A further evaluation of the EGM96 geopotential model based on deflections of the vertical

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Abstract. The deflection of the vertical is a gravitational signal with significant high spatial frequency informational content. However, no such data enter the typical global geopotential model development. Therefore, deflections of the vertical represent an ideal independent data source to assess global gravity models, since they are weakest at high frequencies. A comparison of the deflections of the vertical computed from the recently developed EGM96 geopotential model with astrogeodetic deflections at about 3600 points in the conterminous United States show that the EGM96 model may be under-powered at degrees higher than 200.

1. Introduction

High-degree spherical harmonic models of the Earth's geopotential may be used to compute the deflection of the vertical. Such gravimetric deflections can then be compared to astrogeodetic deflections to assess the accuracy of the geopotential model. A rigorous theoretical and numerical comparison between astrogeodetic and gravimetric deflections is given by Jekeli (1998). The numerical comparison is extended here to provide further evidence of a slight loss in power in the recently computed spherical harmonic model, EGM96 (Lemoine et al., 1998), at high frequencies.

2. The deflection of the vertical

The deflection of the vertical is an angle that describes the deviation of the true vertical, as defined by the direction of Earth's gravity vector, with respect to some reference direction. The

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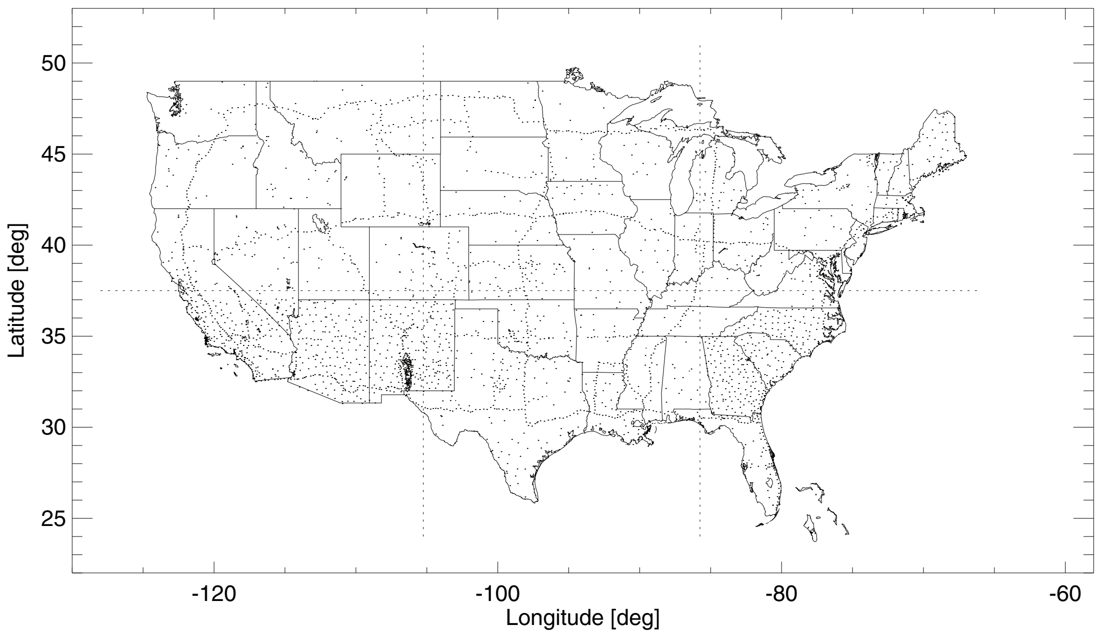

Fig. 1 - Locations of 3561 astronomic deflections of the vertical.

reference direction may be defined purely geometrically or physically. The *astrogeodetic* (or, *Helmert*) deflection at a point corresponds to the classic geometric definition where the reference direction is the normal to an ellipsoid. To first-order approximation its components are

$$\xi^{astro} = \Phi - \phi;\ \ \eta^{astro} = (\Lambda - \lambda)\cos\phi;\ \ \Theta^{astro} = (\xi^{astro}, \eta^{astro})^T \tag{1}$$

where (Φ, Λ) are astronomic coordinates (latitude and longitude) of the point and (ϕ, λ) are the corresponding geodetic coordinates. Θ^{astro} is a two-component vector whose magnitude is the total deflection angle.

The gravimetric deflections refer (approximately) to the direction of normal gravity; and can be computed, given a geopotential model, such as EGM96, approximately according to

$$\begin{Bmatrix} \xi^{grav}(r,\theta,\lambda) \\ \hat{\eta}^{grav}(r,\theta,\lambda) \end{Bmatrix} = -\frac{1}{\gamma}\ \frac{kM}{r^2} \sum_{n=2}^{n_{\max}} \sum_{m=-n}^{n} \left(\frac{a}{r}\right)^n \hat{C}_{n,m} \begin{Bmatrix} -\dfrac{\partial}{\partial\theta} \\ \dfrac{\partial}{\sin\theta\partial\lambda} \end{Bmatrix} \left(\overline{Y}_{n,m}(\theta,\lambda)\right) \tag{2}$$

where (r, θ, λ) are spherical coordinates, γ is normal gravity, kM is the product of gravitational constant and Earth's mass, a is the semi-major axis of the ellipsoid, and $\overline{Y}_{n,m}$ are the usual fully normalized spherical harmonic functions. The coefficients, $\hat{C}_{n,m}$, of this model are given in a tide-free system; whereas, the astrogeodetic deflections are obtained in a mean-tide system. Other potentially significant differences between the two types of deflections include the effect of the

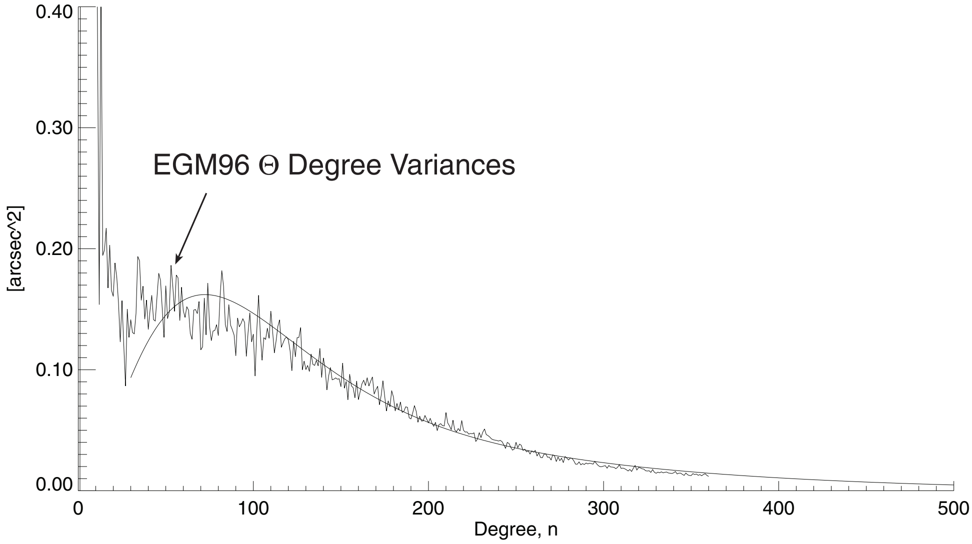

Fig. 2 - Degree variances for the EGM96 vertical deflection and an analytical continuation.

curvature of the normal plumbline, the offset of the center of mass from the ellipsoid center, and the non-parallelism of the coordinate axes.

3. Numerical results

The numerical comparisons are based on 3561 astronomic deflections, provided by the National Imagery and Mapping Agency (NIMA, personal communication). Their locations are shown in Fig. 1, which also shows a delineation of CONUS by types of topographic relief into six areas. The rms (root-mean-square) differences, by region and for all CONUS, between the astrogeodetic and the EGM96 total deflections, $|\Theta|$, are shown in Table 1.

In terms of variances, let

$$\sigma_\Theta^2 \equiv trM(\Theta\Theta^T) = \sigma_\xi^2 + \sigma_\eta^2 \tag{3}$$

where $M(\cdot)$ represents a global average or statistical expectation, as the case may be; and zero mean values are assumed. Then, assuming no correlation between model truncation error and

Table 1 - RMS differences between astrogeodetic and EGM96 deflections, $|\Theta|$ [arcsec].

CONUS: 4.27	North-West (489 pts.): 5.28	North-Central (468 pts.): 2.67	North-East (405 pts.): 2.50
	South-West (1081 pts.): 6.14	South-Central (618 pts.): 2.25	South-East (500 pts.): 2.33

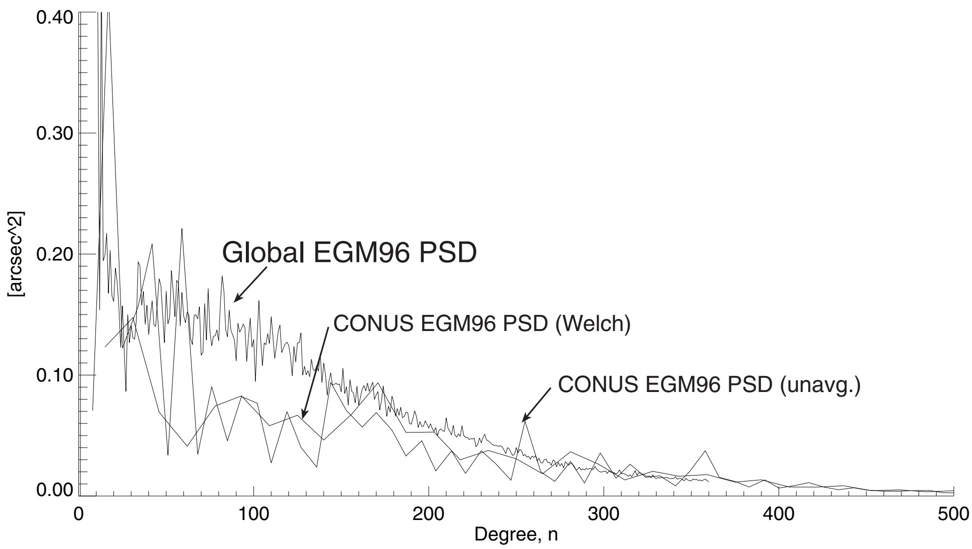

Fig. 3 - Degree variances and local PSD of the EGM96 vertical deflection.

coefficient error, we may write

$$\sigma^2_{\Delta\Theta} \approx \sigma^2_{\Theta,trunc} + \sigma^2_{\Theta,coeff.err} + \sigma^2_{\Theta,astro.err} \tag{4}$$

For EGM96, $\sigma^2_{\Theta,coeff.err}$=(1.80 arcsec)^2, and for the 3561 astronomic deflections, the rms of the error variances is $\sigma^2_{\Theta,astro.err}$=(0.49 arcsec)^2. Therefore, using the entry in the first column of Table 1, a reasonable estimate of the truncation error variance for CONUS is

$$\hat{\sigma}^2_{\Theta,trunc} = (4.27)^2 - (1.80)^2 - (0.49)^2 = (3.84 \ \text{arcsec})^2 \tag{5}$$

Estimates for the truncation error in each of the six regions are estimated similarly and shown in Table 2.

With a view toward evaluating these empirical values of $\sigma^2_{\Theta,trunc}$, Fig. 2 shows the degree variances of the deflection vector for the EGM96 model and a putative analytic approximation and continuation for the degrees higher than n_{max}. The truncation error estimated from this continuation is 1.33 arcsec.

This value which reasonably represents the truncation error of EGM96 is significantly lower than the truncation error estimates based on the astrogeodetic deflections for CONUS and for most of the regions of CONUS. In order for the analytic value of the truncation error to agree

Table 2 - Estimated truncation error of EGM96 deflections, |Θ| [arcsec].

CONUS: 3.84	North-West (489 pts.): 4.85	North-Central (468 pts.): 1.85	North-East (405 pts.): 1.54
	South-West (1081 pts.): 5.85	South-Central (618 pts.): 1.15	South-East (500 pts.): 1.14

with the estimates of Table 2 the high-degree power spectrum of EGM96 model would have to be greater. A note of caution must accompany this speculation, since it is based on only a relatively small part of the globe. Moreover, the distribution of astronomic deflections on which the analysis is based is not uniform even in this small part (namely, CONUS).

One question that can be answered is whether the global degree variances, as shown in Fig. 2, are representative of CONUS. The observed discrepancies in truncation error would be explained if the power spectrum of EGM96 for CONUS is consistently higher than the global degree variances. For then, also the truncation error would be higher than the global value. However, this seems not to be the case. Figure 3 compares the degree variances with the power spectrum of EGM96 deflections evaluated on a 10'×10' grid. The power spectrum was computed using the unaveraged periodogram method, as well as the Welch method, whereby four periodograms of neighboring grid are averaged (Marple, 1987).

4. Summary

A numerical comparison between EGM96 and astrogeodetic deflections in the conterminous U.S. shows that the estimated truncation effect from the astrogeodetic data is inconsistent with the EGM96 model. This result is based on both rms and power spectrum tests and indicates that EGM96 may be under-powered at high frequencies. Further tests and analyses must be conducted, however, to arrive at a definite conclusion.

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