Improving local gravimetric geoid models with external data

R. HAAGMANS[1] and E. DE MIN[2]

[1]*Delf institute for Earth Oriented Space research (DEOS), The Netherlands*
[2]*Survey Department of Rijkswaterstaat, The Netherlands*

(Received August 4, 1998; accepted October 5, 1999)

Abstract. Local gravimetric geoid models from a combination of a global geopotential model and local gravity anomalies, usually contain errors of dm-level on wavelengths longer than 50 km. One of the main causes for this is the limited precision of the global models. External geoid information on discrete points, like GPS and levelling sites on land and altimeter tracks, combined with permanent sea surface to-pography at sea, is often available with cm-precision. These points can be used to correct the medium and longer wavelength errors in the gravimetric geoid. The problem is to find an adequate functional representation of the correction surface. The authors have developed a method of investiging the form of this correction, and finding empirical representations depending on area size. Once a class of functions have been selected the most suitable can be found by a statistical testing procedure.

1. Introduction

In gravimetric geoid computation procedures local gravity data are usually combined with a global geopotential model ((3CM). In principle an optional combination based upon realistic error characteristics of the globally available gravity data and a (3CM should lead to the best possible gravimetric geoid result; however, this is limited due to systematic errors, at regional scales, from terrain reductions, height datums, etc. (cf. Pavlis, 1988). In regional computations, however, dense gravity data are only available in a restricted area. This spatial limitation and the fact that one likes to fully exploit the advantages of the local gravity data and the global model lead to a practical by optimal choice in a combined solution. The advantage of the local gravity is that it provides all details in the geoid solution at small and medium scales. The advantage of

Corresponding author: R.H.N. Haagmans; pesent affiliation: The Agricultural University of Norway, Department of Mapping Sciences, P.O. Box 5034, N-1432 Äs, Norway; phone: +47 64 948846; fax +47 64 948856; e-mail: roger.haagmans@ikf.nlh.no

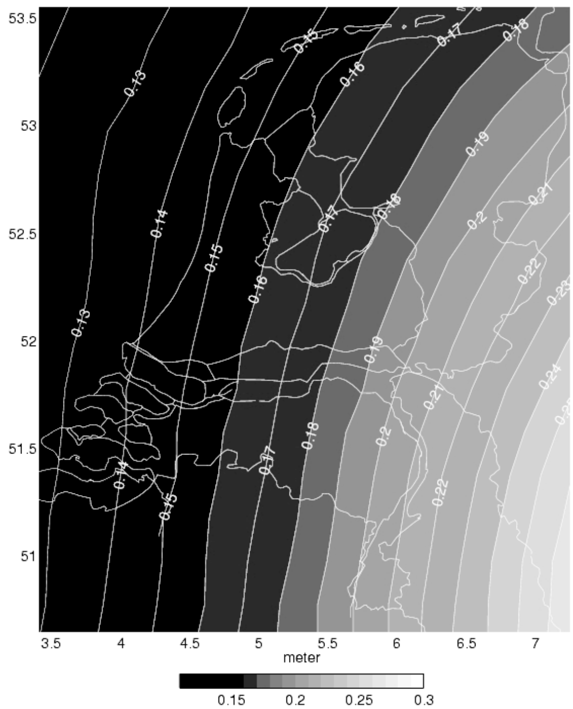

Fig. 1 - Difference between total gravimetric geoid in the Netherlands based on OSU91 and EGM96.

the global models is that the long wavelength solution is best resolved from satellite data instead of gravity data. In the case of a specific weighing, between local gravity data and a global model, the differences in the total gravimetric geoid computations based upon OSU91 and EGM96 for the Netherlands may differ as shown in Fig. 1. From Fig. 1 we can observe that the difference only exhibits a longer wave-length pattern and no smaller scale details which is of practical importance for GPS and levelling applications. Accepting the fact that regional gravimetric geoid solutions may always be contaminated by errors at medium scales (cf. also Sideris and Li, 1992) we try to find a procedure to combine the gravimetric geoid with external geoid data from GPS (h) and levelling (H) on land, and altimetry (h) with a permanent sea surface topography model (H) at sea, all with proper quality measurements. The problem is to find an adequate functional description for the correction surface F_c in Eq. (1).

$$h - H = N = N_g + F_c. \tag{1}$$

Generally, these are chosen as trend functions of bi-linear type or similar (cf. Sideris and She, 1995; Forsberg et al., 1997). Usually, a profound reasoning for choosing such a specific function is missing. Therefore, we tried to find a procedure that can be applied for all gravimetric geoid results and that is applicable and adaptable to areas of different sizes.

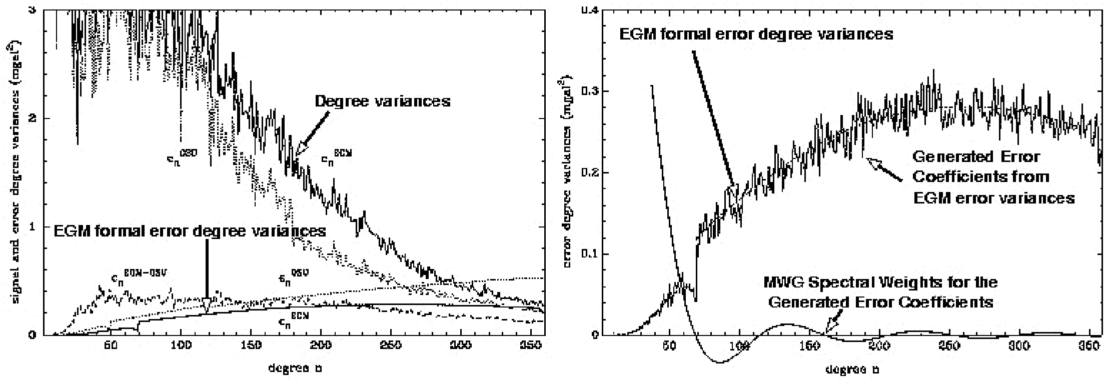

Fig. 2 - Generation of random error coefficients based on EGM96 including the Meissl/Wong&Gore (MWG) weigh s (right) based on the formal coefficient error variances (left).

2. Procedure

The procedure we followed can be divided into several steps:
- generate likely 6CM geoid error surfaces;
- find adequate empirical representations for the surface, out of a set of functions;
- fit the empirical function to the residuals between the gravimetric and external geoid;
- apply a statistical test procedure for finding the best representation.

The possible shape of the correction surface will be analyzed based upon the chosen GGM with its formal error description and the assigned weighing in the procedure. First, several possible sets of error coefficients per degree n and order m (E_{nm}) are generated from the formal standard deviations of the coefficients of EGM96, assuming the errors per coefficient to be normally distributed. For each set error geoid surfaces can be obtained from the error coefficients weighted with w_n, as shown in Eq. (2) (de Min, 1996):

$$N_e = \frac{GM}{\gamma r} \sum_{n=2}^{360} \left(\frac{a}{r}\right)^n \lambda_n w_n \sum_{m=0}^{n} E_{nm} \quad Y_{nm}(\varphi, \lambda). \tag{2}$$

The weights w_n can be in an idealized case Shannon weights, 1, up to 360 and 0 for higher degrees, or Molodenskii weights in case of spatial truncation of Stokes' function, or weights according to the kernel modifications like Meissl, and Meissl/Wong&Gore (Heck and Grüninger, 1987; de Min, 1996; de Bruijne et al., 1997). An example of degree variances based on generated error coefficients for EGM96 is shown in the right part of Fig. 2, together with the Meissl/Wong&Gore weights for coefficients. In the left part of Fig. 2, the signal and error degree variances of OSU91 and EGM96, and the difference between OSU91 and EGM96 are shown for comparison. For the North Sea area, 10 error surfaces were randomly generated according to Eq. (2) with MWG weights for degree parameter 32 and spherical capsize of 4°, the surfaces show a range of 16-23 cm and a RMS of 3.6-6.4 cm, *cf.* Fig. 3.

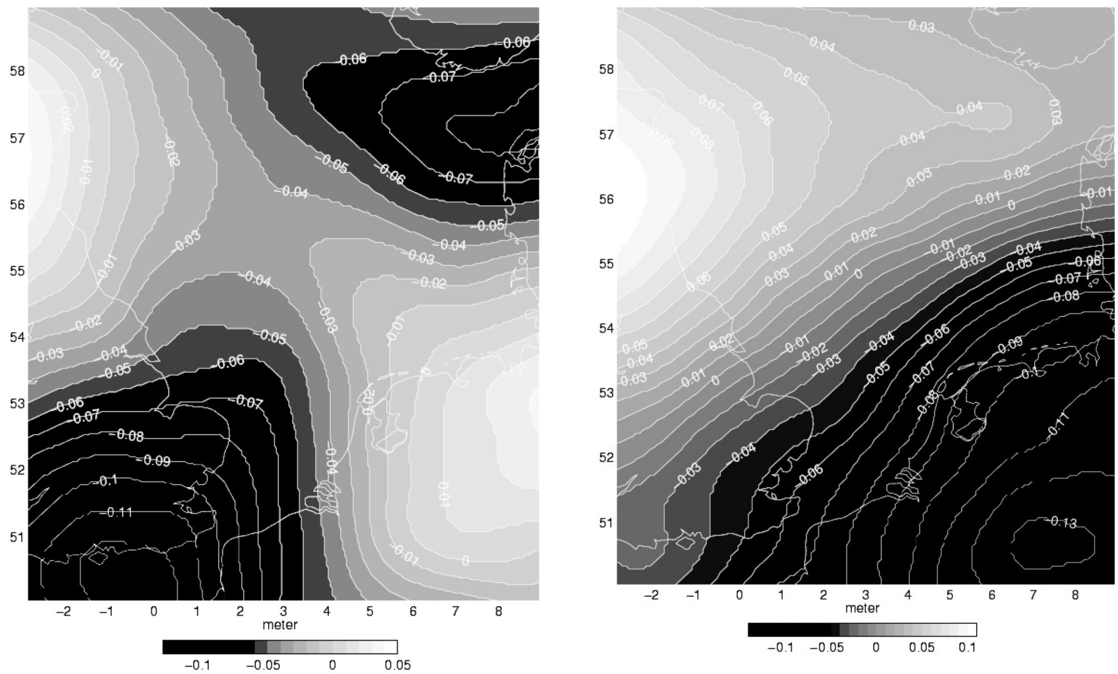

Fig. 3 - Examples of randomly generated geoid error surfaces based on EGM96 and MWG modification.

The next step is to select a class of functions for empirical modelling of these surfaces. Generally, this can be e.g. polynomials, wavelets, harmonic base functions depending on the surface characteristics and the area extent. For the North Sea a bi-linear trend function and trigonometric functions are selected, based on a Fourier analysis, which are symbolically represented in Eqs. (3) and (4). λ_l and φ_k indicate longitude and latitude increments relative to a chosen origin in the area. From the Fourier analysis of the 10 surfaces it appeared that the maximum limit for I and J is 2. In 60%, a 12-parameter model and in 40%, a 28-parameter model was necessary to reduce the unmodelled negligible residual below a 1 cm rms:

$$a_{00} + b_{00}\lambda_l + c_{00}\varphi_k + d_{00}\varphi_k\lambda_l, \tag{3}$$

and

$$\sum_{i=1}^{i}\sum_{j=1}^{J} a_{ij}\cos(i\lambda_l)\cos(j\varphi_k) + b_{ij}\sin(i\lambda_l)\cos(j\varphi_k) + c_{ij}\cos(i\lambda_l)\sin(j\varphi_k) + \\ + d_{ij}\sin(i\lambda_l)\sin(j\varphi_k). \tag{4}$$

Examination of Fig. 2 reveals that the error estimates for EGM96 may be too optimistic by a factor of 2-3, in the range between degree 2-70 from comparison with OSU91. Thus, the previous results need to be scaled to a 2-3 cm unmodelled residual, which is in the range of the

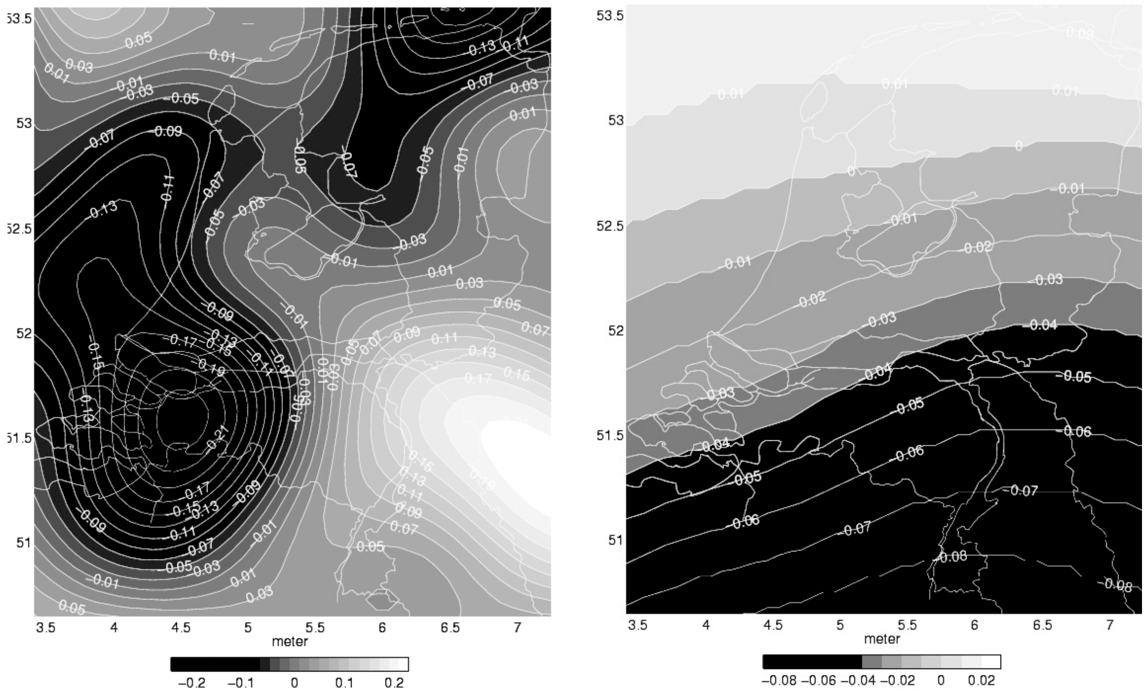

Fig. 4 - Randomly generated geoid error surfaces based on EGM96 anbd with Molodenskii weights (left) and with MWG modification (right).

precision of the external geoid data, so that no extension of the correction model is necessary. N.B. it is in principle possible to extend the model with more bias parameters when land data from different height datum are involved.

The final step is to fit the parameters to the residuals N-N_9 of Eq. (1) in a least squares adjustment, with an overall model test, and iterative data snooping. The model can be extended and tested against others in order to select the optimal one within the class of functions, following the principles developed for the deformation analysis (de Heus et al., 1995). Careful analysis of the geoid error surface and suitable correction functions limits the number of possible and acceptable correction surface parameters. This procedure has been successfully applied for the computation of the preliminary North Sea geoid GEONZ97 (de Bruijne et al., 1997).

3. Conclusions

A procedure is proposed: to correct the longer wavelength errors in the gravimetric geoid by means of an adequately chosen empirical function, based upon geoid error surfaces generated from the formal errors of a GGM. It mainly depends on the weighing between local data and a global model. A standard approach with Molodenskii weights results in a rather irregular geoid error surface for the Netheriands (see Fig. 4 left), that is rather complex to model. The MWG modification shows a smooth trend surface (see Fig. 4 right). Modelling this by means of external geoid data, results in the elimination of the trend surface, but also in the difference between two gravimetric geoid solutions, as shown in Fig. 1: the final geoids will be practically identical. Thus a proper weighing or kernel modification is important. The procedure can easily be extended to larger areas, avoiding unnatural blending of neighboring solutions.

References

de Bruijne A. J. Th., Haagmans R. H. N. and de Min E. J.; 1997: *A preliminary North Sea Geoid model GEONZ97*. MD report, Survey Department, Rijkswaterstaat, MDGAP-9735.

Forsberg R., Kaininskis J. and Soiheim D.; 1997: *Geoid of Nordic and Baltic Region from gravimetry and satellite Altimetry.* IAG no. 117 Gravity, geoid and marine geodesy, Springer Berlin, pp. 540-547.

Heck B. and Grüninger W.; 1987: *Modification of Stokes' integral formula by combining two classical approaches.* Proc. of the XIX IUGG General Assembly, IAG vol. 11, pp. 319-337.

de Heus H., Joosten P., Martens M. and Verhoef H.; 1995: *Strategy for the analysis of the Groningergasfield levellings: an overview.* In: Barends, Brouwer, Schröder (eds), Landsubsidence pp. 301-311.

de Min E. J.; 1996: *De geoïde voor Nederland.* Ph. D. Thesis, Delft University of Technology.

Pavlis N. K.; 1988: *Modeling and estimation of a low degree geopotential model from terrestrial gravity data.* Department of Geodetic Science and Surveying, OSU Report no. 386, Columbus Ohio.

Sideris M. G. and Li Y.; 1992: *Improved geoid determination for levelling by GPS.* In: Proc. of the Sixth Int. Geodetic Symposium on Satellite Positioning Vol. II, Columbus Ohio, pp. 873-882.

Sideris, M. G. and She B. B.; 1995: *A new, high-resolution geoid for Canada and part of the U.S. by the 1D-FFT method.* Bull. Geod., **69**, 2, 92-108.