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Improvement of Japanese geoid with 1D-FFT method and its comparison with altimetry-derived geoid

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Abstract. An improvement of the Japanese gravimetric geoid over a pre-existing model, JGEOID93, is studied with the 1D-FFT method as a strict realization of Stokes' spherical integration in a remove-restore manner and in consideration of possible mean offset errors in the network-adjusted ship gravity data used. Simulation shows that the offset errors of about 4.9 mGal in a limited distribution in space, result in geoid errors of a few meters of amplitude in the medium wavelength, a situation which should not be ignored. The differences between the 1D-FFT and single-band 2D-FFT geoids range from 15 cm to – 25 cm with a 2-3 cm RMS. We prepare an approximated error response function of Stokes' kernel, which well indicates the geographical distribution of the differences. The 1D-FFT geoid obtained, provides better accuracy in magnitude and a slight improvement in precision, in comparison with a nation-wide network and several local networks of GPS at benchmarks. Comparison of the gravity field and geoid with an altimeter-derived global model (Sandwell and Smith, 1997) reveals the existence of significant discrepancies in the southeastern edge of the Sea of Japan, which is attributed to the altimeter model errors.

1. Introduction

Since Japan and its surrounding sea are located in an active region, tectonically and oceanographically speaking, a precision geoid of Japan is strongly demanded by both the geodetic and the oceanographic communities. Kuroishi (1995) determined the gravimetric geoid of Japan on a 3'×3' grid, JGEOID93, with reference to the OSU91A geopotential model (Rapp et al., 1991) by a spherical 2D-FFT method (Strang van Hees, 1990) with surface gravity data: about 38 000 data on land and about 480 000 at sea. In comparisons with GPS ellipsoidal heights on benchmarks, JGEOID93 has an accuracy better than 10 cm in wavelengths shorter than 15 arc

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degrees and consistent discrepancies exist in the approximate form of tilted planes of 2-4 ppm down in the east, suggesting long wavelength errors.

An improvement study of the geoid has been underway. As an initial approach, Kuroishi (1998) evaluated the performance of a newer global geopotential model, EGM96 (Lemoine et al., 1997), as the reference model over Japan. Comparisons with GPS/leveling geoidal heights reveal the following facts: the EGM96 geoid holds superiority in short wavelength resolution to OSU91A over wide areas in Japan, but not always over smaller areas, and no substantial improvement comes out in the replacement of reference models, in the area covered by the surface gravity data.

Further steps are discussed in this paper. The items considered are as follows:

1. possible mean offset errors in the network-adjusted ship gravity data, and
2. removal of an approximation of the spherical Stokes, integral kernel in the 2D-FFT method.

The former errors are specific to the adjustment method. Altimeter-derived gravity field models are of interest in controlling the quality of ship gravity data. Here only a global model by Sandwell and Smith (1997) is compared with the surface data used.

2. Method of geoid determination

The basic method of geoid determination in a remove-restore manner is the same as that in Kuroishi (1998): Stokes' integral is processed for residual geoid recovery in the region, by either 2D or 1D-FFT method, with reference to EGM96. Fay anomalies are the input gravity anomalies in the Molodenskii sense and the indirect effect induced is corrected with Grushinsky's formula. The singularity of Stokes' kernel at the origin is handled by computing the contribution of gravity anomaly at the innermost cell separately.

GRS80 (Moritz, 1980) is employed as the normal Earth model and ITRF89 (Boucher and Altamimi, 1991) co-ordinates are used as a geocentric system. The same co-ordinate conversion method as that in Kuroishi (1995) is applied to the data. The differences in semi-majors and geocentric gravitational constants between GRS80 and EGM96 are accounted for according to Smith and Milbert (1997), in the computation of the reference model anomaly/geoid.

3. Geoid improvement

As mentioned in the introduction, the mean offset errors are first evaluated. In the JGEOID93 determination, a simultaneous network adjustment was performed on the whole ship gravity data with crossover errors (Kuroishi, 1995). A linear drift model in time was assumed. The absolute gravity values are anchored to a few cruises that contain four or less crossover points.

When compared with EGM96 gravity anomalies, the mean offset of the ship data is estimated at about - 4.9 mGal. The effect of the offset can be evaluated by geoid computation with simulated data or, by comparison between geoid models with, and without the offset correction. Both results agree with each other.

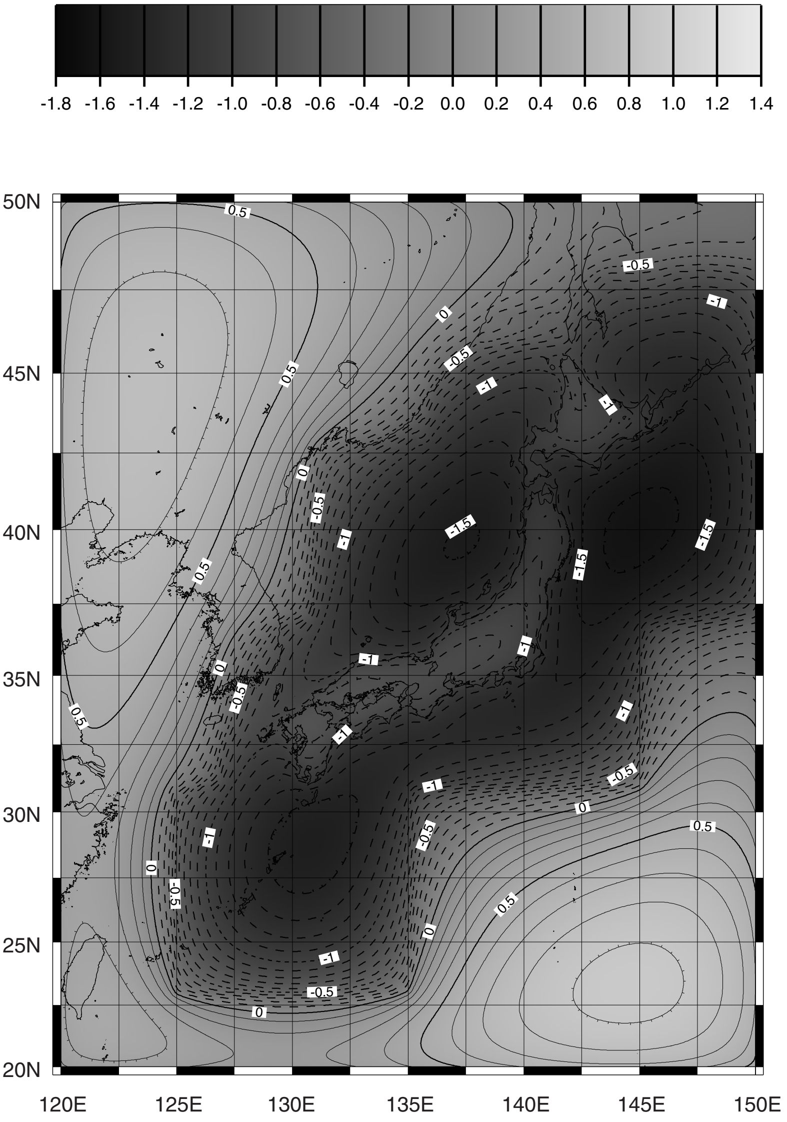

Fig. 1 - Geoid changes associated with gravity-offset corrections. Contour intervals are 0.1m. Unit in m.

The geoid differences given by the latter result are shown in Fig. 1. The ship data distribution determines the geographical pattern of errors. We can easily recognize the boundary of ship data coverage. The errors range from – 1.2 m to – 0.8 m inland, – 1.3 m to – 0.8 m at the coast, and – 1.6 m to + 0.9 m at sea. We do not say this offset estimation is perfect, but the correction should be included, since such a big amplitude of geoid errors cannot be ignored.

In the 2D-FFT method, a spherical distance between the point of interest and a gravity anomaly cell is approximately calculated with the mean latitude of the area. When the major term only is taken into account, differentiation yields a kind of response function of Stokes' kernel errors for residual gravity anomalies:

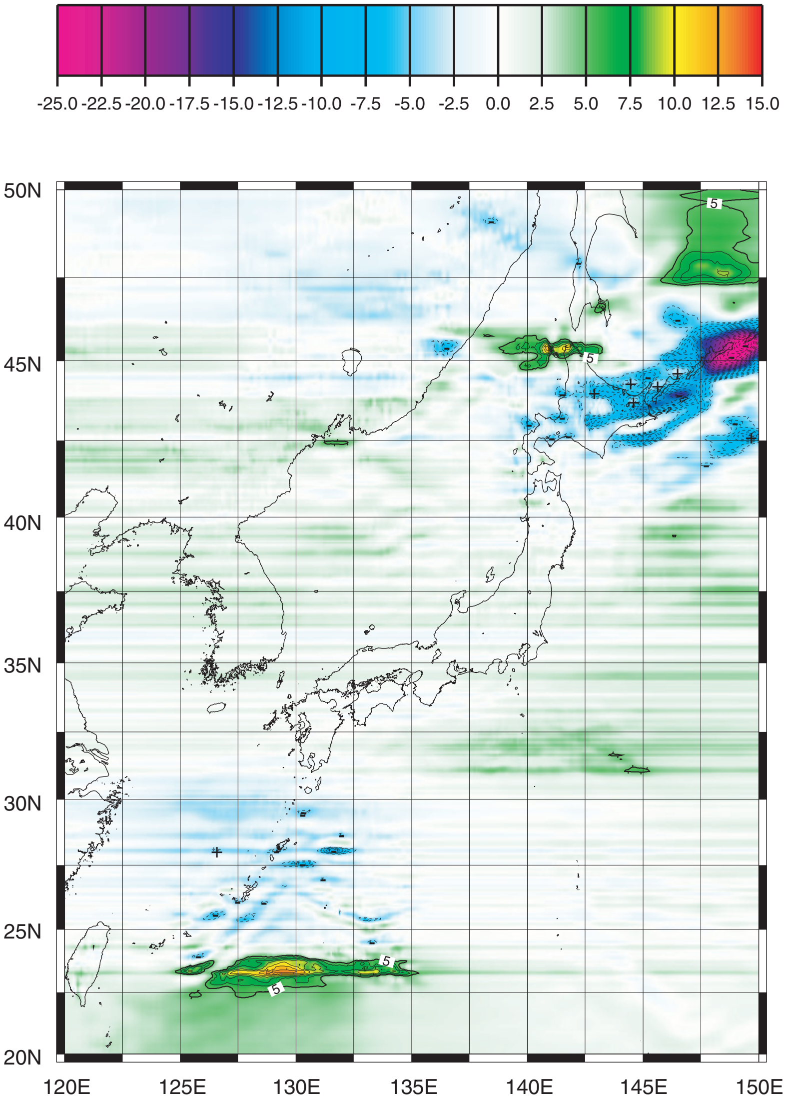

Fig. 2 - Geoid differences between 2D-FFT and 1D-FFT geoids. Unit in cm.

$$\Delta S\cos\varphi_Q = -\frac{\sin^2\dfrac{\Delta\lambda}{2}\cos\varphi_Q\left(\cos^2\dfrac{\varphi_P+\varphi_Q}{2}-\cos^2\varphi_M\right)}{2\left(\sin^2\dfrac{\Delta\varphi}{2}+\sin^2\dfrac{\Delta\lambda}{2}\cos\varphi_P\cos\varphi_Q\right)}$$

where ΔS is the error of Stokes' function, $\Delta\varphi$ and $\Delta\lambda$ are grid intervals in latitude and longitude, φ and λ with a subscript P or Q are geodetic latitude and longitude at the point of interest, P, or a gravity anomaly cell, Q, and φ_M stands for the mean latitude. Although it is not strict, this

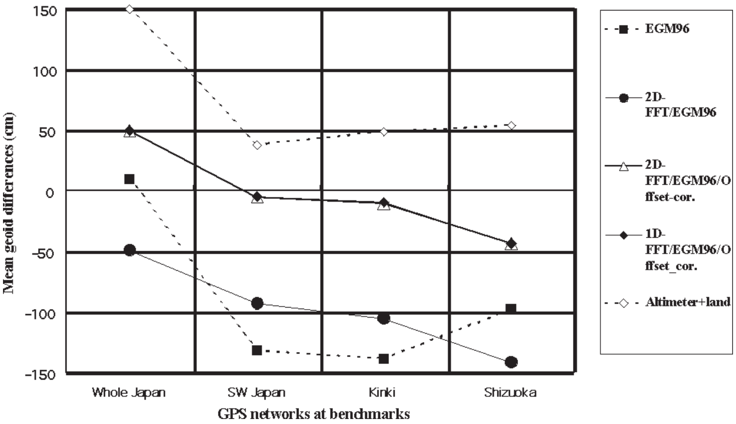

Fig. 3 - Mean geoid differences between models and GPS leveling data.

equation gives geographical features of error propagation: residual gravity anomalies within about 5 degrees around the point have significant contributions where the point is located away from the mean latitude, and the response function has positive values in the northern part and negative ones in the southern part.

The errors induced by the Stokes, kernel approximation in the 2D-FFT method are evaluated by comparing it with the 1D-FFT geoid (Haagmans et al., 1993). The geoid differences are presented in Fig. 2. Solid contour lines and broken ones are plotted for the absolute magnitudes of 5 cm or larger in the positive and negative values, respectively. The differences range from 15 cm to – 25 cm with a 2-3 cm RMS. Looking at the Japanese islands, errors exceed 5 cm only in east Hokkaido and at the north edge of Hokkaido. The geographical distribution of the geoid differences in Fig. 2 is well understood from the response function features and the residual gravity distribution.

The geoid improvement can be evaluated by external comparisons with GPS/leveling geoid undulations. Available GPS networks are the same as those in Kuroishi (1998): one national network of 806 sites (Kuroda et al., 1997) and five local networks of different area sizes. GPS ellipsoidal heights are determined absolutely in the national network and three local ones: SW Japan, Kinki, and Shizuoka. The mean differences are shown in Fig. 3. Fig. 4 presents the standard deviations (SDs) of the differences after best-fit-planes are removed to exclude long wavelength errors in the geoids.

It can be clearly seen that the offset corrections significantly improve the absolute values of the geoid. The mean differences decrease by 6-31 % in the three local networks, and the SD after plane-fitting decreses by 22 % in the national network. The removal of the kernel approximation also leads to improvements: the mean differences decrease by 1-16 % and the SDs after plane-fitting by 1-5 % at the three local networks. The SD after plane-fitting has no substantial change in the national network.

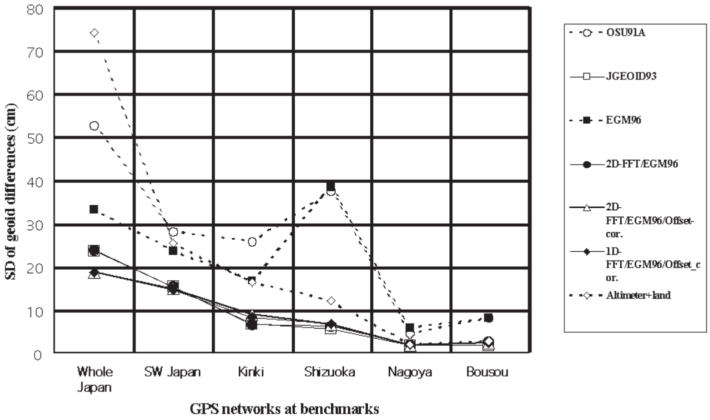

Fig. 4 - Geoid differences between models and GPS leveling data after best-fit-plane removed.

4. Comparison with an altimeter-derived model

The network-adjusted ship gravity data not only contain offset errors, but also have heterogeneity in both coverage and accuracy. Continual satellite altimeter missions, on the other hand, have been providing homogeneous information on ocean gravity with improved resolution and accuracy. Because of the nature of sensing techniques, the gravity information has weaknesses close to coastlines.

As an initial step to include altimeter-derived gravity, the quality of an altimeter-derived global gravity grid by Sandwell and Smith (1997) was assessed by comparing it with the surface gravity grid (Kuroishi, 1998). The altimeter grid shows large deviations off the coast: several tens to a hundred of mgal too large to the west-off the coastline along the Sea of Japan, and several tens of mgal too small to the east-off the coast along the Pacific Ocean in east Japan.

The altimeter data are considered of lower quality in those areas, but the widths of largely deviated areas are wider than expected. When compared with the national GPS/leveling geoid undulation data, a FFT geoid from the altimeter gravity grid with land gravity data on the Japanese islands shows discrepancies of the peak-to-peak amplitude of about 3 m from the west to the east coast in north Japan. This suggests that the gravity errors may be attributed to the altimeter data. We should be careful in handling the altimeter gravity data in such areas for geoid improvement studies.

5. Conclusions

An improvement of the Japanese geoid over the pre-existing model, JGEOID93, is discussed. Offset errors in the network-adjusted ship data are estimated at about 4.9 mGal by comparison with the EGM96 gravity field. This amount in the offset errors produces geoid undulation errors of a few meters: -1.2 m to – 0.8 m inland, - 1.3 m to – 0.8 m at the Japanese coast, and – 1.6 m to + 0.9 m at sea. Errors induced by the Stokes' kernel approximation in the 2D-FFT method range from 15 cm to – 25 cm with a 2-3 cm RMS. Looking at the Japanese islands, errors exceed 5 cm only in East Hokkaido and on the north edge of Hokkaido. The geographical distribution of the errors is well understood from the approximated error response function proposed and the residual gravity anomaly field.

The external evaluation of the geoid improvement is performed by comparisons with the national network and five local networks of GPS at benchmarks. The offset corrections to the ship gravity data yield a 6-31% improvement in the mean geoid undulations at three local networks where absolute ellipsoidal heights were determined, and a 22% reduction of the SD differences around the best-fit-plane in the national network. The removal of the kernel approximation also gives slight improvements: decreases of 1-16% in the mean differences, and of 1-5% in the SD differences around the best-fit planes at the three local networks. In the national network, this removal provides no substantial change in geoid undulations at short wavelengths.

The quality of an altimeter-derived gravity grid by Sandwell and Smith (1997) is assessed over Japan. The grid shows large deviations off the coast: several tens to a hundred of mgal too large west-off the coastline along the Sea of Japan, and several tens of mGal too small east-off the coast along the Pacific Ocean in east Japan. The errors are attributed to the altimeter grid when compared to the national network of GPS/leveling. We should be careful when including the altimeter-derived grid in the geoid determination for Japan, although altimeter data provide homogenous information on gravity with improving accuracy.

Further study is expected, especially in the expansion of gravity data coverage over the ocean.

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