale national or international geoid computations on fine grids.

2. Data description

The gravity material included in the geoid computation can be divided into the two following main parts:

Table 1 - Statistics of data grids.

	max	min	mean	rms	sd
Free-air anomalies [mgal]	101.088	-38.592	17.251	24.739	±17.732
DTM data [m]	2428	21	338.34	447.16	± 292.38

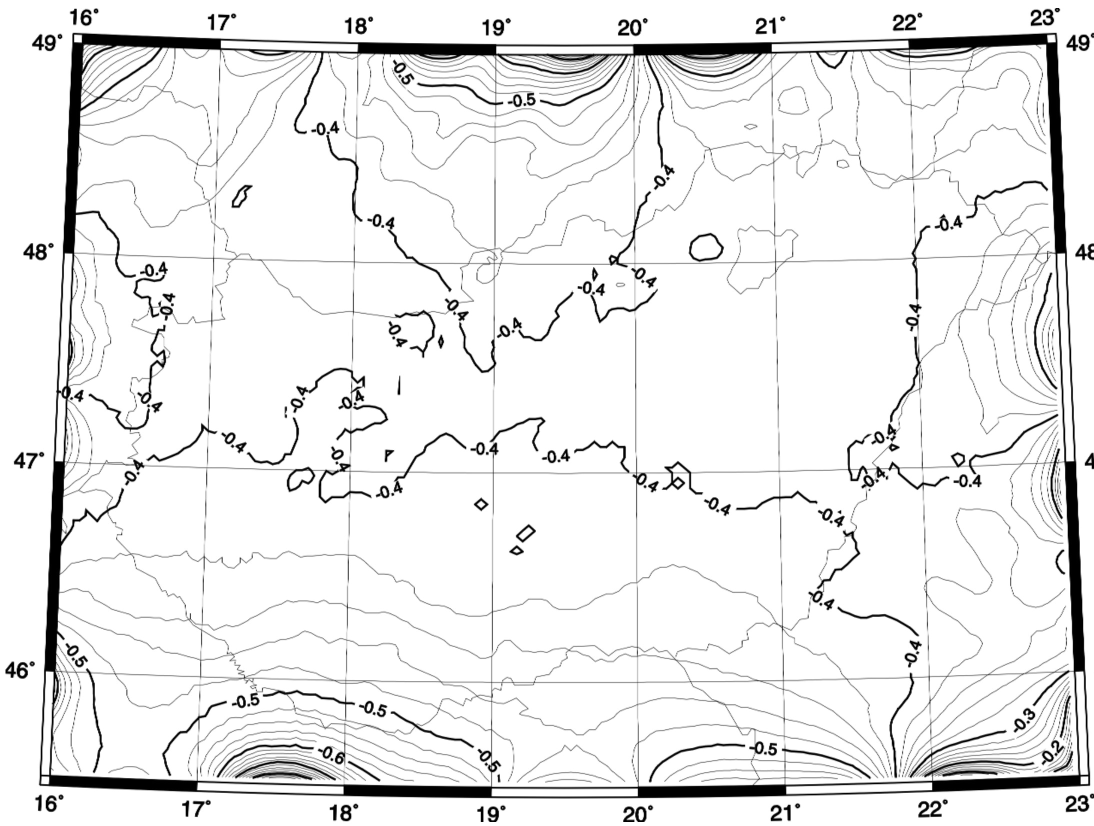

Fig. 2 - Geoid height differences of the FFT and FCOL solution (meter).

1. Mean free-air anomalies for the region of Hungary only from the Etvs Lornd Geophysical Institute’s (ELGI) database and;
2. mean free-air anomalies in the surrounding region from various other data sources.

All the above terrestrial gravity anomalies refer to the International Gravity Standardisation Net 1971 (IGSN71) and to the Geodetic Reference System 1980 (GRS 80).

The gravity data in (1) consists of 13 089 mean 1.5' × 2.5' free-air anomalies, derived from 1.5' × 2.5' mean Bouguer anomalies and elevations. This data set is the same one Hungary provided to the determination of a Precise European Reference Geoid . The estimated rms error is better than 0.1 mGal for all data in the data base (1).

The gravity data in (2) has the following resolution. For Austria, Slovakia, West Ukraine,

Table 2 - Statistics of Faye anomalies.

	max	min	mean	rms	sd
Faye anomaly, mGal	72.821	-79.079	-3.464	13.736	±13.292

Table 3 - Comparison of various geoid solutions in Hungary and with GPS/Leveling data (meter).

	No bias+tilt fit				bias+tilt fit			
Method	**mean**	**min**	**max**	**sd**	**mean**	**min**	**max**	**sd**
FFT°	0.251	0.045	0.750	±0.118	0.000	-0.167	0.338	±0.084
FFT*	0.275	0.137	0.477	±0.072	0.000	-0.098	0.151	±0.056
FCOL°	-0.158	-0.331	0.346	±0.116	0.000	-0.160	0.364	±0.090
FCOL*	-0.135	-0.253	0.052	±0.074	0.000	-0.098	0.127	±0.058
EGG97°	0.244	0.020	0.559	±0.107	0.000	-0.147	0.235	±0.084
EGG97*	0.229	0.020	0.428	±0.093	0.000	-0.127	0.147	±0.060

° 43 GPS/leveling points
* 29 GPS/leveling points

Romania and the former Republic of Yugoslavia the resolutions are 3' × 5', 5' × 7.5', 5' × 7.5', 5' × 7.5' and 5' × 5', respectively. The estimated rms error of the above data is about 3 mGal.

The DTM data used was the same as for our previous solution . The statistics of gravity and DTM grids are presented in Table 1.

3. Methods

In the numerical tests of this study the spectral 1D FFT method and the stochastic fast collocation (FCOL) procedure were used for geoid height calculations. Since both methods are based on a remove-restore procedure, the geoid height computations were performed based on the equations:

$$N = N_{GM} + N_{\Delta g} + N_h, \tag{1}$$

where

$$\Delta g = \Delta g_{FA} - \Delta g_{GM} - \Delta g_h. \tag{2}$$

The 1st term in Eq. (1) gives the contribution of the geopotential model coefficients, while the 2nd term gives the contribution of the reduced free-air gravity anomalies by Eq. (2) with the effects of the geopotential model and the topography removed. The 3rd term gives the contribution of the topography on N (indirect effect). As in our previous geoid solution , the methods we used were the 1D FFT spherical technique and the Fast Collocation (FCOL).

4. Numerical results

Terrain corrections (T.C.) were determined for the whole DTM grid with mass-prism model

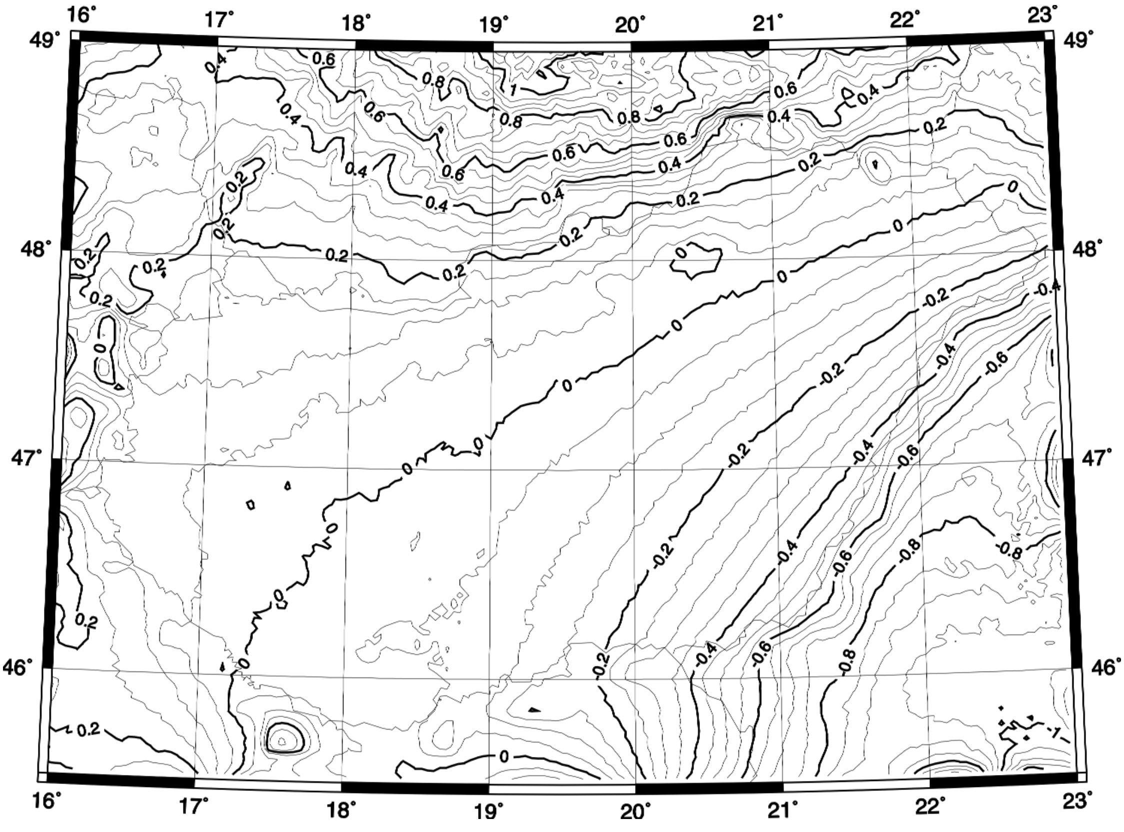

Fig. 3 - Geoid height differences of the FFT and EGG97 solution (meter).

using constant densities up to the 3rd order terms included (for statistics see Tziavos et al., 1998). The Faye anomalies were prepared using the EGM96 geopotential model as a reference and the above terrain corrections. The statistics of these residual Faye anomalies can be found in Table 2. A 27% decrease of the gravity anomaly signal was achieved in terms of standard deviation with respect to the original free-air anomalies. Since the FFT technique requires the rows and columns to be even, the grid of residual Faye anomalies became one row and one column less, i.e. it contains 140 rows 168 columns, 23 520 data.

The 1D spherical FFT method of geoid computation was used with no cap size, and also with 0.5°, 1°, 1.5°, 2° and 3° cap sizes for the continental area above. Fig. 1 shows the final geoid solution for Hungary and the surrounding area with no cap size used. We have evaluated our results with 49 Hungarian EUREF89 stations (Adam and Borza, 1995). For an independent geoid determination we used the Fast Collocation (FCOL) procedure.

The empirical covariance function was determined only from the Hungarian 13 089 residual Faye anomalies (which is better). The resulting geoid solution agreed well with the FFT solution, the standard deviation of differences between the two was only 4 cm. These differences are shown in Fig. 2. Moreover, we have compared our geoid solution in the entire test area to the European geoid recently computed by the Institute of Geodesy, University of Hannover

Table 4 - Comparison of FFT geoid solutions of different cap sizes with GPS/Leveling data, after bias+tilt fit (meter).

	43 GPS/Leveling stations				29 GPS/Leveling stations			
Cap size (degree)	**mean**	**min**	**max**	**sd**	**mean**	**min**	**max**	**sd**
0.5	0.000	-0.187	0.328	±0.097	0.000	-0.138	0.192	±0.080
1.0	0.000	-0.156	0.349	±0.093	0.000	-0.121	0.228	±0.082
1.5	0.000	-0.175	0.277	±0.094	0.000	-0.165	0.127	±0.078
2.0	0.000	-0.164	0.353	±0.089	0.000	-0.088	0.197	±0.063
3.0	0.000	-0.209	0.298	±0.103	0.000	-0.158	0.170	±0.079

(European Gravimetric Geoid 1997, EGG97). The standard deviation of differences was 0.41 m and 0.21 m before and after bias and tilt fit, respectively. The differences are plotted in Fig. 3. The three figures were derived by the Generic Mapping Tools software (GMT) version 3 (Wessel and Smith,1995).

We can see that the differences are reduced from 2 meters down to 1 meter outside the SE borders of Hungary where the resolution of our data was improved with respect of our previous solution (Tziavos et al., 1998). All the statistics were also evaluated for the inner area of Hungary, where we have excluded 14 GPS stations located at the borders of the Hungarian territory. The statistics of these comparisons can be found in Table 3.

As mentioned, we have experimented with various cap sizes. It can be seen clearly from the results of Table 4 that different cap sizes do not improve the solution.

5. Conclusions and discussion

Two gravimetric geoid solutions were computed for Hungary and the surrounding region by FFT- and FCOL-based algorithms using heterogeneous data sources. The results of the two solutions were found to agree at a 4 cm level in terms of standard deviation of the differences, while a significant bias was detected between the two solutions, which reached a level of 88 cm. Using a bias and tilt regression model the aforementioned accuracy dropped to 3 cm. Comparing the geoid heights from both solutions with corresponding heights from the EGG97 European geoid solution a standard deviation of the differences was found close to 20 cm with a bias value close to 27 cm. After applying the above mentioned regression model the standard deviation of the differences decreased to 6 cm.

The comparison of the gravimetric geoid heights with 43 corresponding GPS/leveling heights showed standard deviations of the differences of 14 and 10 cm before and after the fitting with the regression model, respectively. Avoiding several GPS stations at the borders of the Hungarian territory we repeated the same comparisons at 29 GPS benchmarks. The comparisons between the gravimetric and the GPS/leveling heights showed accuracies of differences close to 9 and 6 cm in terms of standard deviation before and after the use of the regression model.

All the accuracies obtained could be further improved if the databases in the neighbouring countries were also updated with respect to accuracy and resolution. Further improvement by using a 500 m × 500 m DTM and 1 km × 1 km Free-air gravity data for Hungary is also possible. In the northern mountainous part of the country a denser GPS network of high accuracy and the upgrading of the existing gravity base could contribute to the improvement of the accuracies obtained.

A numerical test for the gravity data with different cap sizes has not shown any clear improvement in the results. However, more research is needed in this direction by optimally combining the limited cap sizes with modified kernel functions.

It is demonstrated that the geoid solutions, on a national scale, can estimate geoid heights and geoid height differences to an accuracy comparable to that derived by GPS and leveling, at least in the flat areas. The geoid height accuracies obtained in this study meet the requirements of the most current geodetic applications and the demands of a wide variety of engineering, mapping and surveying projects.

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