Development and evaluation of a new Canadian geoid model

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Abstract. The purpose of this paper is to present the development and evaluation of a new gravimetric geoid model (GARR98) for Canada and parts of the U.S., created in the Dept. of Geomatics Engineering at the University of Calgary. The methodology applied for the computation of the new geoid, and the data types that were used, are discussed. GARR98 uses the most current databases available for Canada, namely, new additional surface gravity data, high resolution DEM model, and a more accurate geopotential model (EGM96). Comparisons among the new geoid, geopotential models (OSU91A and EGM96), the latest GSD95 Canadian geoid model, and GPS/levelling data, are also presented. Absolute and relative differences at 1300 GPS benchmarks are computed, on both national and regional scales. These external comparisons reveal interesting information regarding the behavior of the Canadian gravity field, the quality of the geoid models, and the achievable accuracy in view of future GPS/levelling applications.

1. Introduction

The benefits achieved from combining GPS observations with geoid information was the main motivation behind the pursuit of computing a new gravimetric geoid model for Canada and parts of the U.S. Since the last Canadian geoid model was created by the Geodetic Survey Division in 1995 (GSD95; see, Véronneau, 1997), new gravity field data have been obtained which could be used to update the geoid information, resulting in a more accurate representation of the Canadian gravity field. The geoid heights obtained from this model could then be used, in conjunction with GPS ellipsoidal heights, in order to compute orthometric heights practically everywhere in Canada.

In the past, traditional spirit levelling has been used to obtain orthometric heights with very

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high accuracy. However it is not a feasible method for obtaining absolute orthometric heights in large unsurveyed territories, such as northern Canada, due to the absence of absolute vertical control points. On the other hand, the versatility and accuracy of GPS has brought to the forefront of the surveying industry the importance of an accurate geoid, which allows for the use of GPS-based levelling techniques as an efficient alternative over traditional levelling. This paper outlines the data and methodology used for the development of the new geoid model, and evaluates its accuracy based on comparisons with already existing geoid models and GPS/levelling data at 1300 GPS benchmarks across Canada.

2. Computational methodology

The computation of the new Canadian geoid model (GARR98) was based on the classic "remove-compute-restore" technique (see, e.g., Rapp and Rummel, 1975; Mainville et al., 1992; Sideris et al., 1992). The treatment of the terrain effects was also based on Helmert's second method of condensation (see, e.g., Wichiencharoen, 1982; Heiskanen and Moritz, 1967, p.145). The underlying procedure can be summarized in three basic steps as follows:

1. *remove* a long-wavelength gravity anomaly field (determined by a global spherical harmonic model) from Helmert gravity anomalies that are computed from local surface gravity measurements and digital elevation data. In this way, "residual Helmert anomalies" are obtained. Faye anomalies were actually used to approximate the Helmert gravity anomalies in this study. For details regarding the definition and properties of Helmert and Faye gravity anomalies, see Heiskanen and Moritz (1967), and Wichiencharoen (1982);
2. *compute* "residual co-geoid undulations" $N_{\Delta g}$ by a spherical Fourier representation of Stokes' convolution integral using the residual Faye gravity anomalies;
3. *restore* a long-wavelength geoid undulation field N_{GM} (determined by a global spherical harmonic model) to the residual co-geoid undulations, and add a topographic indirect effect term N_H (computed from digital elevation data) to form the final geoid undulations.

These steps can be combined in a single formula as follows:

$$N = N_{GM} + N_{\Delta g} + N_H \tag{1}$$

The computation of the long-wavelength geoid component N_{GM} was made on a 5'× 5' grid, within the following geographical boundaries: northern 72°N, southern 41°N, western 142°W, and eastern 53°W. This is also the grid configuration on which the final GARR98 geoid heights are given. The EGM96 global geopotential model (Lemoine et al., 1997), complete to degree and order 360, was used for these computations according to the following formula (Heiskanen and Moritz, 1967):

$$N_{GM}(\phi\lambda) = R\sum_{n=2}^{360}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\bar{P}_{nm}(\sin\phi) \tag{2}$$

where $\bar{P}_{nm}$ are fully normalized Legendre functions, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully normalized unitless coefficients of the geopotential model (from which the contribution of a normal gravity field, based on the GRS80 geodetic reference system, has been subtracted), and R is the mean radius of the Earth. An initial comparison at the above grid between EGM96 and the older global model OSU91A (Rapp et al., 1991), which was used in the development of the latest GSD95 Canadian geoid, showed an RMS geoid undulation difference of 97 cm. Further information and tests for the performance of the EGM96 geopotential model over the Canadian region can be found in the special issue of the bulletin of the International Geoid Service (IGeS, 1997).

The medium-wavelength contributions to the total geoid heights were computed from the local gravity anomaly data according to Stokes' formula (Heiskanen and Moritz, 1967)

$$N_{\Delta g}(\phi_P, \lambda_P) = \frac{R}{4\pi\gamma} \int\limits_{\lambda_Q}\int\limits_{\phi_Q} \Delta g(\phi_Q, \lambda_Q) S(\psi_{PQ}) \cos\phi_Q d\phi_Q d\lambda_Q \tag{3}$$

where $S(\psi_{PQ})$ is the Stokes function, γ is a representative mean normal gravity value on the reference ellipsoid, and the local data Δg are residual Faye anomalies, i.e.

$$\Delta g = \Delta g_{FA} + c - \Delta g_{GM} \tag{4}$$

In the last equation, Δg_{FA} are the usual free-air anomalies, c is the classic *terrain correction* term (Heiskanen and Moritz, 1967; Mainville et al., 1994), and Δg_{GM} is the removed long-wavelength contribution of the global geopotential model which is computed from the expression,

$$\Delta g_{GM} = \gamma \sum_{n=2}^{360} (n-1) \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\sin\phi) \tag{5}$$

The gravity data used in the computations were obtained from the Geodetic Survey Division (GSD) of Geomatics Canada in the form of a 5' × 5' grid of mean Faye anomalies (corrected also for atmospheric effects), within the geographical boundaries mentioned above. This is essentially a smaller part of the gravity anomaly grid used in the development of the GSD95 geoid model. The original GSD grid covered a slightly larger region, which extended its eastern boundary to W46° and included half of the Greenland area and most of the Labrador sea. The average spacing of the surface gravity measurements used for the gridding was approximately 10 km on land, and 1 km over the oceans in Canada. A description of the Canadian gravity database and the followed gridding procedures can be found in Véronneau (1997). A discussion of the treatment and computation of all the necessary reductions (atmospheric, free-air gradient, downward continuation, terrain reduction) applied to the original gravity data is included in Véronneau (1997) and Mainville et al. (1994). The evaluation of Stokes' integral in Eq. (3) was performed by the 1D spherical FFT algorithm (Haagmans et al., 1993), according to the expression

$$N_{\Delta g}(\phi_P,\lambda_P)=\frac{R\Delta\phi\Delta\lambda}{4\pi\gamma}\mathbf{F}^{-1}\left\{\sum_{\phi_Q=\phi_1}^{\phi_{\max}}\mathbf{F}\left[S(\psi_{PQ})\right]\mathbf{F}\left[\Delta g(\phi_Q,\lambda_Q)\cos\phi_Q\right]\right\} \tag{6}$$

where the operators **F** and $\mathbf{F}^{-1}$ denote the forward and inverse 1D discrete Fourier transform, $\Delta\phi=\Delta\lambda=5'$ is the used grid spacing, and ϕ_1, ϕ_{max} are the southern and northern grid boundaries respectively. The gravity anomaly input grid had 50% zero padding applied on its east and west edges, but none on the north and south sides. This is because Eq. (6) performs the FFT in the east/west direction, and thus zero padding is only needed on those two edges to eliminate circular convolution effects. Also, the values of the Stokes spherical kernel S were analytically computed at all points of the rectangular zero-padded grid, and its discrete spectrum was subsequently used in Eq.(6).

The shorter wavelength information for the GARR98 geoid model was obtained from the computation of the indirect effect term N_H, induced by using Helmert's second condensation method for the gravity data reduction on the geoid surface. In general, the formulation of the topographic indirect effect on the geoid, according to Helmert's second condensation method, is made in terms of a Taylor series expansion from which only the first three terms are usually considered:

$$N_H=\delta N_0+\delta N_1+\delta N_2 \tag{7}$$

Wichiencharoen (1982) should be consulted for all the detailed formulas related to the three above terms. In our case, only the zero-order term

$$\delta N_0=-\frac{\pi G\rho H_{DEM}^2}{\gamma} \tag{8}$$

was used for the geoid computation. The same approximation was also adopted in the construction of the GSD95 geoid model. The height data used to evaluate Eq. (8) were obtained from a 1km × 1km Digital Elevation Model (DEM) which covered most of the western Canadian region (N67°, N47°, W135°, W110°). The resulting indirect effect values were then averaged to form a 5' × 5' grid. In contrast to the GSD95 geoid solution, which additionally used the global ETOPO5-DEM and the Digital Terrain Elevation Data (DTED) Level 1 (DMA, 1981) to obtain a terrain coverage for all of Canada, no other height data sources were incorporated in the GARR98 model. In this way, no indirect effect values for the geoid undulations have been computed in central/eastern Canada for GARR98. The resulting geoid error caused from neglecting the indirect effect in these relatively flat areas is studied in the next sections. The final GARR98 geoid model on a 5 × 5 grid is illustrated in Fig. 1.

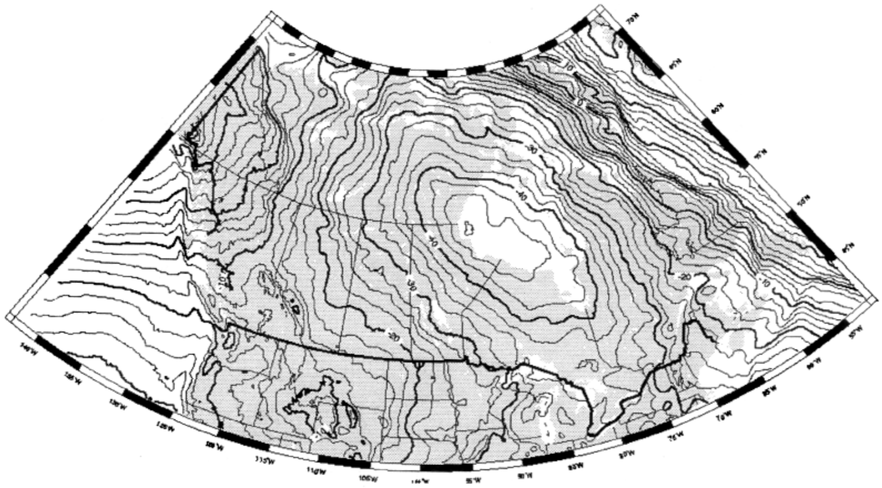

Fig. 1 - The GARR98 Geoid Model (contour interval: 2.5 m).

3. Comparisons of various geoid models

Various comparisons among different geoid models were made on the 5' × 5' grid used to compute GARR98. Some of these differences are shown on the shaded contour plots in Figures 2 and 3, and their statistics are given in Table 1. The comparison between EGM96 and OSU91A (Fig. 2) reveals the strong effect of the additional surface gravity data in the western and the northeastern parts of Canada, which were incorporated in the determination of EGM96. This new gravity data result in up to 10 m geoid difference in northwestern Canada and the area around Greenland. Extreme differences are also seen in the Rocky Mountain region due to its rugged terrain features, which suggests the importance of using dense gravity coverage in order to recover the lower-wavelength gravity field information. The highest level of agreement between the two GMs is found in central Canada (~50 cm), which decreases to 1.5 m in parts of eastern Canada.

The differences between the two gravimetric solutions, GARR98 and GSD95, are shown in Fig. 3. The RMS agreement of the two models is at the 37 cm level. The maximum difference values averaging 1-2 m (with extreme values up to ~4 m) are seen in the Hudson's Bay area, as well as around the shores of the Grand Banks and in the area of the Davis Strait, off the northeastern coast of Canada. These differences can be partially correlated with the differences

Table 1 - Statistics for various geoid model differences.

Geoid model	Min (m)	Max (m)	Mean (m)	RMS (m)	σ (m)
EGM96-OSU91A	-10.57	6.95	-0.07	0.97	0.96
GSD95-EGM96	-5.45	5.99	-0.01	0.58	0.58
GARR98-GSD95	-1.62	3.62	0.00	0.37	0.37

between the two GMs (see Fig. 2), and they may also result from the extended local gravity grid used in GSD95 (see section 2). The two gravimetric geoids have the highest level of agreement in the Canadian Shield area, central British Columbia, southern Alberta, Saskatchewan and northern Quebec. High levels of disagreement are seen in the Northwest Territories, the northern tip of Labrador. The coastal range surrounding the Vancouver area is also fairly poor in terms of agreement. The range of differences in the western region is approximately 25-50 cm in parts of British Columbia, and reduces to 5-25 cm moving north.

These discrepancies may be partly attributed to the extended DEM in western Canada, which was used in the computation of GARR98, as well as to the additional surface gravity information. The large differences between the two supporting GMs in the western regions are also seen, on a smaller scale, between the two gravimetric solutions.

4. Comparisons at GPS benchmarks

An external evaluation of the quality of a gravimetric geoid model can be performed by comparing its interpolated values (N) at a network of GPS benchmarks with the corresponding GPS/levelling-derived geoid heights (N^{GPS}). Such a comparison is traditionally based on the following simple transformation model (see, e.g., Forsberg and Madsen, 1990; Sideris et al., 1992):

$$N_i^{GPS} - N_i = h_i - H_i - N_i = x_0 + x_1 \cos\phi_i \cos\lambda_i + x_2 \cos\phi_i \sin\lambda_i + x_3 \sin\phi_i + v_i \qquad (9)$$

which is solved for the unknown parameters (x_0, x_1, x_2, x_3,) by minimizing the sum of the squared residuals v_i. The adjusted values for the residuals give a realistic picture of the level of absolute agreement between the gravimetric geoid and the GPS/levelling data. The four-parameter model in Eq.(9) absorbs most of the datum inconsistencies among the available height data sets, as well as possible long-wavelength geoid biases. In such comparisons, it should always be kept in mind that the final residual values v_i are not just purely gravimetric geoid errors, but they contain levelling and GPS positioning errors as well; for some more sophisticated adjustment models that can be employed in such 1D multi-data networks, see Kotsakis and Sideris (1999). A detailed description of the above transformation model, along with its geometrical interpretation, can be found in Heiskanen and Moritz (1967, p. 213).

A total of 1300 GPS stations, which are all part of the first-order Canadian levelling network, were used for the evaluation. Their known ellipsoidal heights refer to the ITRF92 reference frame, whereas their Helmert orthometric heights were obtained from the latest adjustment of the Canadian levelling network by GSD in 1995. Four geoid models were used for these comparisons, namely, GARR98, GSD95, EGM96 and OSU91A.

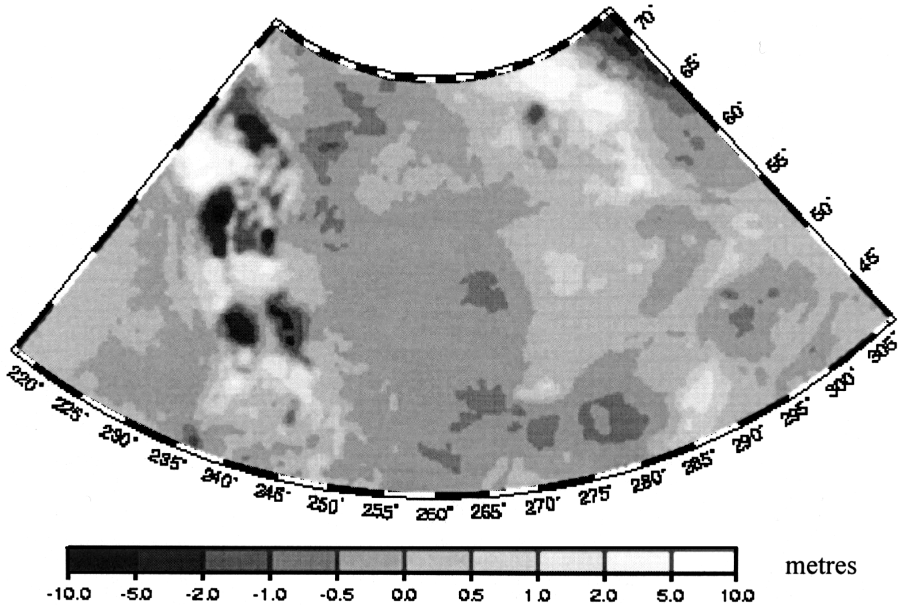

Fig. 2 - Geoid differences between EGM96 and OSU91A models.

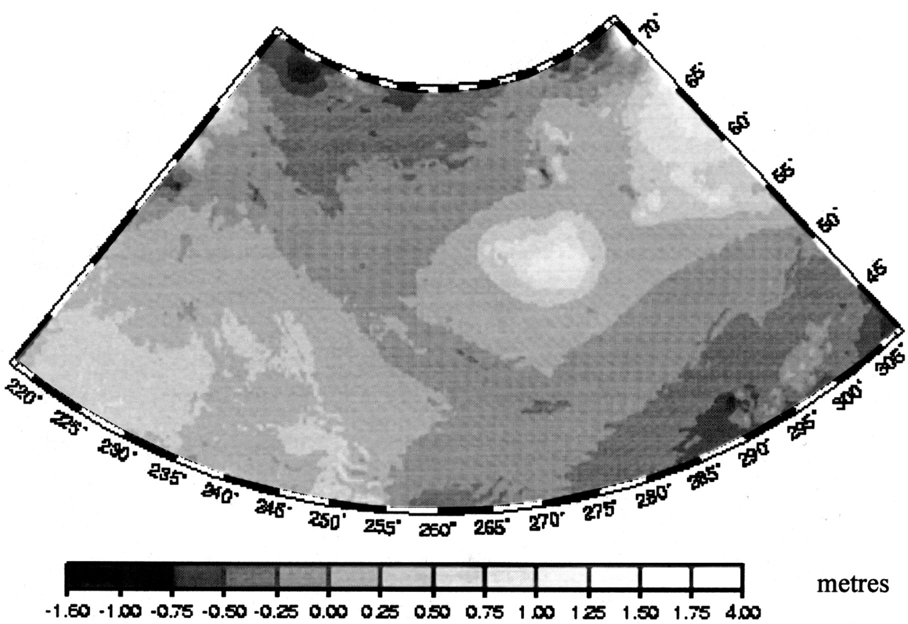

Fig. 3 - Geoid differences between GARR98 and GSD95 models.

4.1 Absolute agreement of geoid models with respect to GPS/levelling.

Table 2 shows the statistics of the absolute differences for the entire Canadian region, as well as for three separate major regions, namely (i) British Columbia and Alberta, (ii) central Canada (95°W<λ<110°W), and (iii) eastern Canada (east of 95°W). The values inside the parentheses (see Table 2) are the results after the fitting of the 4-parameter transformation model. For all comparisons presented in this section, the GPS benchmarks showing large residual values (i.e. > 3σ level) before and after the transformation model was applied were removed from the analysis, since they were suspected for possible blunders.

From the statistics shown in Table 2, it can be seen that the GARR98 gravimetrically derived geoid, with the support of the EGM96 global model, drastically improves the overall agreement with the GPS/levelling-derived geoid in Canada, from a σ value of 44 cm to 20 cm. This significant amount is due to the more accurate long-wavelength information contained in EGM96, compared to OSU91A which supports the GSD95 geoid model. After the fit, however, both GARR98 and GSD95 present approximately the same overall external accuracy, which is at the 13-14 cm level. This would suggest that, even with the use of EGM96, the present gravity data accuracy and resolution still needs to be improved in Canada, in order to bring the absolute geoid consistency with the GPS/levelling data down to the cm-level. It is also interesting to note the superiority of the EGM96 global model, over OSU91A, for describing the long-wavelength structure of the Canadian gravity field. After the four-parameter transformation, EGM96 alone fits the Canadian GPS/levelling geoid with an overall RMS accuracy of 31 cm, whereas OSU91A cannot perform better than 67 cm on a national scale.

A regional analysis of the differences reveals the interesting result of having the same level of absolute agreement (after the datum fit) for the gravimetric geoid solutions, in both western and eastern Canada. This is not surprising and it simply confirms that the strong terrain gravity signal in the western region has been properly modeled in both GSD95 and GARR98 models. In the western area, the two global geoid models provide an RMS agreement with the GPS/levelling data of 38 cm (EGM96) and 67 cm (OSU91A), illustrating again the superiority of the EGM96 model.

In central Canada, GSD95 seems to perform slightly better than GARR98, with the former model giving an RMS accuracy at the GPS benchmarks of 6 cm and the latter 9 cm. A possible reason for this difference in accuracy between the two gravimetric models is the incorporation in the GSD95 solution of height data for this region (i.e. indirect effect), in contrast to GARR98 which uses a DEM only for western Canada. In addition, no improvement in EGM96 over OSU91A occurs in this area, which might have been reflected in the corresponding local gravimetric solutions. More interesting, however, is the fact that the use of GPS, in conjunction with a global geoid model alone, seems to be sufficient for levelling applications in central Canada requiring dm-level accuracy. Both global models represent the gravity field in the central flat areas quite well, with an overall agreement level of 12 cm for both EGM96 and OSU91A. All four geoid models achieve their best performance in this area.

In the eastern part of Canada both gravimetric geoids show similar results, with GSD95 (12 cm) being marginally better than GARR98 (13 cm). Again, the fact that additional gravity and

Table 2 - Comparison of various geoid models with the GPS/levelling-derived geoid.

Geoid model	Min (m)	Max (m)	Mean (m)	RMS (m)	σ (m)
All of Canada					
N^{GPS}-*GARR98*	-2.31 **(-0.64)**	-1.20 **(0.50)**	-1.77 **(0.00)**	1.78 **(0.14)**	0.20 **(0.13)**
N^{GPS}-*GSD95*	-1.89 **(-0.45)**	0.04 **(0.54)**	-1.13 **(0.00)**	1.21 **(0.13)**	0.44 **(0.14)**
N^{GPS}-*EGM96*	-2.55 **(-1.27)**	0.41 **(1.63)**	-1.09 **(0.00)**	1.15 **(0.31)**	0.37 **(0.31)**
N^{GPS}-*OSU91A*	-5.92 **(-4.73)**	4.04 **(5.13)**	-1.14 **(0.00)**	1.34 **(0.67)**	0.69 **(0.67)**
BC and Alberta					
N^{GPS}-*GARR98*	-2.18 **(-0.35)**	-1.47 **(0.36)**	-1.84 **(0.00)**	1.85 **(0.11)**	0.11 **(0.11)**
N^{GPS}-*GSD95*	-1.89 **(-0.40)**	-0.92 **(0.39)**	-1.39 **(0.00)**	1.39 **(0.11)**	0.13 **(0.11)**
N^{GPS}-*EGM96*	-2.55 **(-1.24)**	0.41 **(1.56)**	-1.24 **(0.00)**	1.30 **(0.38)**	0.39 **(0.38)**
N^{GPS}-*OSU91A*	-5.92 **(-4.67)**	1.49 **(2.82)**	-1.35 **(0.00)**	1.52 **(0.66)**	0.68 **(0.67)**
Central Canada					
N^{GPS}-*GARR98*	-2.14 **(-0.18)**	-1.59 **(0.31)**	-1.91 **(0.00)**	1.92 **(0.09)**	0.10 **(0.09)**
N^{GPS}-*GSD95*	-1.65 **(-0.22)**	-1.15 **(0.16)**	-1.37 **(0.00)**	1.37 **(0.06)**	0.08 **(0.06)**
N^{GPS}-*EGM96*	-1.63 **(-0.49)**	-0.79 **(0.28)**	-1.05 **(0.00)**	1.06 **(0.12)**	0.13 **(0.12)**
N^{GPS}-*OSU91A*	-1.48 **(-0.40)**	-0.83 **(0.31)**	-1.12 **(0.00)**	1.13 **(0.12)**	0.14 **(0.12)**
Eastern Canada					
N^{GPS}-*GARR98*	-2.31 **(-0.49)**	-1.20 **(0.44)**	-1.58 **(0.00)**	1.60 **(0.14)**	0.22 **(0.13)**
N^{GPS}-*GSD95*	-1.31 **(-0.49)**	0.04 **(0.43)**	-0.55 **(0.00)**	0.63 **(0.12)**	0.30 **(0.12)**
N^{GPS}-*EGM96*	-1.59 **(-0.67)**	0.12 **(0.89)**	-0.83 **(0.00)**	0.87 **(0.24)**	0.25 **(0.24)**
N^{GPS}-*OSU91A*	-1.61 **(-0.64)**	-0.05 **(0.83)**	-0.86 **(0.00)**	0.90 **(0.24)**	0.28 **(0.24)**

height data were used for this region in the GSD95 solution (see section 2) is probably causing this difference. In the case where a global geoid model is only employed for the eastern region, the level of agreement worsens by more than 10 cm, reaching 24 cm for both EGM96 and OSU91A.

An interesting observation can also be made by comparing the σ values shown in Table 2 for the three regional areas in Canada, before and after the four-parameter datum fit. It should be noted that the results for these individual regions correspond to three separate least-squares adjustments performed according to the basic model of Eq. (9), and using only the GPS benchmarks inside the corresponding regions. The accuracy improvement due to the datum fit is, in almost all cases, approximately 1-4 cm. The only exception occurs for GSD95 and GARR98 in Eastern Canada, where the sigma values are reduced by 18 cm and 9 cm, respectively. This deviation is possibly due to the fact that the parametric model of Eq. (9), combined with improper weighting of the original residuals, not only eliminates the datum differences among the available data sets, but also absorbs a part of gravimetric geoid *random* error caused by the extended amount of low quality shipborne data used in both GSD95 and GARR98 gravity grids. Concluding this final remark, we could say that further investigations need to be done and more attention should be paid to the actual behavior of the adjustment model in Eq. (9), which has

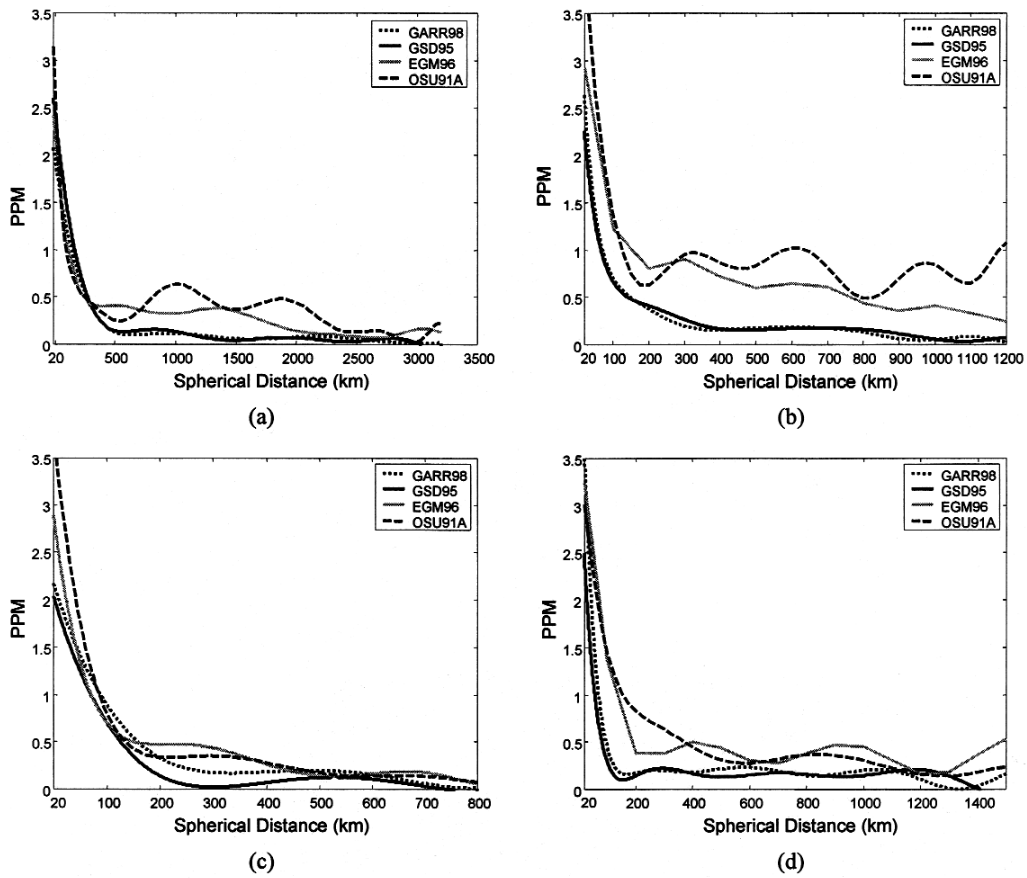

Fig. 4 - Relative Accuracy of the Geoid Models (a) across Canada, (b) Western Canada, (c) Central Canada, (d) Eastern Canada.

routinely been used for "external" geoid evaluation studies in the geodetic community; for more details see Kotsakis and Sideris (1999).

4.2 Relative agreement of geoid models with respect to GPS/levelling.

In order to evaluate the relative accuracy of the four geoid models with respect to the GPS/levelling data, relative undulation differences (ΔN-ΔN^{GPS}) in parts per million (ppm) were formed, and plotted as functions of the baseline lengths. Fig. 4 illustrates these results for all of Canada, and for the three separate regions previously considered. The relative differences refer to the values obtained after checking the GPS/levelling data for outliers and after the fitting of the transformation model of Eq. (9), as described previously.

At a national scale, the two global geopotential models show similar relative accuracies for baseline lengths up to 350 km, ranging from 3.5 ppm to 0.5 ppm. For longer baselines, we begin to see the improved long wavelength structure of EGM96 as compared to OSU91A, with EGM96 giving approximately 0.3 ppm for 1500 km baselines, whereas OSU91A averages to about 0.5 ppm for the same baseline length. GARR98 and GSD95 exhibit similar behavior across Canada for all baseline lengths, which is approximately 2.5 ppm for baselines of 20 km, 1.3 ppm for baselines of 100 km, dropping down to 1 ppm at approximately 250 km, and 0.2 ppm for baseline lengths over 500 km.

On a regional level, there are also some differences in the relative agreement of the geoid models with respect to the GPS/levelling benchmarks. In British Columbia and Alberta, GARR98 and GSD95 perform essentially the same. For baseline lengths up to 20 km their relative accuracy is at the 2.5 ppm level, decreasing to 0.8 ppm for baseline lengths of 100 km. The large difference between the performance of the two global geoid models in this region can be seen in the long baseline lengths (>300 km), where there is 0.4 ppm improvement in EGM96's relative accuracy over OSU91A. In central Canada, GSD95 performs better than GARR98 by approximately 0.2 ppm for baseline lengths up to 480 km, and this difference decreases to an average of 0.1 ppm as the baseline length increases. In eastern Canada, GSD95 performs better than GARR98 for shorter baselines, averaging 2.5 ppm at 20 km while GARR98 gives 3.5 ppm for the same baseline length. The difference in relative accuracy between the two models decreases as the baseline length increases. The performance of the two global geopotential models is similar for all baselines, with the exception of baseline lengths of 100-450 km where EGM96 seems to perform approximately 0.3 ppm better than OSU91A.

5. Summary

A new geoid model (GARR98) has been computed for Canada and parts of the U.S., using the EGM96 global geopotential model as a reference field, and approximately the same gravity data used in the GSD95 geoid model. Terrain effects were modeled through the use of a 1 km × 1 km DEM for all of western Canada. The overall agreement of GARR98 with GPS/levelling is approximately 13 cm, after the datum inconsistencies have been removed via a four-parameter datum transformation, which agrees very well with the older GSD95 model. On a regional level, the various geoid models showed some differences. For levelling applications requiring dm-level of accuracy (10-20 cm), even the sole use of a global geoid model might be feasible in the central parts, while for other regions a rigorously computed gravimetric geoid, such as GARR98 or GSD95, should always be used.

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