Two-step data analysis for future satellite gravity field solutions: a simulation study

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Abstract. A two-step approach for gravity field recovery from future SST- or SGG-missions is discussed, where space-localizing base functions serve for modelling the anomalous field. Results of a simulation study regarding scenarios of CHAMP type (high-low GPS-SST) and GRACE type (high precision low-low SST) are presented.

1. Introduction

The task of the envisaged SST/SGG-missions like CHAMP, GRACE, and GOCE is the computation of a global gravitational field with high resolution and precision and - if possible - with repetition in time. Global recovery approaches are aimed at the computation of spherical harmonic models. But open questions are related to a violation of an ideal data coverage or, to not well-defined boundary surfaces. The global support of the spherical harmonics does not allow us to adapt precision to areas of geodynamical interest or to time-dependent phenomena.

The authors focus on a two-step approach for satellite data analysis (Fig. 1): first, the true gravitational field is approximated by space-localizing kernel functions. In combination with terrestrial data, this could supplement the satellite derived regional field information at this level. In a second step, whether the "regional" solution covers the whole earth, or independent regional solutions are merged into a global one in an appropriate way, spherical harmonic coefficients may be derived by a simple summation process.

2. Modelling

The primary observation, Eq. (1), used in this study is based on the eigenfunction expansion (Schneider 1984) of the intersatellite or gradiometry baseline Eq. (2)

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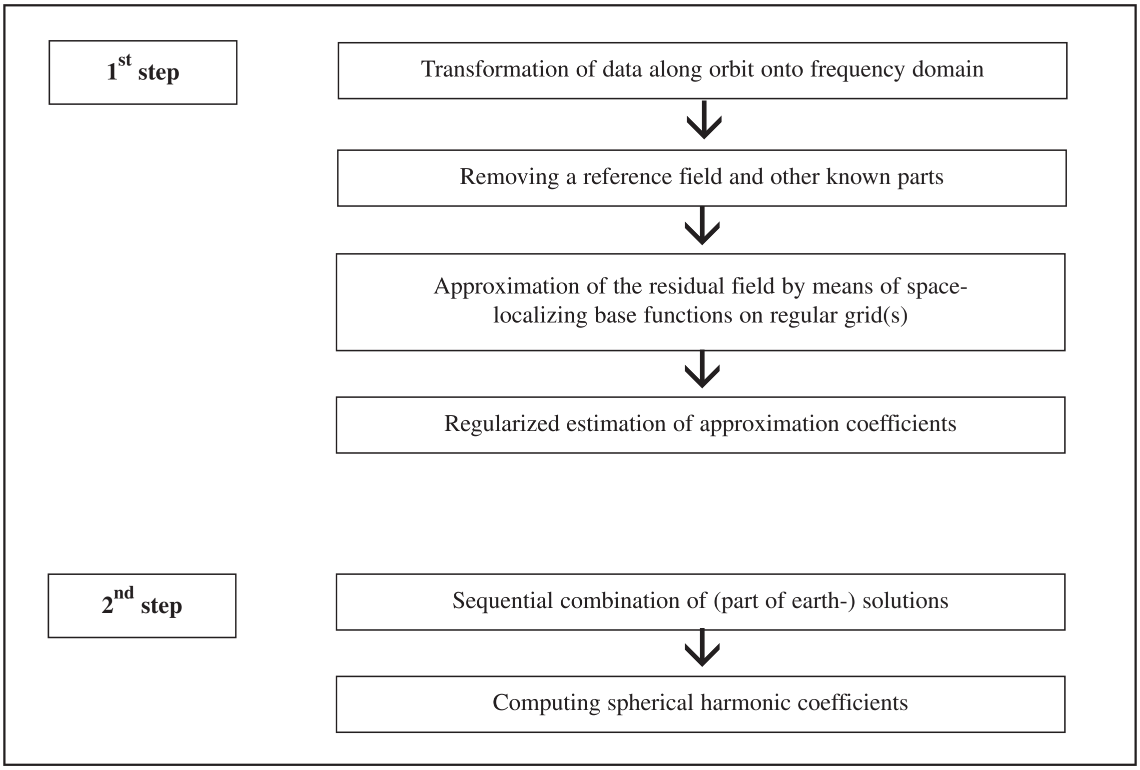

Fig. 1 - Scheme of two-step analysis.

$$r_\nu = -2\left(\frac{\Delta t}{\nu\pi}\right)^2 \int_0^1 \sin(\nu\pi\tau')\left(\frac{1}{r}(|\dot{r}|^2 - \dot{r}^2) + e \cdot D(V_{ref} + T)\right) d\tau', \tag{1}$$

$$r(\tau) = \bar{r}(\tau) + \sum_\nu r_\nu \sin(\nu\pi\tau), \tag{2}$$

where

$$D_{SST} = \nabla_2 - \nabla_1, \quad D_{SGG} = r \cdot \nabla\nabla. \tag{3}$$

These (pseudo-) observations can be computed from original SST range r, range-rates $\dot{r}$, or SGG differential acceleration data $\ddot{r}$ by numerical integration

$$r_\nu = 2\int_0^1 \sin(\nu\pi\tau)\left(r(\tau) - \bar{r}(\tau)\right) d\tau \tag{4}$$

$$r_\nu = 2\frac{\Delta t}{\nu\pi}\int_0^1 \cos(\nu\pi\tau)\dot{r}(\tau) d\tau \tag{5}$$

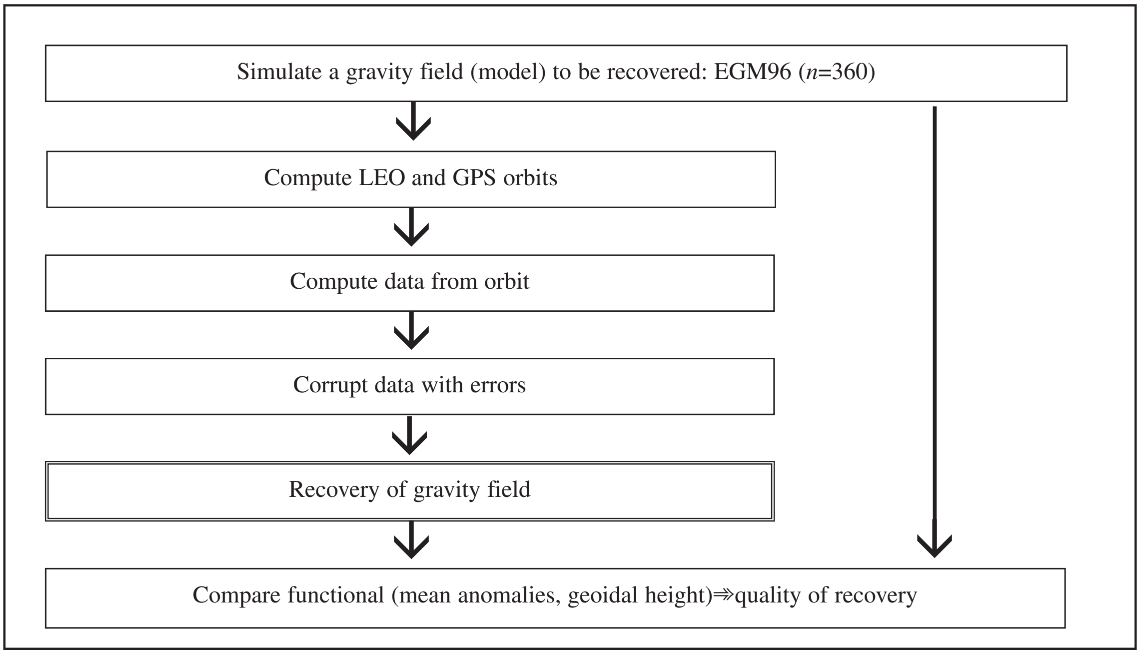

Fig. 2 - Scheme of simulation study.

$$r_\nu = -2\left(\frac{\Delta t}{\nu\pi}\right)^2 \int_0^1 \sin(\nu\pi\tau)\ddot{r}(\tau)d\tau, \tag{6}$$

allowing a simple common processing of different data types as well as a certain degree of data compression (Ilk et al 1995). On the model side, the anomalous field T is approximated using isotropic space-localizing base functions B_j

$$\tilde{T} = \sum_j \chi_j B_j, \quad B_j(Q) = \sum_{n=0}^{\infty} \frac{2n+1}{4\pi} b_n P_n(Q,Q_j), \tag{7}$$

which unlike spherical harmonics are non-orthogonal, i.e. they possess a full Gram matrix $(B_j, B_k)=(\boldsymbol{P})_{jk}$. Noisy observations are considered as bounded linear functionals in a reproducing kernel Hilbert space H,

$$l^j + \varepsilon^j = A^j T = (A_j, T)_H. \tag{8}$$

Minimizing a weighted sum of least-squares (from data noise as well as projection error) and H-norm of T

$$F_{\gamma^2} = \left\| A_\chi - l \right\|_{P_{ll}}^2 + \gamma^2 \left\| \tilde{T} \right\|_H^2, \tag{9}$$

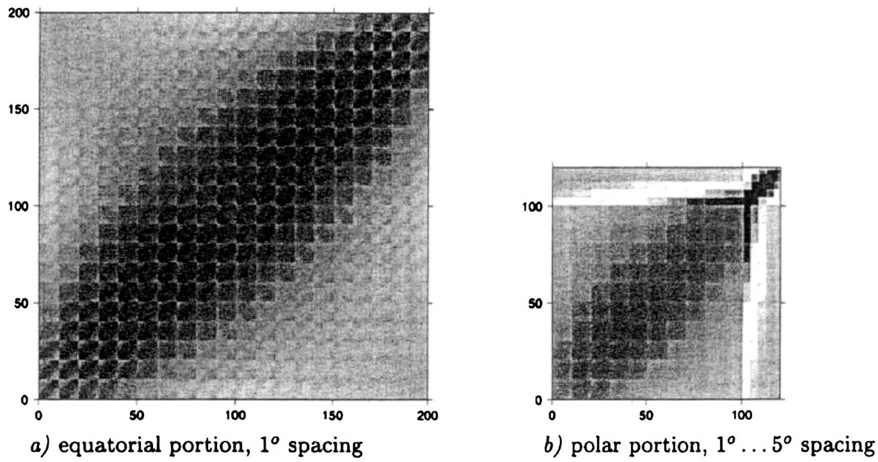

Fig. 3 - GRACE normal equation matrix.

leads to the well-known Tykhonov-regularized solution

$$\hat{\chi} = \left(A^T P_{ll} A + \gamma^2 P\right)^{-1} A^T P_{ll} l \tag{10}$$

with variance-covariance matrix

$$\hat{C}_{\hat{\chi}\hat{\chi}} = \hat{\sigma}^2 \left(A^T P_{ll} A + \gamma^2 P\right)^{-1} A^T P_{ll} A \left(A^T P_{ll} A + \gamma^2 P\right)^{-1}. \tag{11}$$

Finally Eq. (7) implies that spherical harmonic coefficients for $\tilde{T}$ are given by

$$\tilde{T}_{nm} = (\tilde{T}, Y_{nm})_{L^2(\Omega)} = b_n \sum_j \chi_j Y_{nm}(Q_j). \tag{12}$$

3. Simulation study

The concept described above was verified in a simulation study regarding two idealized SST mission scenarios. Fig. 2 provides a sketch of the simulation principle.

The basic mission configurations were:

1. GRACE scenario: 2 LEO's, baseline 300 km, 25 GPS satellites;
2. CHAMP scenario: 1 LEO, 25 GPS satellites. A LEO orbit was chosen by a=6 732 266.20 m, e=0.001, i=97.29°, mean altitude 354 km. According to the "snapshot" objective of GRACE, a mission duration of 31 days was simulated. The white noise level was σ=1 μm/s for LEO-

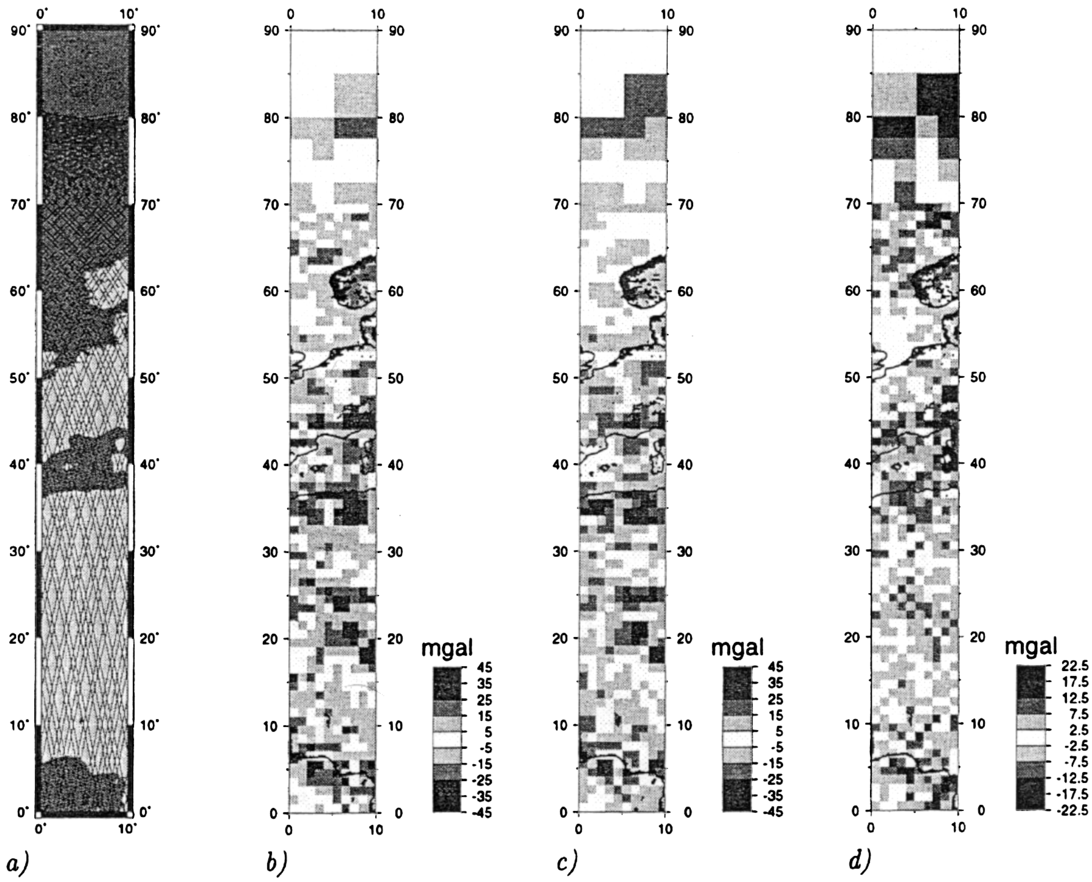

Fig. 4 - Recovery of 1° ... 5° mean gravity anomalies from GRACE simulation.

LEO range-rates, σ=3 *cm* for LEO-GPS ranges. All orbits were integrated numerically applying the EGM96 gravity field model. The area under consideration was $(\lambda, \varphi) \in$ [0°, 10°] × [0°, 90°] with base function grid spacing of 1° × 1°.

Fig. 3 shows *a)* an "equatorial portion" as well as *b)* a "polar portion" of the resulting GRACE normal equation matrix, the greyscale corresponding to $|(\boldsymbol{A}^T \boldsymbol{P}_{ll} \boldsymbol{A})_{ij}|$. Clearly visible is the stabilizing effect of the broadened grid spacing in the northernmost area.

Fig. 4 illustrates *a)* the LEO ground track pattern, *b)* mean residual gravity anomalies computed from the EGM96 model with reference to the low-degree model, *c)* the recovery result from the GRACE scenario, and *d)* absolute deviations. In Fig. 5 results are displayed for the idealized CHAMP mission: *a)* gives the 5° × 5° mean anomalies from the EGM96 input, *b)* the recovery result, and *c)* absolute deviations.

4. Conclusions

Aiming for a spatial resolution of 1° × 1° (100 *km*) in a GRACE/31*days* recovery simulation,

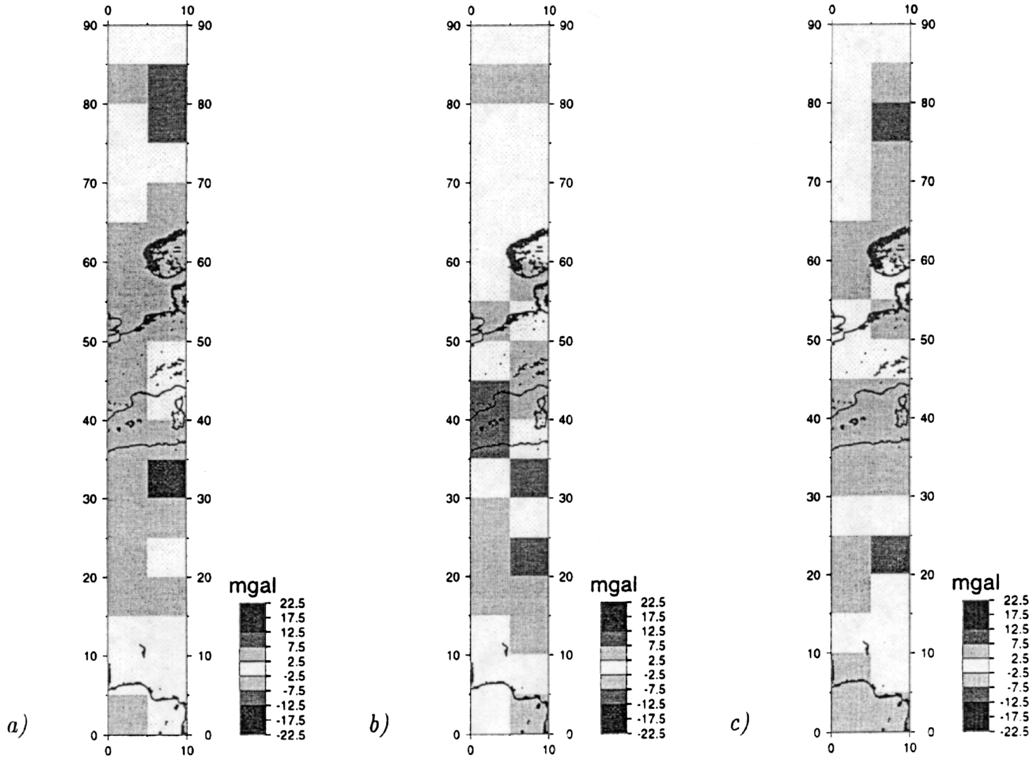

Fig. 5 - Recovery of 5° mean gravity anomalies from CHAMP simulation.

we found an overall rms value of 6-7 mGal from the total deviations. The accuracy degrades in areas of a rough gravity field, as clearly indicated in Fig. 4d. When removing features of less than 300 *km* from the EGM96 anomalies by truncating the expansion at n=120, we found the deviations distributed randomly with an rms value of 3-4 mGal. For the CHAMP scenario we had an rms value of 4 mGal for 5° mean anomalies.

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