Bollettino di Geofisica Teorica ed Applicata Vol. 40, n. 3-4, pp. 211-217; Sep.-Dec. 1999

New gravimetric quasigeoid of Slovakia

M. Mojzeš and J. Janák

Departament of Theoretical Geodesy, Slovak University of Technology, Bratislava, Slovak Republic

(Received October 4, 1998; accepted August 5, 1999)

Abstract. The gravimetric model of the Slovak quasigeoid (GMSQ98A) was determined using FFT techniques for numerical integration of Stokes formula. The solution was tested at 42 GPS/leveling points. The 4th degree polynomial model was used for reducing differences. The RMS after reduction of differences, was 6 cm (Mojzeš and Janák, 1998). For the computation of the new gravimetric quasigeoid of Slovakia (GMSQ98BF), the same point gravity data were used but with a 20" × 30" (600 m × 600 m) grid and with a 6th degree polynomial model with 18 coefficients for fitting. After rejecting 4 GPS/leveling points, the RMS value was 2.4 cm.

1. Introduction

The gravimetric computation of the quasigeoid is influenced by errors in the terrestrial gravity data, errors in the reference global model and other errors due to various theoretical approximations. All these errors can be studied by comparisons with GPS/leveling data and appropriate modeling of the residuals (e.g. by surface polynomial fittings).

2. Basic gravimetric and physical characteristics of the area of Slovakia

The area of Slovakia is about 49 000 km^2, and is located between 47.6° N, 49.6° N, 16.3° E, and 22.5° E boundaries. The altitude varies from 100 m to 2660 m above sea-level. The free-air gravity anomalies' range is from -25 to 130 mGal, and the terrain correction reaches values of up to 35 mGal.

Corresponding author: M. Mojzeš; Department of Theoretical Geodesy, Slovak University of Technology, 81368 Bratislava, Slovak Republic, phone: + 421 7 52494401; fax: + 421 7 52925476, e-mail: mojzes@svf.stuba.sk

3. Mathematical model used

For the computation of a gravimetric quasigeoid in Slovakia, a remove-restore technique is used, i.e.

$$\zeta_{GRV} = \zeta_{GM} + \zeta_{REZ} \tag{1}$$

The first part in equation (1) is the contribution of a global spherical harmonic model, and the second part comes from a residual gravity anomaly field (i.e., point free-air gravity anomaly, plus terrain correction, minus global free-air gravity anomaly). The residual height anomaly ζ_{RES} was computed with the formula

$$\zeta_{RES} = \frac{4}{4\pi\bar{\gamma}} \int_s\int \Delta g_{RES}(\varphi,\lambda) S(\psi) ds, \tag{2}$$

where

$$\Delta g_{RES}(\varphi,\lambda) = \Delta g^*(\varphi,\lambda) + \delta g_t(\varphi,\lambda) - \Delta g_{GM}(\varphi,\lambda), \tag{3}$$

Δg^* (φ,λ) is Molodensky's surface free-air gravity anomaly with atmospheric correction, δg_t (φ, λ) is the terrain correction, Δg_{GM} (φ,λ) is the contribution from the global model anomaly, φ is ellipsoidal latitude, λ is ellipsoidal longitude, $\bar{\gamma}$ is a mean normal gravity value and S (ψ) is Stokes' function. For the solution of equation (3), the 2D planar FFT method was used (Schwarz

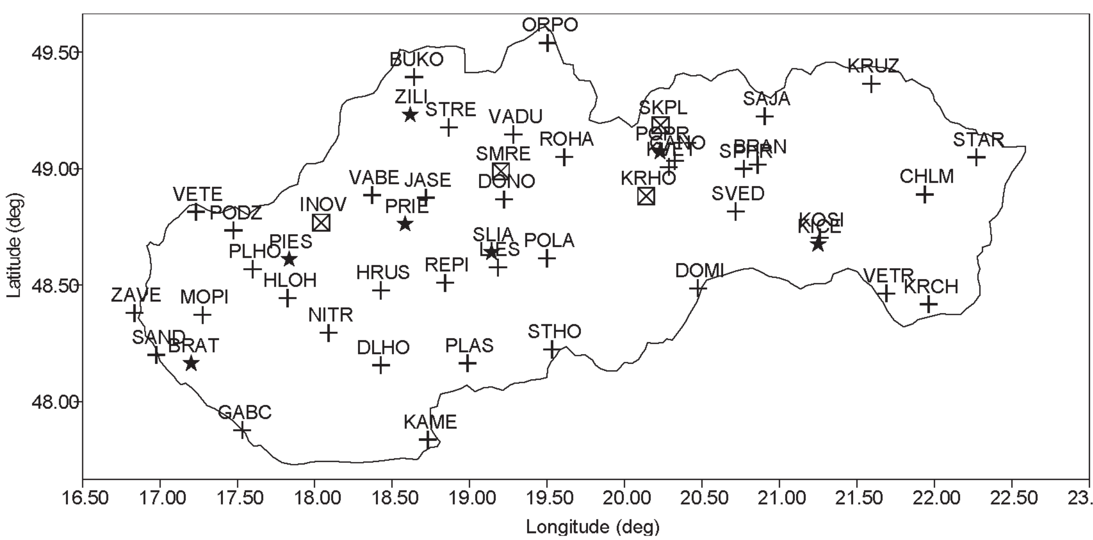

Fig. 1 - Plus marks correspond to SLOVGERENET points used to test GMSQ98B. Marks in squares show rejected points. Asterisks show test points.

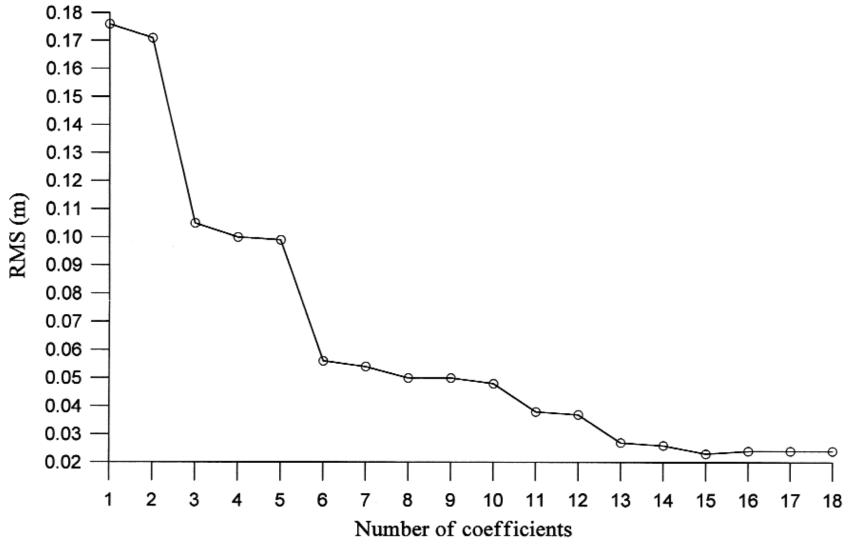

Fig. 2 - Dependence of the RMS of the residual on the number of polynomial coefficients.

et al., 1990)

$$\zeta_{RES} = \frac{1}{\bar{\gamma}} F^{-1}\left(\Delta G(u,v) \frac{1}{\sqrt{u^2+v^2}} \right), \tag{4}$$

where ΔG *(u, v)* is the 2D Fourier transform of the Δg grid, and u and v are the corresponding spatial wavenumbers. The quasigeoid obtained by this method refers to a global datum. Height anomalies at the GPS points were also computed as the difference between ellipsoidal height h_{GPS} and normal height H_{LEV} by

$$\zeta_{GPS} = h_{GPS} - H_{LEV}. \tag{5}$$

The difference in height anomaly computed from GPS/leveling data and from the quasigeoid model

$$\delta\zeta = \zeta_{GPS} - \zeta_{GRV} \tag{6}$$

may be quite large as a result of errors in the used global GM model, in the local gravity data, in the GPS and leveling data, and the datum differences. These differences were modeled using a general polynomial form

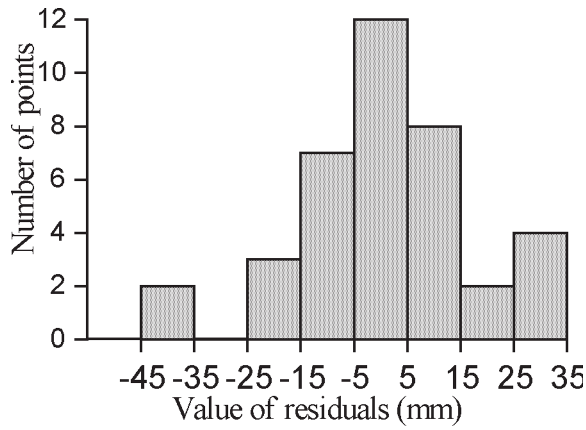

Fig. 3 - Histogram of residuals.

$$\delta\zeta = Q_{0,0} + Q_{1,0}u + Q_{0,1}v + Q_{1,1}uv + Q_{2,0}u^2 + \ldots + Q_{n,n}u^n v^n, \tag{7}$$

where $u=\varphi-\varphi_0$, $v=(\lambda-\lambda_0)\cos\varphi$, and φ_0, λ_0 are ellipsoidal coordinates of the center of the area, and $Q_{n,n}$ are unknown coefficients determined by least squares fitting.

4. Data and software used

The following types of data were used for the computation of the quasigeoid model:
- point free-air gravity anomalies (222 225 points), together with terrain corrections in the area of Slovakia,
- mean Bouguer gravity anomalies and mean heights and 5' × 7.5' grid spacing for areas outside Slovakia up to 100 km from the borders.

The terrain correction was computed separately for each gravity point using a digital terrain model with a 100 m × 100 m grid interval. For point and mean free air gravity anomalies the grid interval 20" × 30" (600 m × 600m) was used. The horizontal position of anomalies was

Table 1 - Values after fitting.

Parameter	Value
Number of points	42
Rejected points	4
Min. residual	-0.044 m
Max. residual	0.035 m
RMS	0.024 m

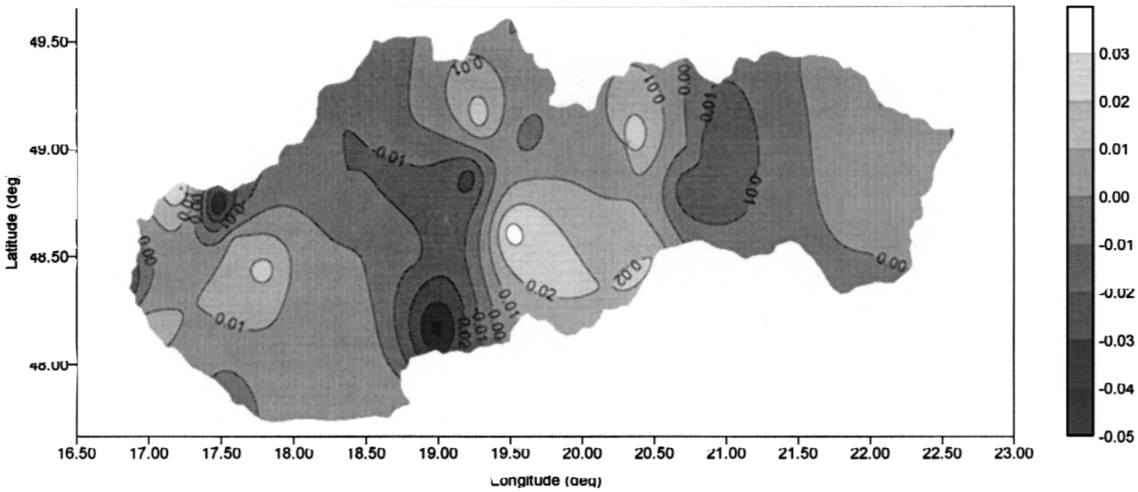

Fig. 4 - Residuals after polynomial fitting. Contour interval 0.01 m.

transformed to the ETRF89 by a linear transformation model. The EGM96 global model, up to degree and order 360 (Lemoine et al., 1996), was used for the computation of the global free-air gravity anomalies. The quasigeoid solution was finally computed using the program “f388b.f” (Rapp, 1994) and software package “GRAVSOFT” (Tscherning et. al., 1994).

5. Computation and fitting of quasigeoid with GPS/leveling heights

The computation principles outlined in Section 2 and the data described in Section 4 have been used for the computation of a new gravimetric model of Slovak quasigeoid (GMSQ98B) in the area between the boundaries 46°50' N, 50°30' N, 15°30' E, and 24°00' E. For the testing of the quasigeoid, 42 GPS/leveled points of the Slovak geodynamic reference network (SLOVGERENET), shown in Fig. 1, were used. The average distance between the points was 45 km. The GPS observations in the SLOVGERENET were performed in 1995, using Trimble dual-

Table 2 - Results of test.

Point	$\zeta_{GPS} - \zeta_{GMSQ98BF}$ (m)
BRAT	0.042
PIES	0.004
SLIA	-0.004
POPR	0.045
KICE	0.030
PRIE	0.002
ZILI	0.027

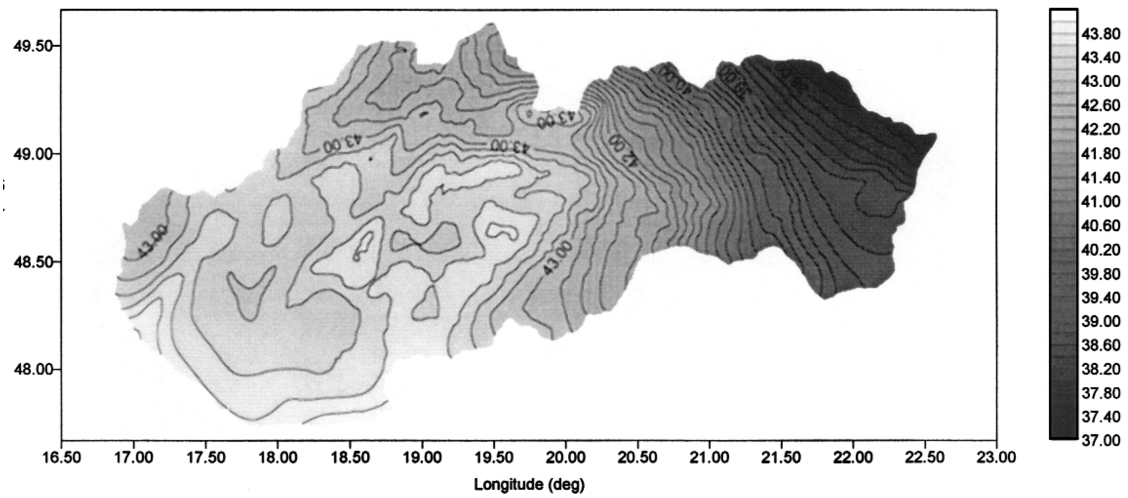

Fig. 5 - Gravimetric model of Slovak quasigeoid - GMSQ98BF. Contour interval 0.2m.

frequency type receivers only. The length of the observation session was 36 hours. The GPS observations were processed using the Bernese, Ver. 4.0 software package (Beutler et al., 1996). The Precise Ephemeris from the IGS/Centre for Orbit Determination in Europe was used for the processing. The computation was performed as a free solution, which was connected to the Wettzell ITRF site. The final solution was obtained by a linear transformation of the free solution to the EUREF89 using the Wettzell, Graz, Onsala, Jezefoslav and Matera ITRF sites. The standard deviation of the ellipsoidal heights was 1 cm.

All 42 GPS points were connected to benchmarks of the Slovak leveling network using precise spirit leveling. The a posteriori standard deviation of the Slovak leveling network is $1.39 mm/\sqrt{km}$ (Seeger, 1997). The degree of the polynomial model used for the modeling of residuals was chosen according to their corresponding RMS value. The dependence of the RMS on the number of coefficients is shown in Fig. 2. Four GPS/leveling points KRHO, SKPL, SMRE and INOV with residual values larger than 2.5* sigma were rejected. Results of this fitting are summarized in Table 1.

The histogram of residuals is shown in Fig. 3 and their distribution on the area of Slovakia is shown in Fig. 4. The final "corrected" quasigeoid GMSQ98BF is shown in Fig. 5.

6. Test of GMSQ98BF

The quasigeoid GMSQ98BF was also tested at 7 GPS points, shown in Fig. 1. The GPS observations were carried out in a 24-hour time interval with a Trimble dual frequency receiver at each point. Every GPS observation was processed by Bernese, Ver.4.0 software together with Graz, Modra-Piesky and Jozefoslav observations.

The GPS points were connected to benchmarks of the Slovak leveling network by precise spirit leveling. The differences between determined and interpolated height anomalies ranged from -0.004 m to 0.045 m. The values of these differences are given in Table 2.

7. Conclusions

The GMSQ98BF is not the ultimate solution for the quasigeoid in Slovakia. Other GPS/leveling points are needed for a more detailed analysis. Major improvements may be reached by removing errors in the gravity data through corrections in old gravity measurements in the area of Slovakia and abroad (in the surrounding countries).

Acknowledgements. The authors gratefully thank the Geocomplex Ltd. Bratislava for the gravity data, the Geodetic and Cartographic Institute Bratislava for SLOVGERENET GPS and spirit leveling data, and the Grant Agency of Slovakia for financial support for this research. The authors particularly thank Professor M.G. Sideris for reviewing the manuscript.

References

Beutler G., Brockmann E., Fankhauser S., Gurtner W., Johnson J., Mervart L., Rothacher M., Schaer S., Springer T. and Weber R.; 1996: *Bernese GPS Software, Version 4.0*. Astronomical Institute of Bern, 1-415.

Lemoine F. F., Smith D. E., Smith R., Kunz L., Pavlis E. C., Pavlis N. K., Klosko S. M., Chinn D. S., Torrence M. H., Williamson R. G., Cox C. M., Rachlin K. E., Wang Y. M., Kenyon S. C., Salman R., Trimmer R., Rapp R. H. and Nerem R. S.; 1996: *The Development of NASA GSFC and NIMA Joint Geopotential Model.* Proc. Symp. On Gravity, Geoid and Marine Geodesy, Tokyo, 461-469.

Mojzeš M. and Janák J.; 1998: *Gravimetric Model of Slovak Quasigeoid.* Reports of the Finnish Geodetic Institute, 98; **4**, 277-280.

Rapp R. H.; 1994: *The Use of Potential Coefficient Models in Computing Geoid Undulations.* International School for the Determination and Use of the Geoid, Milan, 71-100.

Seeger H.; 1997: *The Current Status and Perspectives of EUREF.* Reports on Geodesy, **3** (26), 31-76.

Schwarz K. P., Sideris M. G. and Forsberg R.; 1990: *The use of FFT Technique in Physical Geodesy.* Geophys. J. Int., **100**, 485-514.

Tscherning C. C., Forsberg R. and Knudsen P.; 1992: *The GRAVSOFT Package for Geoid Determination.* Proc. First Continental Workshop on the Geoid in Europe, Prague, 327-334.