BOLLETTINO DI GEOFISICA TEORICA ED APPLICATA VOL. 40, N. 3-4, pp. 555-562; SEP.-DEC. 1999

The influence of different crust models on the gravity field of the Earth

M. KUHN

Geodetic Institute, University of Karlsruhe, Germany

(Received October 4, 1998; accepted August 8, 1999)

Abstract. By describing the geoid in relation to a reference ellipsoid as a particular equipotential surface of the Earth's gravity field, we end up with the well-known problem that the density distribution of the Earth is not exactly known. The Earth's crust, in particular, is of major interest because it produces the medium and short wavelength constituents in the variation of the geoid. To model the crust, we can use the three famous isostatic models of Pratt-Hayford, Airy-Heiskanen and Vening Meinesz, or, additional information about the density distribution and the depth of the Mohorovičić discontinuity. The influence of different crust models on the gravity field of the Earth, especially on the geoid, can reach several centimeters. This wiIl be illustrated with the isostatic models of Pratt-Hayford and Airy-Heiskanen for the area of Baden-Württemberg in Germany.

1. Introduction

In recent years the requirement of a precise geoid, or quasigeoid, has become more and more important in Geodesy to connect GPS heights with conventional height systems. The relative accuracy of the geoid must be in the range of one centimeter over hundreds of kilometres, which is achievable with relative heights derived from GPS. The well-known Stokes' integration formula (Heiskanen and Moritz, 1967) as a solution of the Stokes' Problem can be used to compute the geoid as a particular equipotential surface of the Earth's gravity field from gravity anomalies. To find the solution, no masses must be outside the geoid, and the gravity data are given on the geoid itself. Therefore, theoretically, an exact knowledge of the density distribution of the topographic masses between the Earth's surface and the geoid is necessary. For smoothing purposes in particular, the density distribution of the masses below the geoid must be known. The problem is, that the density distribution of the whole crust is only approximately known. But there exist some isostatic models for the Earth's crust, or additional information from geology

Corresponding author: M. Kuhn (former University of Karlsruhe) Deutsches Geodätisches Forschungsinstitut, Marstallplatz 8, D80539 München, Germany; phone: +49 89 23031 216; fax: +49 89 23031 240; e-mail: kuhn@dgfi.badw.de

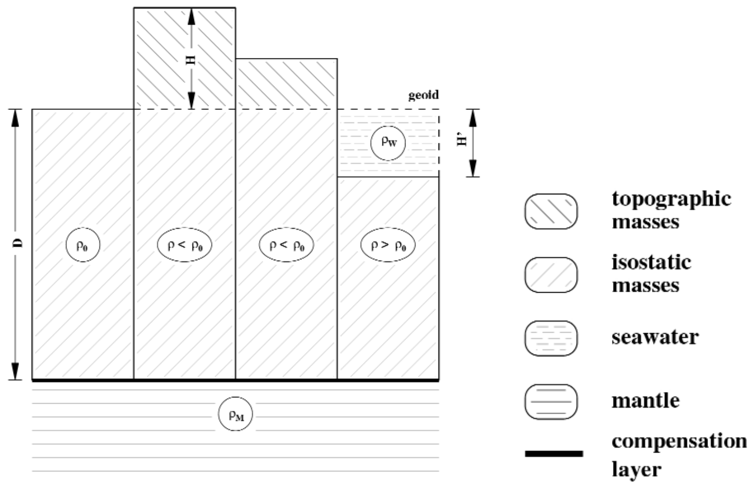

Fig. 1 - Isostatic model of Pratt-Hayford.

and geophysics. In this connection, the impact of the differences between the models on the geoid is of great interest. In the sequel, the principle possibilities for modelling the Earth's crust will be described and the results for a geoid computation for the isostatic models of Pratt-Hayford and Airy-Heiskanen will be shown for the region of Baden-Württemberg in Germany.

2. Modelling the Earth's crust

The first way is to model the topography by using digital terrain models (DGM) with different resolutions (Forsberg and Tscherning, 1997). A precise description of the topographic masses is necessary because they produce the short, and very short wavelength constituents in the variation of the geoid. By taking only the topographic masses into account, the results are Bouguer anomalies which are not very suited for geoid determination because of their large, indirect, effect of about several hundred meters in mountainous regions. To reduce the indirect effect the use of the isostatic models of Pratt-Hayford, Airy-Heiskanen or Vening Meinesz is more reasonable because they model the masses of the entire crust.

$$\text{original model of Pratt-Hayford: } \rho = \frac{D-H}{D} \cdot \rho_0 \tag{1}$$

$$\text{planar model of Pratt-Hayford: } \rho = \frac{D}{D+H} \cdot \rho_0 \tag{2}$$

$$\text{planar model of Airy-Heiskanen: } t = \frac{\rho_0}{\rho_M - \rho_0} \cdot H \tag{3}$$

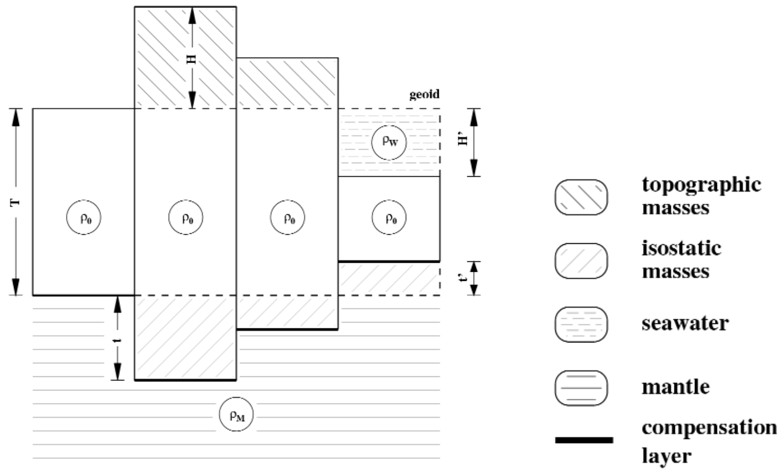

Fig. 2 - Isostatic model of Airy-Heiskanen.

In the model of Pratt-Hayford (Fig. 1), the compensation depth D is constant (D=100 km) and the density varies laterally. The original model also treats the topographic masses with the constant density ρ_0 of the column with the topographic height H = 0 m and only the masses between the geoid and the compensation depth with the model density ρ (1). In the planar model, all masses between the Earth's surface and the compensation depth are treated with the model density ρ (2). In the model of Airy-Heiskanen (Fig. 2) the compensation depth varies laterally and the density is constant. The corresponding formula for the compensation depth in the planar model of Airy-Heiskanen is given in (3). Apart from the isostatic models, additional information about the Earth's crust can be used. Better information about density distribution can be taken from geological maps or 2d or 3d digital density models (DDM) as shown in Tanimoto (1995) and Marti (1997). From seismology we get the Moho depth as the real compensation depth (Martinec, 1994).

3. The geoid from different crust models

To show the influence of different crust models on the geoid we use the isostatic models of Pratt-Hayford and Airy-Heiskanen with the corresponding formulas (1) - (3). The region of Baden-Württemberg in SW-Germany is chosen as a numerical illustration. In this region, more than 10 000 gravity points, from different sources, exist which means a resolution of approximately one gravity point each 2 km. The topographic heights for this region are illustrated in Fig. 3. An insight into the behavior of the gravity field is given by the isostatic gravity anomaly for the planar model of Airy-Heiskanen in Fig. 4. The principle behaviour of the model of Pratt-Hayford is the same, only the absolute numbers are different. All the effects on gravity and potential from the masses are computed by point, prism and tesseroid formulas according to Mader (1951) and Grüninger (1990). Prism formulas are used in the local vicinity around the

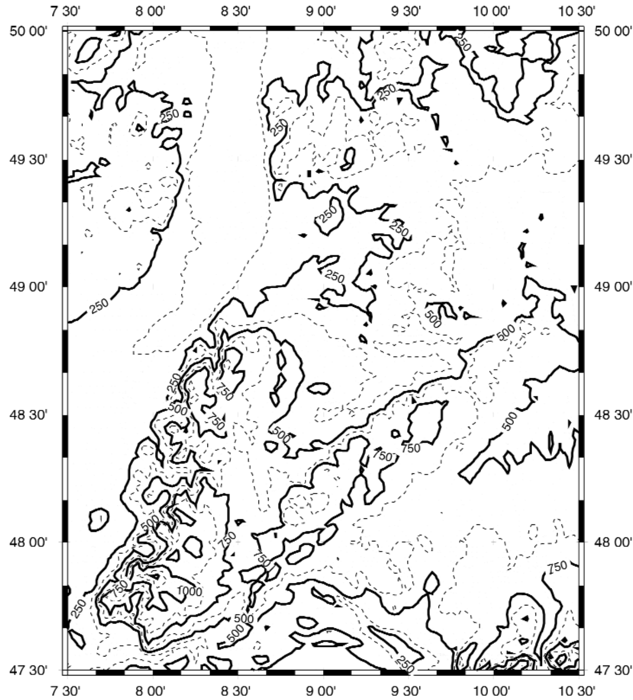

Fig. 3 - Heights in Baden-Wtirttemberg [m].

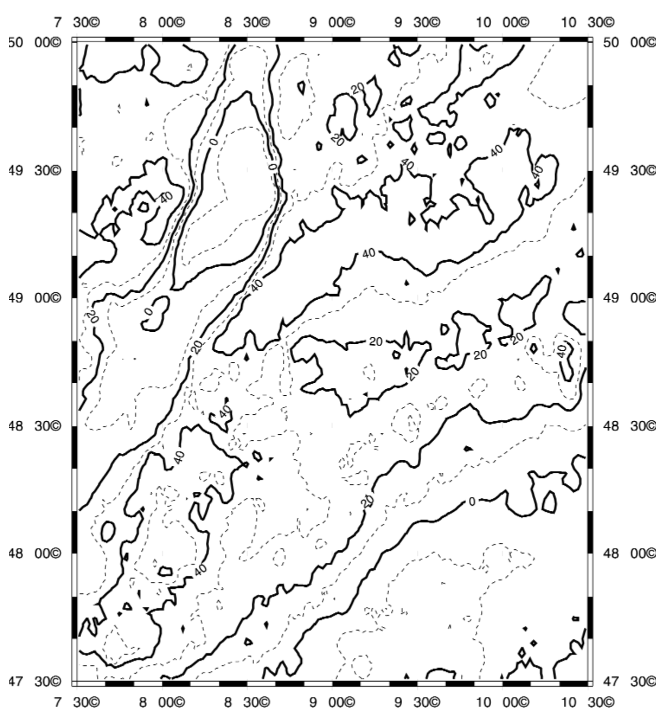

Fig. 4 - Airy-Heiskanen gravity anomalies [10^{-5}m/s^2] mean: 20.802 st.dev.: 17.544.

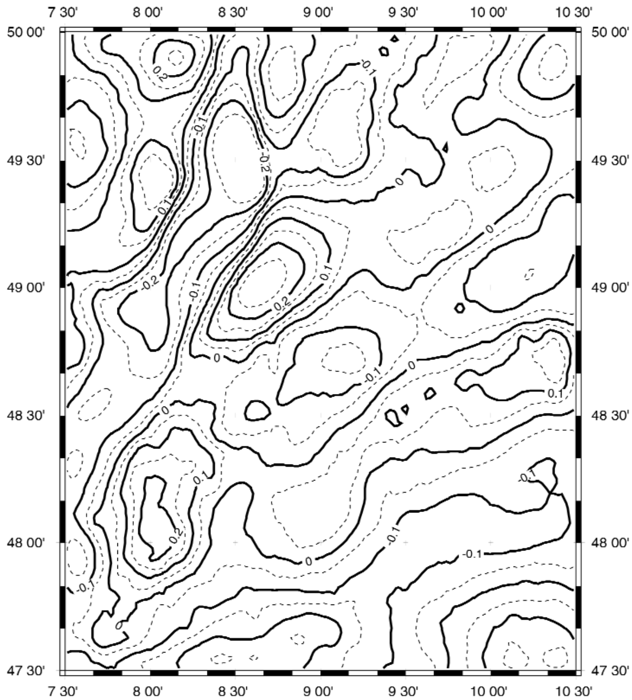

Fig. 5 - Airy-Heiskanen planar model part N_2 [m] mean: -0.028 st.dev.: 0.107.

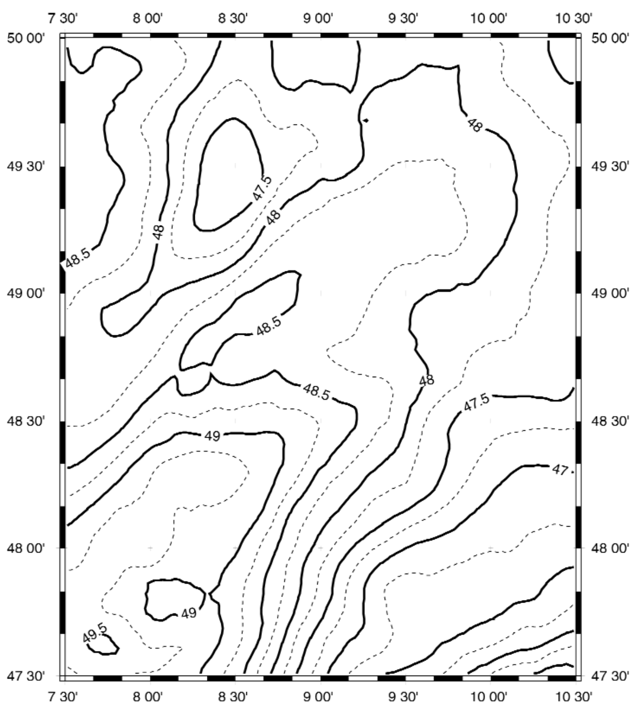

Fig. 6 - Geoid Aiiy-Heiskanen planar model [m] mean: 48.038 st.dev.: 0.739.

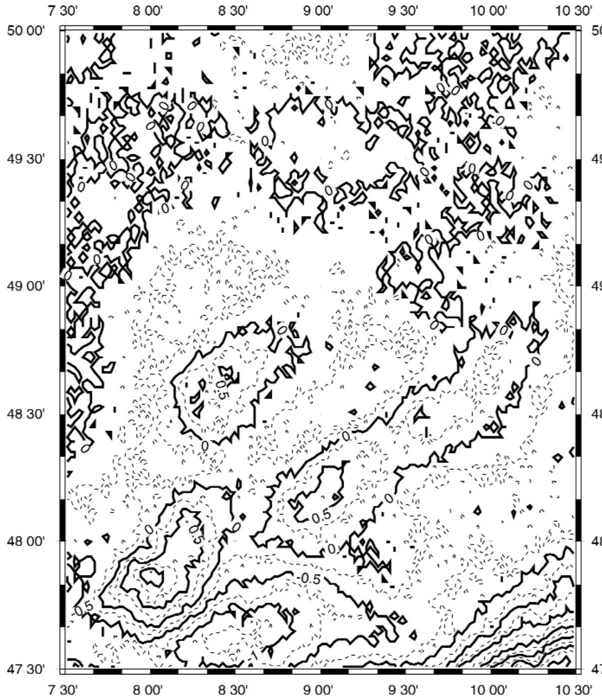

Fig. 7 - Geoid difference [cm] Pratt-Hayford original model - planar model.

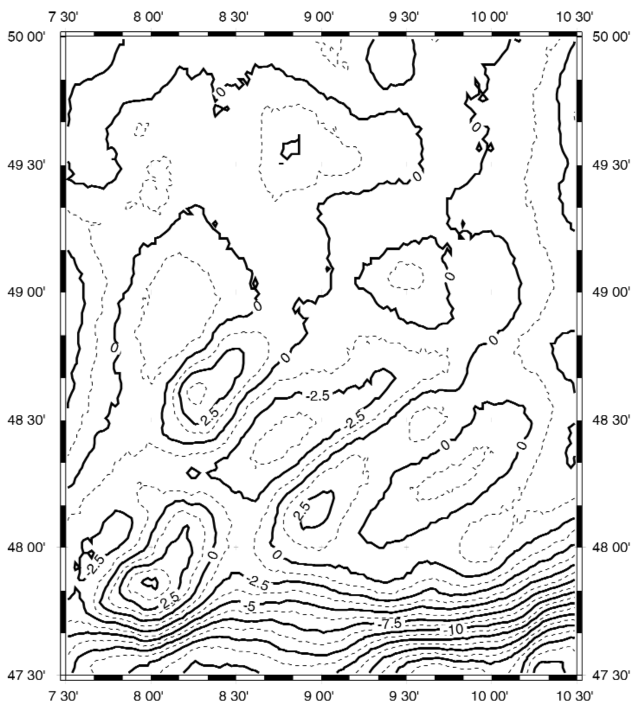

Fig. 8 - Geoid difference [cm] Pratt-Hayford planar - Airy-Heiskanen planar.

computation point up to a radius of approximately 10 km. The farer masses are regarded as tesseroid or point masses. The global topography is taken from the 5' × 5' DGM of the Technical University of Graz (ETOPO5). In Baden-Wiirttemberg a DGM compiled by BKG (Federal Agency for Cartography and Geodesy, Germany) with a resolution of 30" × 50" is used. The gravity anomaly shows a strong correlation with the topography which means that the density of the topographic and isostatic masses is not modelled in a sufficiently correct way, or the isostatic compensation is not completely achieved in this region. The minimum in the valley of the river Rhine and at the beginning of the Alps region in the south-eastern part, corresponds to thick tertiary and quartiary sedimental layers with a mean density of about 2200 kg/m^3 instead of a density of 2670 kg/m^3 for ρ_0. The maximum corresponds to the highest mountain.

In the next step different geoids are computed in relation to the different isostatic models of Pratt-Hayford and Airy-Heiskanen. To compute the geoid, the principles of the spectral combination method (Rapp and Rummel, 1975) are used. Here the geoidal height N is composed of four different parts according to (4). N_1 is the long wavelength part from a regional or global gravity field model. The part N_2 is the short wavelength part from local gravity anomalies transformed via Stokes' formula (5) in geoidal heights. The respective gravity anomaly Δg in Eq. (5) is corrected by the indirect effect on gravity $\delta\Delta g$ (Wichiencharoen, 1982) and is reduced

$$N = N_1 + N_2 + N_3 + \delta N \tag{4}$$

$$N_2 = \frac{R}{4\pi\gamma_P}\int_\sigma\int\left[(\Delta g + \delta\Delta g) - \Delta g_M\right]\cdot S(\psi)\cdot d\sigma \tag{5}$$

by subtracting the long wavelength part Δg_M in the gravity anomaly from the gravity model. The integration in Eq. (5) is done over a spherical cap with the radius $r \leq r_\sigma$ with $r_\sigma \approx 2°$. The short wavelength part N_3 from Stokes integration for the far zone $r \geq r_\sigma$ is assumed to be zero. The term δN in Eq. (4) is the short wavelength part of the indirect effect. The problem in this method is the long wavelength part Δg_M for the respective gravity anomaly. Normally, the gravity field models such as EGM96 or EGG97 are based on free-air gravity anomalies and therefore also Δg_M in this model. In this paper the long wavelength part Δg_M comes from the gravity data themselves by using a simple mean operator for a wavelength of about 30 minutes. To get realistic figures for the geoid the long wavelength part of the European Geoid, EGG97 was added for the term N_1. In Fig. 5 the short wavelength part N_2 in the planar model of Airy-Heiskanen is illustrated. Here too the correlation with the topography is significant. Putting all the effects together, also including the short wavelength part of the indirect effect, we get the geoid for the planar model of Airy-Heiskanen (Fig. 6).

The geoid for the isostatic model of Pratt-Hayford shows approximately the same behaviour. But now, the most important result is the difference between these models in the geoidal heights. Fig. 7 shows the difference between the original and planar model of Pratt-Hayford. There is a small correlation with the topography that shows the influence of different density models. The maximum lies in the range of one centimeter. The difference between the planar models of Pratt-Hayford and Airy-Heiskanen shows the same behaviour (Fig. 8), only the magnitude is much larger and can reach several centimeters. This shows a significant difference in the calculated geoidal heights arising from different models of the Earth's crust.

4. Conclusion

The numerical example has shown that the influence of different crust models on the geoid could reach a magnitude of several centimeters. If we speak about the "One Centimeter Geoid", a sufficiently good representation of the Earth's crust is necessary. The treatment of the topographic and isostatic masses with a constant density ρ_0 or a simple isostatic model, is not sufficient. To keep of the "One Centimeter Geoid" in view all available data about gravity, density and Moho depth in the region of interest must be taken into account.

References

Forsberg R. and Tscherning C. C.; 1997: *Topographic Effects in Gravity Fieid Modelling for BVP*. In: F. Sansó and R. Rummei (eds.) Geodetic Boundary Value Probiems in View of the One Centimeter Geoid. Springer, Berlin Heidelberg (Germany) pp. 241-272.

Griininger W.; 1990: *Zur topographisch-isostatischen Reduktion der Schwere*. PhD-thesis at the university of Kadsruhe.

Heiskanen W. A. and Moritz H.; 1967: *Physical Geodesy.* W.H. Freeman & Co, San Francisco.

Mader K.; 1951: *Das Newtonsche Raumpotential prismatischer Körper und seme Abieitungen bis zur dritten Ordnung.* Òsterreichische ZfV, Sonderheft 11.

Marti U.; 1997: *Geoid der Schweiz 1997.* Geodiitisch-geophysikalische Arbeiten der Schweiz. Band 65, Voi. 56.

Martinec Z.; 1994: *The minimum depth of compensation of topographic masses*. Geophys. J. mt. **117**, 545-554.

Rapp R. H. and Runnnel R.; 1975: *Methods for Computation of Detailed Geoids and their Accuracy.* Reports of the Department of Geodetic Science No. 233, The Ohio State University, Coiumbus/Ohio 1975.

Tanimoto T.; 1995: *Crustal Structure of the Earth.* In: Thomas J. Ahrens (ed.) Global Earth Physics. A Handbook of Physical Constants. AGU Reference Shelf 1, pp. 214-224.

Wicliienclìaroen C.; 1982: *The indirect effects on the computation of geoid unciulation.* OSU Rep. No 336.