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Combining gravity anomalies and deflections of the vertical for a precise Austrian geoid

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Abstract. The combination of a rich geomorphology which includes lowlands as well as high mountains and the high quality of the available source data proves Austria to be an excellent test area for geoid computations. A dense field of gravity anomalies and deflections of the vertical together with a high-resolution height model are the ideal base to study different algorithms and methods for geoid computation. The following case study, within the project of the recomputation of the Austrian GEOID2000, investigates the combination of different data-sources for a high-resolution geoid for areas with rough topography. The computations are performed using Least Squares Collocation (LSC) and the remove-restore technique.

1. Introduction

The following geoid computation is done by LSC. The given gravity anomalies are used to determine the parameters of the covariance function. This covariance function is used to compute a purely gravimetric as well as a purely astrogeodetic geoid of Austria. The accuracy of each solution is checked by fitting the geoid model to 40 GPS-reference points. The advantage of combining the gravimetric and the astrogeodetic solution is subsequently shown.

The computation is performed by applying the well-known remove-restore technique. The main idea of the remove-restore procedure is to model the long and short wavelengths of the gravity field and to subtract their influence from the measurements.

Long wavelengths are modelled using an earth gravity model. In order to remove short wavelengths, a high resolution height model is needed. Geoidal heights are computed based on the remaining residual field of gravity anomalies or deflections of the vertical. Finally, the effect of the removal is restored to the geoidal heights.

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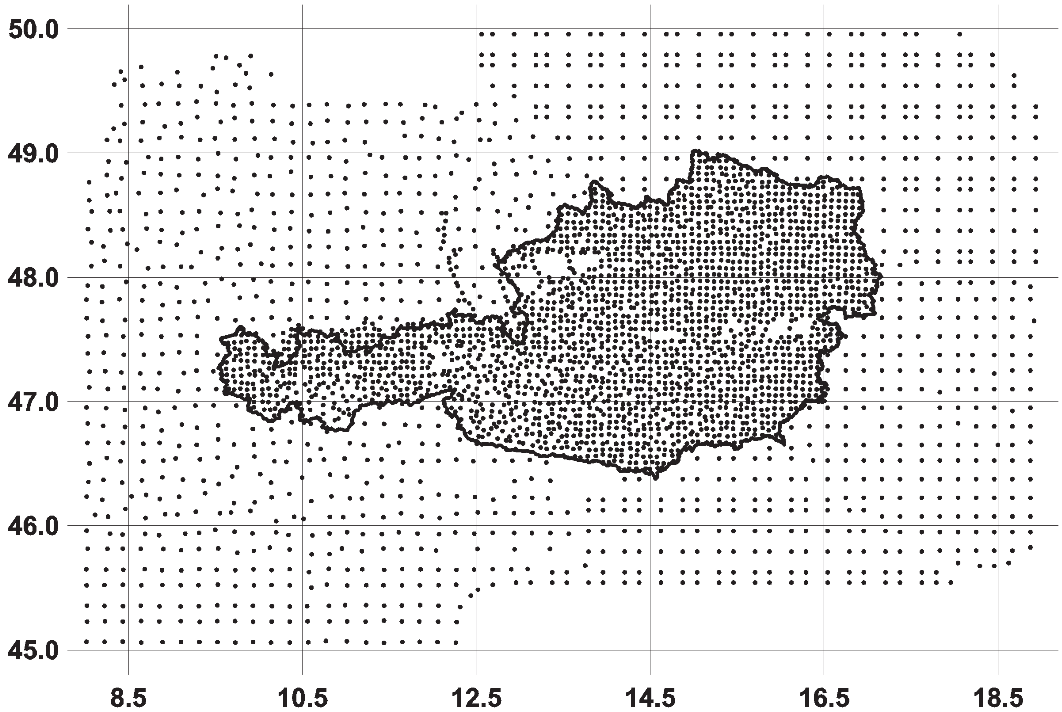

Fig. 1 - Gravity anomalies used for the computation.

2. Gravimetric geoid using LSC

In the following, the steps for the gravimetric geoid computation by LSC are given. As the LSC method is not described in full detail, the reader is referred to Moritz (1980) for the theoretical background, and to Sünkel (1987) for the application of the formulas for the geoid computation. A purley gravimetric geoid solution using LSC was already presented by Kühtreiber and Rautz (1996).

2.1. Gravity anomalies

A data set of 3671 gravity anomalies is used. Fig. 1 shows the data set which includes irregularly distributed point values and gridded mean values. Within Austria, a sample of 2397 point gravity anomalies, out of all known gravity points, was selected to get a coverage of approximately one point per 6 km × 6 km. Outside Austria, the mean distance between neighbouring data points was extended to 16 km.

The actual amount of data for Austria has increased since this test data set was established. The improved data set will be used for the final computation of Geoid2000. Italy, Switzerland and Germany provided point values, while Hungary, Czech Republic, Slovakia and Slovenia provided mean values with different block sizes. The difference between point values and mean

Table 1 - Gravity reduction using the standard density of 2.67g/cm^3 and the geopotential model EGM96. Anomalies are given in mGal. Statistics based on 3671 points.

	Min	Max	Mean	Std.dev.
Δg_{FA}	-144.3	200.7	9.2	±43.3
$\Delta g_{EGM\text{-}red}$	-217.6	133.8	-14.2	±38.7
Δg_{ISO}	-181.6	38.7	-18.9	±24.9

values was neglected. In fact, the quality of the Slovenian data needs to be especially improved for the final geoid solution of Geoid2000.

2.2. Gravity reduction

In the remove-restore concept, the effect of an earth gravity model and the effect of the topography are removed from the observed gravity by

$$\Delta g_{ISO} = g + F - \gamma - \Delta g_{EGM} - \Delta g_{TI}, \tag{1}$$

with Δg_{ISO} , the topographic-isostatically reduced gravity anomaly, derived from the measurements g by applying the free-air reduction F, the reduction due to an earth gravity model Δg_{EGM} and the topographic-isostatic reduction Δg_{TI} . The normal gravity computed on the ellipsoid of GRS80 is denoted by γ. According to Heiskanen and Moritz (1967), $g + F - \gamma$ is called free-air anomaly Δg_{FA}.

The earth gravity model EGM96 was used to compute Δg_{EGM}. The topographic-isostatic reduction Δg_{TI} was computed by an adapted version of TC from Forsberg (1984). The digital elevation model used as input for TC was established by the Federal Office of Metrology and Surveying for the project GEOID2000. The elevation model is based on grids with different resolutions determined photogrammetrically. The grid distances of these grids vary from 30 m × 30 m, for rough topography to 160 m × 160 m in flat areas. From these base models, a digital elevation model was derived with the uniform resolution of 1"40625 × 2"34375, or approximately, 44 m × 49 m, see Graf (1996). Table 1 shows that the standard deviation of the free-air anomalies is lowered considerably to ±24.9 mGal by the reduction process. Nevertheless, a trend remains in the isostatic anomalies. It has its origin in the incomplete isostatic compensation of the Eastern Alps.

2.3. Gravimetric geoid by LSC

Covariance function. - The well-known Tscherning-Rapp covariance function model was used for the following LSC solutions. The global covariance function of the gravity anomalies

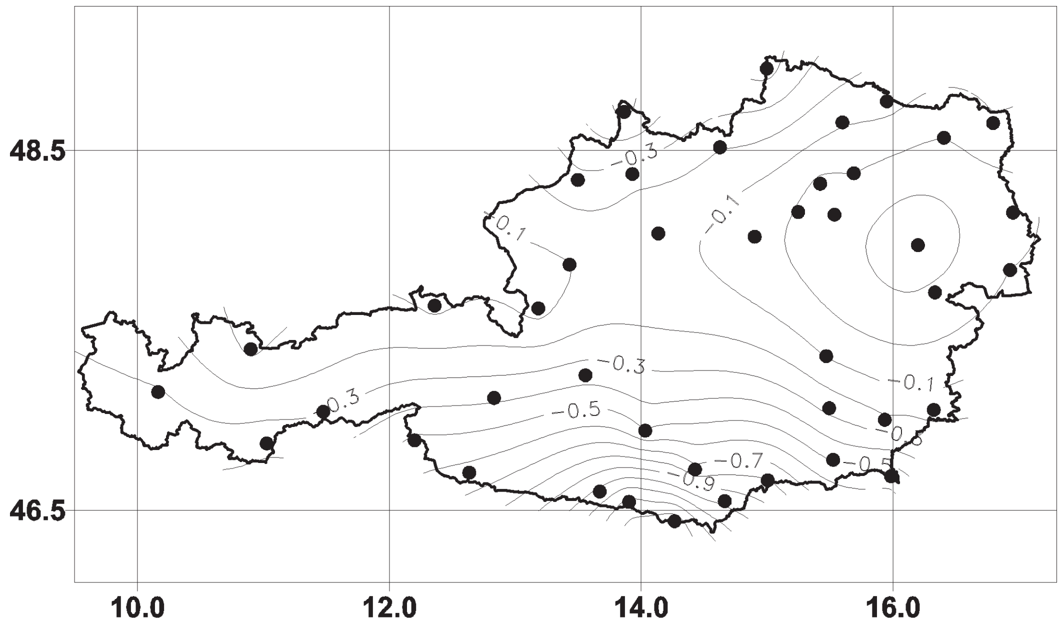

Fig. 2 - Difference between the gravimetric geoid and the geoid obtained by GPS and levelling. Black dots mark the 40 GPS-reference points. Contour interval 0.1 m.

$C_g(P,Q)$ given by Tscherning and Rapp (1974, p. 29) is written as

$$C_g(P,Q) = A\sum_{n=3}^{\infty}\frac{n-1}{(n-2)\ (n+B)}\ s^{n+2}P_n(\cos\psi), \tag{2}$$

where $P_n(\cos\psi)$ denotes the Legendre polynomial of degree n, ψ is the spherical distance between P and Q and A, B and s are the model parameters. A closed expression for (2) is available in (Tscherning and Rapp, 1974, p. 45).

The local covariance function of gravity anomalies $C(P,Q)$ given by Tscherning-Rapp can be defined as

$$C_g(P,Q) = A\sum_{NN+1}^{\infty}\frac{n-1}{(n-2)\ (n+B)}\ s^{n+2}P_n(\cos\psi). \tag{3}$$

This expression may be written in the form

$$\begin{aligned} C(P,Q) &= A\sum_{n=3}^{\infty}\frac{n-1}{(n-2)\ (n+B)}\ s^{n+2}P_n(\cos\psi) - A\sum_{3}^{NN}\frac{n-1}{(n-2)\ (n+B)}s^{n+2}P_n(\cos\psi) \\ &= C_g(P,Q) - A\sum_{n=3}^{NN}\frac{n-1}{(n-2)\ (n+B)}s^{n+2}P_n(\cos\psi), \end{aligned} \tag{4}$$

where $C_g(P,Q)$ is given by (2) and its closed form.

Modelling the covariance function means in practice fitting the empirically determined covariance function (through its three essential parameters; the variance C_0, the correlation length ξ and the variance of the horizontal gradient G_{0H}) to the covariance function model. Hence the four parameters *A, B, NN* and *s* are to be determined through this fitting procedure.

The essential parameters of the empirical covariance parameters for 2397 gravity stations in Austria are 580.4 $mGal^2$ for the variance C_0 and 40.2 km for the correlation length ξ. The value for the horizontal gradient G_{0H} was roughly estimated by 100 E^2.

For the fixed value B=24 the following Tscherning-Rapp covariance function model parameters were fitted: s = 0.998747, A=529.74 $mGal^2$ and NN=78.

FITTING THE GRAVIMETRIC GEOID TO GPS-REFERENCE POINTS. - The gravimetric geoid is obtained after restoring the effect of the earth gravity model EGM96 and the indirect effect caused by the topographic-isostatic reduction to the predicted geoid (LSC-output). The difference between the gravimetric geoid and the geoid derived from GPS and levelling is shown in Fig. 2. The difference shows a long-wavelength character with bigger values near the borders of Austria. The character of the difference is mainly caused by the influence of distant zones on the gravimetric geoid. If the difference is modelled by a fourth order polynomial regression surface, the remaining residuals ΔN_{GRAV} are small with a standard deviation of ±3.5 cm. For 50% of the points the difference is less than ±2.0 cm (see details in Table 3).

3. Astrogeodetic geoid using LSC

3.1. Deflections of the vertical

A data set of 706 deflections of the vertical is used. Fig. 3 shows all the measurements. This data set was already used for the Austrian geoid computation by Sünkel (1987).

3.2. Reduction of the deflections of the vertical

The topographic-isostatically reduced deflections of the vertical can be expressed by

$$\xi_{ISO} = \xi_{obs} - \xi_{TI} - \xi_{EGM} - \delta\phi \ , \tag{5}$$

$$\eta_{ISO} = \eta_{obs} - \eta_{TI} - \eta_{EGM} \ , \tag{6}$$

where the subscript *obs* refers to the observed deflections, the subscript *TI* refers to the effect

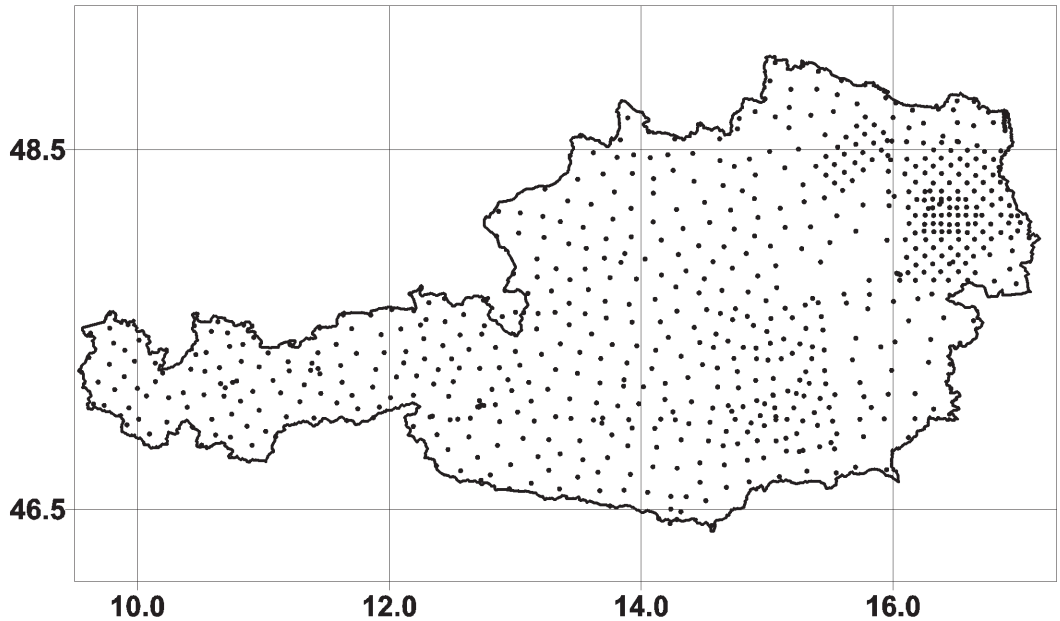

Fig. 3 - Measured deflections of the vertical used for the computation.

of the topography and its compensation on the deflection components and the subscript *EGM* refers to the effect of the reference field on the deflection components. $\delta\phi$ is the reduction according to the curvature of the plumb line. Table 2 shows the detailed statistics of the reduction for ξ and η.

3.3. Astrogeodetic geoid by LSC

The astrogeodetic geoid was computed using the covariance function determined in Section 2.3.

In analogy to the gravimetric geoid, the astrogeodetic geoid is obtained after restoring the

Table 2 - Reduction of the deflections of the vertical using the standard density of 2.67g/cm^3 and the geopotential model EGM96. Statistics based on 706 points.

	ξ (arcsec)				η (arcsec)			
	Min	**Max**	**Mean**	**Std.dev.**	**Min**	**Max**	**Mean**	**Std.dev.**
ξ,η_{obs}	-14.2	24.8	3.7	±6.3	-15.5	17.7	2.8	±5.6
$\xi,\eta_{EGM\text{-}red}$	-12.8	16.1	0.7	±4.1	-14.6	13.7	0.9	±3.7
ξ,η_{ISO}	-12.4	10.0	-1.4	±4.5	-9.4	8.8	-0.8	±2.7

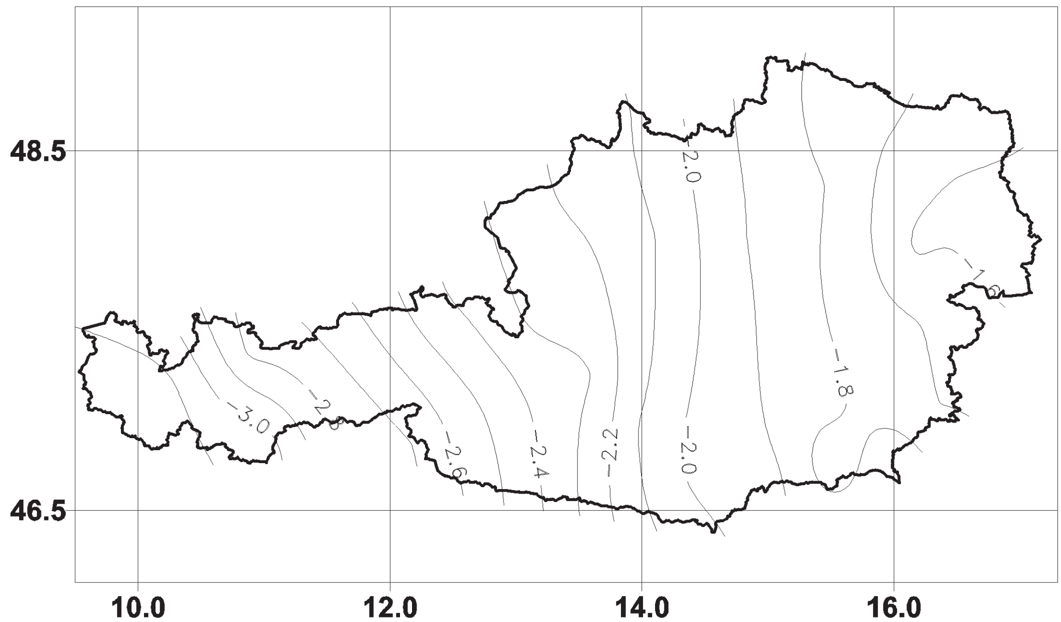

Fig. 4 - Difference between the astrogeodetic geoid and the geoid obtained by GPS and levelling. Contour interval 0.1 m.

effect of the earth gravity model EGM96 and the indirect effect caused by topographic-isostatic reduction to the predicted geoid (LSC-output). Fig. 4 shows the difference between the astrogeodetic geoid and GPS-derived geoidal heights. While the differences are bigger, as in the gravimetric case, they are smoother with a mainly west-east trend.

4. Combination of astrogeodetic and gravimetric geoid solution

Before the combination of the gravimetric and astrogeodetic solution is performed, the agreement of both solutions is checked. The main difference can be expressed by a high order polynomial trend surface. Fig. 5 shows the residuals after removing a fourth order polynomial regression surface from the differences. For more than 70% of the area the agreement is better

Table 3 - Remaining differences between the gravimetric geoid N_{GRAV}, the atrogeodetic geoid N_{ASTRO} and the combined geoid solution N_{COMB} and a geoid derived by GPS and levelling after subtracting a fourth order polynomial trend. Statistics based on 40 points given in cm.

	Min	Max	Std.dev.	No. of points with $\Delta N < \pm 2cm$
ΔN_{GRAV}	-6.9	8.1	±3.5	20
ΔN_{ASTRO}	-7.5	10.5	±3.4	23
ΔN_{COMB}	-6.3	7.1	±3.1	25

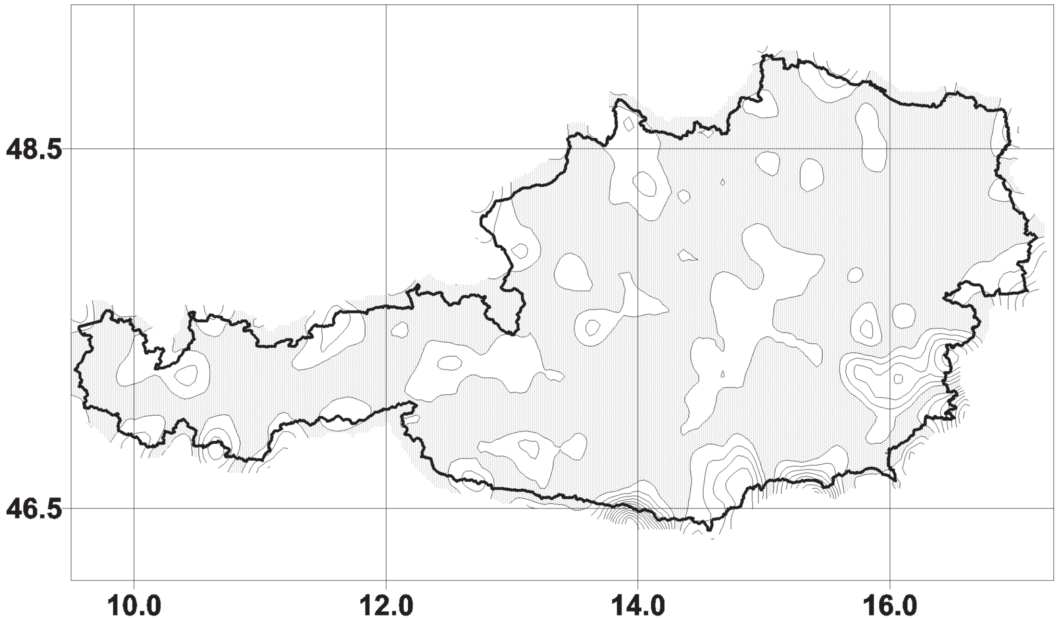

Fig. 5 - Residual difference between gravimetric and astrogeodetic geoid solutions after subtracting a fourth order polynomial trend surface. The shaded areas denote regions with a difference less than ±3 cm. Contour interval 3 cm.

than ±3 cm (shaded area in Fig. 5). The biggest differences occur at the southern border. The reason for this is the lack of gravity data, as well as the extensive uncertainties of the Slovenian gravity data. Differences near the border are also influenced by the fact that deflections of the vertical were used only inside Austria. Overall, the differences show no correlation with topography. Further investigations are needed to distinguish between differences caused by measurement/errors and differences caused by an uneven data distribution.

The combination between the two geoid solutions was done by forming the simple arithmetic mean between the gravimetric solution and the astrogeodetic solution with the help of the previously described regression surface. Table 3 shows the accuracy of the three different geoid solutions, which is expressed by the remaining residuals after fitting the geoid solutions to the geoid derived by GPS and levelling, and neglecting a fourth order polynomial. Although the ratio: number of deflections to number of gravity measurements, is one to five, the accuracy is about the same. The best accuracy is reached for the combined solution, where more than 60% of the points have residuals at less than ±2 cm.

5. Conclusions

The combination of the astrogeodetic and gravimetric geoid solutions improves the accuracy of the geoid. Even a simple chosen combination of measurements has the advantage that on the one hand an external check of the measurements is possible, and on the other the resulting geoid accuracy can be improved. Nevertheless, further investigation into better algorithms, for combining gravity and deflections of the vertical, is needed.

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