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An analysis of geopotential difference determination from satellite-to-satellite tracking

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Abstract. The determination of the Earth's gravitational field from a dedicated satellite-to-satellite (SST) tracking mission has been considered for more than two decades. The usual assumed approach to gravitational field modeling from SST data is by spherical harmonic series development. On the other hand, many feasibility studies based on covariance analyses have assumed a direct relationship between geopotential and intersatellite range-rate. The latter would make local gravity field determination possible, without the need for computationally massive global spherical harmonic solutions. However, the relationship between the actual observable and the potential is not as simple as assumed in previous studies. Alternatively, three-dimensional velocity differences may be determined with GPS baseline tracking, and then the relationship to potential is, in fact, straightforward. But the measurement accuracy of the observable is less than with intersatellite ranging. This paper examines in detail the relationship between geopotential differences and the observables in intersatellite ranging and in GPS relative velocity measurements. Error analyses and simulations based on the GRACE mission are presented to show the feasibility for local gravity field determination in either case.

1. Introduction

A satellite mission dedicated to the improvement of our knowledge of the Earth's gravitational field by the use of a direct measurement system has now been approved and is expected to be realized in 2001: GRACE, the Gravity Recovery And Climate Experiment. GRACE is a variant of the erstwhile GRAVSAT and GRM mission concepts (Keating et al., 1986) in that two low-altitude satellites will track each other as they circle the Earth in identical

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near polar orbits. Unlike GRM, the satellites are not "drag-free" and non-gravitational accelerations must be measured independently using on-board accelerometers. Also, the altitude of the GRACE satellites is significantly higher (450 km) than that proposed for GRM (160 km). Another significant departure from the older concept is that each satellite will carry a geodetic quality GPS receiver to aid in orbit determination, as well as provide GPS satellite occultation measurements.

These receivers (combined with the accelerometers) will also provide an independent, direct measurement of the gravitational potential difference between the satellites. This implies a local measurement of the gravitational field in the true spirit of in situ satellite gravimetry. The primary measurement system, satellite-to-satellite tracking (SST), also provides an in situ measurement, but theoretically it is only an approximate measure of the geopotential difference.

Many analyses regarding gravity field improvement from SST exist in the literature (e.g., Rummel, 1979). This paper quantifies the differences between the range (SST) and the vector difference (GPS) measurements in a *time-varying* potential field for the expressed purpose of determining the geopotential. This has direct application in improving local and regional geoid models in remote areas not currently well mapped gravimetrically, in providing a means to calibrate the GRACE data using local high-accuracy models, and in suggesting alternative methods to process the data for global modeling.

2. The model

For drag-free orbits in a static gravitational field, the law of conservation of energy is given by

$$\frac{1}{2}v^2 - V = E \tag{1}$$

where V is the gravitational potential energy, E is the total (constant) energy of the satellite in its orbit, and v is the magnitude of its velocity. For two co-orbiting satellites, with position vectors x_1 and x_2, the gravitational potential difference is given by

$$V_{12} = V_2 - V_1 = \mathbf{x}_{12}^{\mathbf{T}}\mathbf{x}_1 + \frac{1}{2}\mathbf{x}_{12}^{\mathbf{T}}\mathbf{x}_{12} - E_{12} \tag{2}$$

where E_{12} is a constant, $v_j^2 = \mathbf{x}_j^T\mathbf{x}_j$ and $\mathbf{x}_{12}=\mathbf{x}_2-\mathbf{x}_1$. With Earth rotation (and neglecting all other time-varying potentials (Jekeli, 1999)), and still omitting non-gravitational terms, the potential difference becomes

$$V_{12} = \dot{\mathbf{x}}_{12}^{\mathbf{T}}\dot{\mathbf{x}}_1 + \frac{1}{2}\left|\dot{\mathbf{x}}_{12}\right|^2 - \omega_e(x_{12_1}\dot{x}_{2_2} - x_{2_2}\dot{x}_{12_1} - x_{1_1}\dot{x}_{12_2} + x_{12_2}\dot{x}_{1_1}) - E_{0_{12}} \tag{3}$$

This is an exact relationship between the potential difference and measured velocity vectors and vector differences. The third term, due to Earth's rotation rate, ω_e, is called the potential rotation term.

No such rigorous relationship exists between the range-rate and the potential. However, we

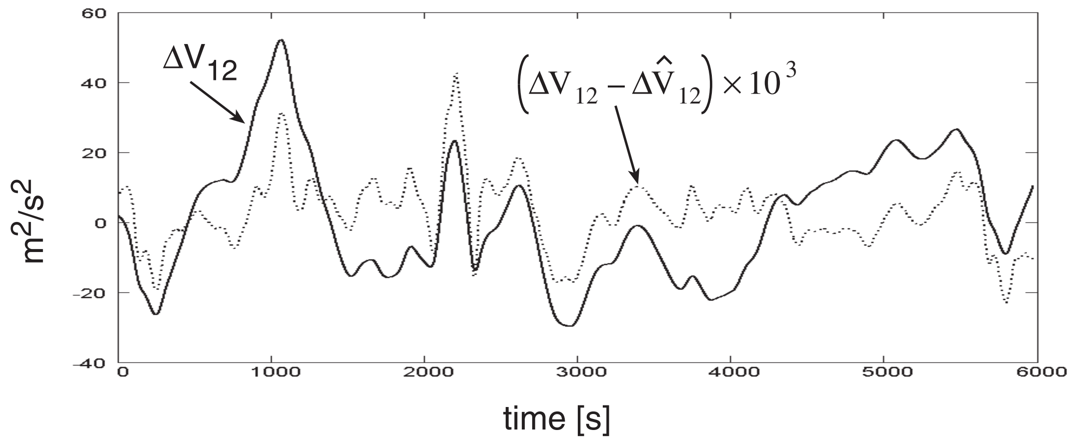

Fig. 1 - Comparison of true residual potential difference to model (4) (no Earth rotation, identical orbits).

will find a good approximation under certain conditions. Utilized in the analyses by Wolff (1969), Rummel (1980), Jekeli and Rapp (1980), and Dickey (1997), among others, this approximation is

$$\Delta\hat{V}_{12} = \left|\dot{\mathbf{x}}_{\mathbf{1}}^{\mathbf{0}}\right|\Delta\dot{\rho}_{12} \tag{4}$$

where a reference potential field has been introduced, identified by the superscript "0", and the residual to this reference is indicated by the Δ; for example, $\Delta\dot{\rho}_{12} = \dot{\rho}_{12} - \dot{\rho}^{0}_{12}$. The error in this model is given by Jekeli (1999) as

$$\varepsilon\Delta V_{12} = \Delta\hat{V}_{12} - \Delta V_{12} = v_1 + v_2 + v_3 + v_4 - \Delta VR_{12} + \Delta E_{0_{12}} \tag{5}$$

where ΔVR_{12} is the residual potential rotation term, and

$$\begin{aligned} v_1 &= \left(\dot{\mathbf{x}}_{\mathbf{2}}^{\mathbf{0}} - \left|\dot{\mathbf{x}}_{\mathbf{1}}^{\mathbf{0}}\right|\mathbf{e}_{\mathbf{12}}\right)^T \Delta\dot{\mathbf{x}}_{\mathbf{12}} \qquad & v_2 &= \left(\Delta\dot{\mathbf{x}}_{\mathbf{1}} - \left|\dot{\mathbf{x}}_{\mathbf{1}}^{\mathbf{0}}\right|\Delta\mathbf{e}_{\mathbf{12}}\right)^T \dot{x}_{12}^{0} \\ v_3 &= \left(\dot{\mathbf{x}}_{\mathbf{1}}\right)^T \Delta\dot{\mathbf{x}}_{\mathbf{12}} & v_4 &= \frac{1}{2}\left|\Delta\dot{\mathbf{x}}_{\mathbf{12}}\right|^2 \end{aligned} \tag{6}$$

3. Numerical results

The error terms in Eq. (5) were quantified using simulated, approximately circular orbits of GRACE satellites in the potential field described by the spherical harmonics of the model EGM96 (Lemoine et al., 1998) up to degree and order n_{max}=180. The altitude of the satellites was about 400 km, their along-track separation about 200 km, and the inclination about 87° . The reference field comprised harmonics up to degree and order n_{max}=2. Nongravitational

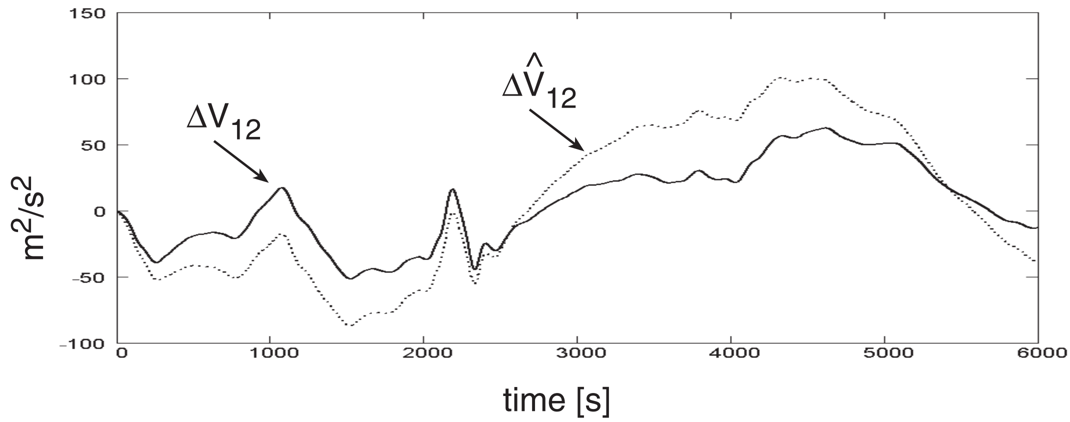

Fig. 2 - Comparison of true residual potential difference to model (4) (Earth rotation, unequal orbits differing by less than 60 m).

accelerations were not included.

In the case of two satellites travelling along identical orbits (implying no Earth rotation), the model error, Eq. (5), is about three orders of magnitude less than the potential difference, itself, as shown in Fig. 1. On the other hand, with more realistic orbits in the presence of the rotating potential field, the model error contains a large long-wavelength (once-per-revolution) component, as shown in Fig. 2. Figs. 3 and 4 give the magnitudes of the individual error terms, ν_1, ν_2, ν_3, ν_4, and ΔVR_{12}., clearly identifying the long-wavelength component, as well as the significance of the potential rotation term. Terms ν_3, and ν_4 are essentially negligible.

Correcting the model, Eq. (4), means measuring (e.g., with GPS) the velocity vector and vector difference components that enter terms ν_1, ν_2, and ΔVR_{12}. Fig. 5a compares the errors in velocity and velocity difference to the errors in range rate that can be tolerated for particular

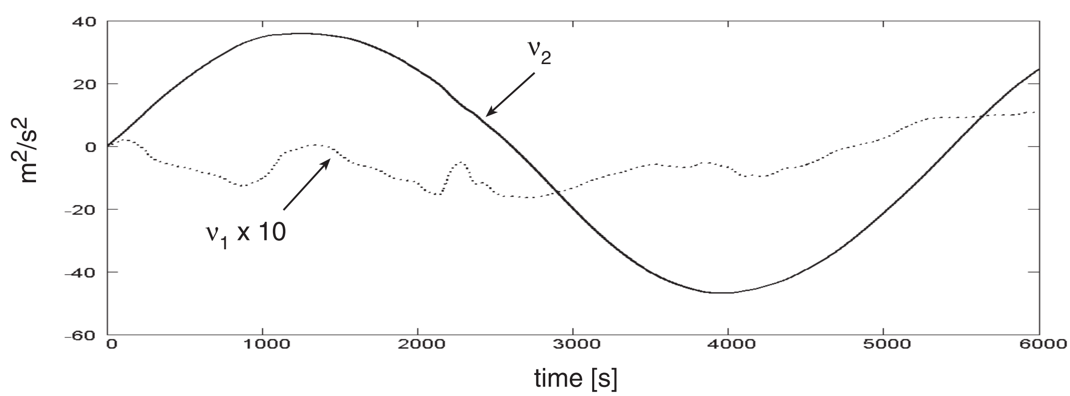

Fig. 3 - Model error terms ν_1 and ν_2 for the case shown in Fig. 2.

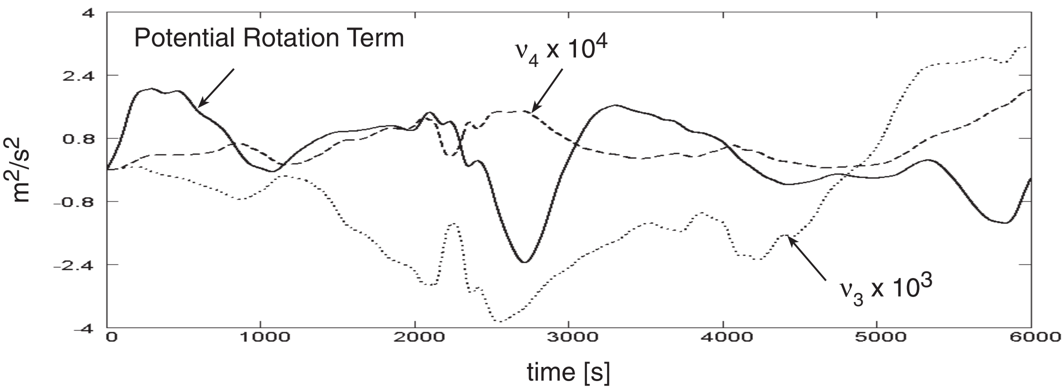

Fig. 4 - Model error terms v_3, v_4, and ΔVR_{12} for the case depicted in Fig. 2.

desired errors in the potential difference. Fig. 5b similarly shows errors in position and position differences as functions of potential difference errors.

4. Summary

It may be concluded from this analysis that accurate in situ potential difference determination from SST can only be accomplished if corresponding vector velocity differences (and absolute velocities) are also measured. The accuracies in the latter, however, may be an order of magnitude less compared to the range-rate accuracy. The simulations also show that the potential rotation term is significant at the level of 1 m^2/s^2 for satellites in near-polar orbits with 400 km altitude.

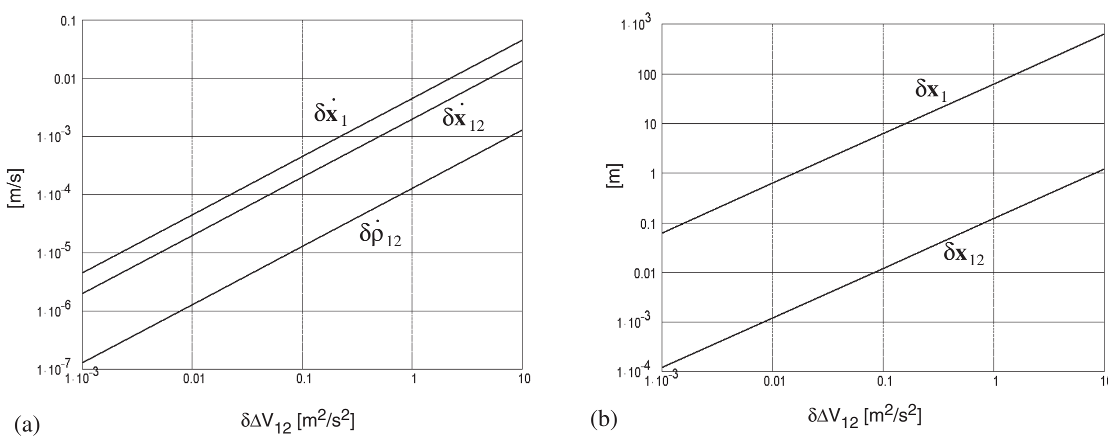

Fig. 5 - Range-rate and velocity accuracy (a) and position accuracy (b) requirements for potential difference determination.

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