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A global high resolution mean sea surface from multi mission satellite altimetry

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Abstract. Satellite altimetry from the GEOSAT and the ERS-1 geodetic missions provide altimeter data with a very dense coverage. Hence, the heights of the sea surface can be recovered in great detail. Satellite altimetry from the 35-day repeat cycle mission of the ERS satellites and, especially, from the 10-day repeat cycle TOPEX/POSEIDON satellite mission provide repeated heights, so that the mean sea surface heights along the ground tracks can be recovered very accurately. In this study a global high resolution mean seas surface (1/8 by 1/8 degrees) is constructed using satellite altimetry from those four missions. The mean sea surface fits the TOPEX/POSEIDON mean sea surface heights within 5 cm and the slopes fit within a 2.5 mm per km.

1. Introduction

An accurate mean sea surface is essential when ocean variability is analysed using satellite altimetry. If repeat mission altimetry such as the TOPEX/POSEIDON mission where sea surface heights are measured along tracks every 10 days, are used, then the mean heights of the sea surface may be obtained by simple averaging. However, since the ground tracks are not exactly collinear the cross track gradients (or slopes) of the sea surface induce an erroneous contribution to the individual height anomalies. Hence, accurate cross-track gradients are needed as well to eliminate errors of that kind.

Accurate mean sea surface heights may be computed using satellite altimetry from the TOPEX/POSEIDON mission. Mesoscale and seasonal variations are effectively eliminated by averaging time series of accurate sea surface heights that are sampled every 10 days for several years. For this study the first 150 cycles of the TOPEX/POSEIDON altimetry were used (September 1992 to September 1996). However, each cycle of 10 days consists of 127 revolutions to cover the surface of the earth. At the equator neighbouring ground tracks have are separated

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by 315 km which make it difficult to compute accurate cross-track height gradients.

The ERS-1 35-day repeat mission covers the surface of the earth more densely. 501 revolutions are completed through each cycle which yields a track spacing of 80 km at the equator. Furthermore, this mission covers an extended region between latitudes -82° and 82° compared to TOPEX/POSEIDON that covers between latitudes -66.5° and 66.5°. The first 18 cycles of the ERS-1 35-day repeat mission covering a period of about 20 months (1992 - 1994) was used.

Satellite altimetry from the geodetic missions of the GEOSAT and the ERS-1 provides the opportunity of making a very detailed mapping of the sea surface heights. The GEOSAT Geodetic Mission altimetry contains data from the first 18 months of the GEOSAT satellite mission (1985-1986) covering marine regions at latitudes between -72° and 72°. The satellite was operated in an non-repeating orbit yielding a very dense, though not completely homogeneous coverage of observations. The ERS-1 satellite completed its 336-day Geodetic Mission in March 1995. The altimetry from this mission provides a very dense and homogeneous coverage. At the equator the spacing between the satellites ground tracks of the ERS-1 Geodetic Mission is 8.3 km.

2. Determination of the mean sea surface

A determination of the mean sea surface from the repeat mission altimetry is straightforward, since ocean variability can be averaged at along-track locations. However, mean sea surfaces from different satellite missions may be different due to different data coverages. A determination of the mean sea surface from the geodetic non-repeating mission altimetry is very difficult, since ocean variability cannot be averaged in a simple manner. The task may be accomplished using different kinds of crossover adjustments (e.g. Knudsen, 1993). As described by Andersen and Knudsen (1998), however, very small regions may be required to filter out the ocean variability properly when bias and tilt terms are applied. Alternatively, polynomials of higher order may be used to obtain a similar resolution, but in any case, errors of such wavelengths are introduced in the recovered mean sea surface.

To avoid inconsistencies along edges of neighbouring cells it was decided to use the Andersen and Knudsen (1998) global grid of marine gravity anomalies to recover the short wavelength parts of the mean sea surface. The determination of a so-called marine geoid was carried out relative to the EGM96 geopotential model (Lemoine et al., 1997) from the global set of altimetric gravity anomalies using the very efficient approximation method based on the Fast Fourier Transform (FFT). In the frequency domain (u,v) geoid undulations, N, are derived from gravity anomalies, Δg, using (Schwarz et al., 1990)

$$\tilde{N}(u,v) \approx \frac{1}{\omega\gamma}\Delta\tilde{g} \qquad (1)$$

where $\omega^2=u^2+v^2$ and γ is the normal gravity. Three parallels at 0°, 40°, and 60° were used throughout this task to compensate for the use of a flat-earth approximation on a spherical earth.

Actually, in this global case the flat-earth approximation is a cylinder approximation, since periodicity in the longitude directions is present. Then the so-called marine geoid was obtained by restoring the EGM96 geoid model.

The marine geoid was removed from the averaged ERS-1 35-day repeat mission sea surface heights and from the averaged TOPEX/POSEIDON repeat mission sea surface heights respectively. The differences between the two sets of mean sea surface heights and the marine geoid were assumed to contain errors in the marine geoid, the sea surface topography, and systematic errors in the two altimeter data sets. The mean values of the ERS-1 and the TOPEX/POSEIDON data were 0.82 m and 0.38 m respectively. To avoid problems that such systematic differences may cause the respective mean values were subtracted from the altimeter data.

The mean sea surfaces associated with the two periods covered by the two repeat missions are not supposed to coincide. Long term sea level changes such as seasonal and interseasonal ocean variability may cause such differences. It was decided to fit the mean sea surface to the TOPEX/POSEIDON altimetry, since these data have the lowest errors and span the longest period of time. Hence, the ERS-1 mean sea surface heights were fitted to the TOPEX/POSEIDON mean sea surface heights.

The differences between the ERS-1 and TOPEX/POSEIDON mean sea surfaces was supposed to have a long wavelength characteristic that may be modelled using a spherical harmonic expansion to degree and order 20. The estimation of this harmonic expansion was carried out in two steps: first, a spherical harmonic expansion to degree and order 20 was fitted to the TOPEX/POSEIDON data; then a second spherical harmonic expansion to degree and order 20 was fitted to the differences between the ERS-1 data and the first harmonic expansion. Due to the different coverages, zero differences were introduced outside latitudes -66.5° and 66.5°. Subsequently, the sea surface of the second harmonic expansion was removed from the ERS-1 mean sea surface heights.

A spherical harmonic expansion to degree and order 20 was then fitted to the merged set of mean sea surface heights in order to obtain a determination of the long wavelength parts of the differences between the mean sea surface and the marine geoid. The dominant contribution to this surface is made by the sea-surface topography. After this surface was removed from the mean sea surface heights the residuals are assumed to contain wavelengths between 2000 km which roughly correspond to harmonic degree 20, and 100 km.

$$\hat{h} = C_P (C + E)^{-1} \underline{h} \tag{2}$$

Those residuals were recovered using the local collocation technique. That is where C and D are covariance matrices associated with the signal and the error covariances respectively. The signal covariances were obtained using the following expression

$$c(r) = C_0 (1 + \frac{r}{\alpha}) e^{(-r/\alpha)} \tag{3}$$

Table 1 - RMS values of heights and slopes of TOPEX/POSEIDON altimeter data, with the EGM96 removed and with the geoid and the harmonic expansion removed, and the residuals with respect to the final mean sea surface.

	h	**h mean**	**Δh**	**s**	**s mean**	**Δs**
	m			**mm/km**		
data	30.825	30.805	0.777	100.3	72.0	50.8
-EGM96	0.776	0.773	0.082	8.2	7.7	4.2
-Geoid	0.711	0.710	0.026	3.6	2.5	1.6
-Harm	0.102	0.096	0.024	3.2	2.3	1.4
Residuals	0.051	0.043	0.019	2.5	1.8	1.0

where the C_0 is the variance. The parameter α and the error variance were empirically determined to a correlation length of 100 km and $(0.1\ \text{m})^2$ respectively to provide smoothness at the required level.

Finally, the mean sea surface was obtained by adding the marine geoid, the long wavelength harmonic expansion, and the medium wavelength residuals. Also the 0.38 m difference between the TOPEX/POSEIDON altimetry and the marine geoid was included to obtain consistency with the TOPEX/POSEIDON reference frame.

3. Results

To evaluate the constructed mean sea surface, comparisons to the TOPEX/POSEIDON mean sea surface heights were carried out. For this set of mean sea surface heights, and after each step through the process Root-Mean-Square (RMS) values were computed. For subsets of seven consecutive data along track, the fourth height, the mean of the seven heights, and their difference were evaluated. The slopes were analysed in similarly. The mean taken over seven heights represents roughly a mean over 50 km along track. Hence, the RMS of this mean height reflects the magnitude of wavelengths longer than about 100-200 km and the difference. The RMS of the difference between the fourth height and the mean height reflects the magnitude of the shorter wavelength parts.

The results for the TOPEX/POSEIDON altimetry are shown in Table 1. Especially, the removal of the EGM96 geoid model which reduces all RMS. The removal of the marine geoid has an effect on the height differences and the slopes. Hence, the marine geoid resolves the short wavelengths of the mean sea surface. The removal of the spherical harmonic expansion (to degree 20) reduces the heights as well as the mean heights.

The final mean sea surface fits the TOPEX/POSEIDON mean sea surface heights with 5 cm. The heights relative to the mean heights are fitted by 2 cm and the slopes fit within a few mm per km (see also Knudsen, 1999).

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