BOLLETTINO DI GEOFISICA TEORICA ED APPLICATA VOL. 40, N. 3-4, pp.451-460; SEP.-DEC. 1999

Investigation of raw variance and variability spectra from non gravimetric geoidal signals

M. G. DOUFEXOPOULOU and V. N. PAGOUNIS

Department of Rural and Surveying Engineering,
National Technical University of Athens, Greece

(Received October 4, 1998; accepted August 5, 1999)

Abstract. Recent investigations on non-gravimetric geoid undulations at 35 GPS points, within the southern part of the Greek territory, showed the existence of lateral inhomogeneity of the local gravity field, that can be classified into 3 tectonic "provinces". The possible harmonic degrees, in spherical expansion within which the main causal sources for these inhomogeneities, were found between the range of n=20 and n=70, thus implying a lithospheric origin. In this paper we examine the information content of variance, variability, relative variability spectra of sample point undulations within these 3 tectonic "provinces". The examination is made in respect to the variation per degree of OSU91A and EGM96 earth models in order to define the depth variation among the main causal sources.

1. Introduction

The values of geoid undulation are a geometrical "measure" of the integrated effect from all existing Newtonian sources in the radial and the lateral sense. The determination of geoid involves assumptions, and/or numerical information of the mass-density.

The Global Positioning System (GPS) provides a vertical control height system that apparently does not depend on Newton's law, at least on the regional scale. When the orthometric heigth H is subtracted from the GPS ellipsoidal height h, and then raw local geoid undulations are received,

$$N_{GPS} = h - H \tag{1}$$

they can be considered as belonging to a non-gravimetric geoid. This type of geoid undulations

---

Corresponding author: M.G. Doufexopoulou; Department of Rural and Surveying Engineering, National Technical University of Athens, Kentavron 14 - 15237 Filothei, Athens, Greece; phone: +30 1 77 22 664; fax: +30 1 7722 670; e-mail:mairiedf@central.ntua.go

is considered as a signal from the Earth's system that can be used to detect the existence of mass in homogeneities in given area. For geodesy and geodynamical control, the detection of mass in homogeneities within the upper layers of the Earth's interior is of interest to define possible existing tectonic blocks in an area, or to contribute to feasible modeling of time-dependent measurements.

The use of non-gravimetric undulations over gravimetric ones has certain advantages:

1. the undulations may be considered as "objective" signals that reflect the effect of regional Newtonian sources;
2. the undulations as "high pass filtered gravity anomalies" carry density information content that comes mainly from the tectonic constellation, while the effect of the near to surface density "sources" is diminished.

Interest to analyze the geoid from this point of view, started with the first published geoidal maps in the mid sixties. A weak global correlation exists between the geoid and major "tectonic provinces" of the Earth (Souriau and Souriau, 1982). Observations of the gravity field spectrum made with the very early geopotential models (Lambeck, 1976) had shown that most of the power in the spherical harmonic expansion above degree n=6, reflects the heterogeneities in the first 300-400 km of the mantle. Parsons and Daly (1983) attempted to connect models of mantle convection to the observed gravity field. Those studies associated the relation of geoid anomalies to the main tectonic features and other geophysical formulations from a geophysical point of view.

In geodesy, attention has been paid to interpret the geoid by already "known" geophysical features, deduced by other disciplines. Bjerhammar (1981) had shown the trace of the Scandinavian uplift as it could be "seen" in geoid undulations computed by low degree n harmonic coefficients of the early GEMB model. In studying the Greek area (Doufexopoulou, 1984; Doufexopoulou and Papafitsorou, 1986; Doufexopoulou et al., 1989) the results revealed the dependence of geoid and of the local gravity field on the local tectonic pattern. (Schwarz,1985) gave overall global scale estimations on the percentage of the density information content in the gravity anomalies and geoid undulations. He concluded that beyond the harmonic expansion degree n=360 the density information is almost exhausted in the geoid undulations, while for the gravity anomalies about 36% of information is maintained.

In this study we use various forms of geoid undulation signals

$$\Delta N = N_{GPS} - N_{model} \tag{2}$$

computed at 35 observed GPS points in the continental Greek area after the coefficients C_{nm},S_{nm} of the geopotential models OSU91A and EGM96. These signals were efficient to determine the existence of 3 main tectonic "provinces" within the study area (Fig. 1) (Doufexopoulou and Pagounis, 1997, 1998). Now these data are used to investigate, numerically, the information content on tectonic mass-density in the vertical sense. The methodology is based on the construction of a number of relative quantities, called variance, variability and relative variability that are built upon various signals δN at individual GPS stations. By scanning these quantities for different bands of spherical harmonic coefficients one yields a characteristic curve, called raw spectrum, at each point. The curves are compared with points that belong to the "provinces" A, B, C. The use of the so-called non-gravimetric geoid undulations has certain advantages:

- the undulations may be considered as "objective" signals that reflect the effect of regional Newtonian sources;
- the undulations as "high pass filtered" gravity anomalies carry density information content, that comes mainly from the tectonic constellation, while the effect of the near to the surface density sources (e.g. topographic effects) is diminished.

In addition to the previous considerations the numerical treatment of the signals δN for the depth estimation of density anomalies provides a way to overpass the restriction related to the harmonic form that the density function must have (Sansó et al., 1986) in the analytical approaches of the inverse gravimetric problem.

The spectral investigations used are actually a numerical signal decomposition, in which the subtraction of the geopotential model "components" at certain degrees of expansion lags is considered as elimination of the regional "trend" that comes from the global scale variations of the gravity field.

2. The general methodological approach

The test area (Fig. 1) is characterized by a wide variability of tectonic features within a limited geographical area. Plateaus interrupt the areas of rough topography, and the sea covers 60% with several islands in beturen. Various unknown effects on the local gravity field have their origin "source" within the lithosphere at depth, and in the lateral sense; they mask shorter wavelengths of the field that are associated to the crustal thickness and the topography (Doufexopoulou, 1984). This familiar situation known in geophysical signals (Hahn et al., 1976), can be treated effectively in the present case, when the dependence of the geoid from the longer wavelengths, is eliminated. Thus, by referring the point GPS undulations to the degrees n=70 - 90 of the OSU91A and EGM96 models, earlier studies found (Doufexopoulou and Pagounis, 1998) that it was an optimal choice for an overall elimination of wavelengths that represent the global scale mass effect upon the region, deduced by the horizontal information (Doufexopoulou and Pagounis, 1997, 1998). However, a previous numerical study of the region, that treated Doppler baseline undulation differences δN in a similar concept (Doufexopoulou and Paradissis, 1988), showed that the degree n of the model expansion, at which the model differences δN along baselines "misfit" to the corresponding Doppler undulation differences, could start at degree of expansion n=30. The apparent difference between n=70 and $n \leq 30$ must not be considered as contradictory. For baseline undulation differences, the degree n=30 corresponds to a reference surface for the "signal" differences in lateral sense, while the n=70 corresponds to an overall reference for the individual undulation "signals".

GPS undulations and recent geopotential models (e.g. EGM96, EGM97) provide a good overall accuracy at the medium wavelength geoid scale. Therefore, strong deviations of the local signal power from the power of the geopotential model at rather low degrees of expansion may be associated to local non-modeled causal "sources". Recently an additional set of 61 GPS points is used for similar than the presented investigations (Pagounis, 1999). The signals δN produced at the 35 GPS points, were combined at various wavelengths and harmonic windows and were investigated by simple statistical tests:

- among different GPS points (i.e. horizontal information) concluding that the traces of 3 main tectonic blocks could be detected by both geopotential models;
- for various truncation degrees n at the same point (i.e. the depth information) by computing the spectra, that are presented here.

The 35 N_{GPS} after Eq. (2) are signals representing the effect of mass-density distribution within the whole Earth. From these the Model Undulations (MU), computed by the coefficients C_{nm}, S_{nm} of both models, from n=10 -360, truncated each 10 expansion degrees, were subtracted. As the harmonic coefficients are computed by standard analytical assumptions about the global-scale structure of the gravity field, while the GPS undulations are received without related analytical or statistical assumptions, the signal differences (Eq. 2) at the various truncation degrees are certainly rich in local mass-density information. The choice of n=360 as the upper truncation limit in previous tests, was chosen intuitively, on the basis of a compromised choice between earlier studies within the test region (e.g. Doufexopoulou and Papafitsorou, 1986), and on the well accepted performance of high degree and order models. This choice agrees with recent evaluations on the EGM97 model (Denker, 1998). The coefficients of the models OSU91A and EGM96 above degree n>36 are considered in this study as mostly independent on local mass effects because:

- their (theoretical) dependence is suppressed by the least squares based methods that are used to estimate the size of C_{nm}, S_{nm};
- the limited geographical extention of the test region operates towards the decrease of possible effects coming from local lateral inhomogeneities upon the coefficients.

The accuracy of the individual GPS undulations, taking into account existing error sources in the vertical coordinate of GPS points (e.g. Kuang et al., 1986) and the accuracy of geodetic methods for the trigonometric height, is estimated at an overall rate of ± 5-10 cm (Pagounis, 1999). These are sufficient for the present investigation, which evaluates the amplitude spectral information in a relative sense (in the tectonic regions).

Several authors (e.g Rapp, 1977; Schwarz, 1985) use conventions on the range of degrees in harmonic expansion, that classify the geoid's spectral components into wavelength parts. Similar conventions are also followed here, under a modified classification suggested from results of previous investigations in the area (Doufexopoulou and Papafitsorou, 1986; Doufexopoulou and Paradissis, 1988). In the present, the undulation spectra are classified into 3 parts:

1. *a low frequency* part for n<30, denoted as LFP - due to upper mantle and lower lithosphere;
2. *a medium frequency* part for 50>n≥30,denoted as MFP - due to lithospheric sources (e.g. Bjerhammar, 1982);
3. *a high frequency* part for n≥50, denoted as HFP. The HFP includes the harmonics beyond n=50 and may be associated to the combined effects that come from the deeper origins in the MFP, (30≤=n≤=50), the variable crustal depth and other unknown micro-tectonics.

Possible datum tilts that may exist between the GPS datum to the geopotential datum's are considered here as not significant (e.g. Pavlis, 1998). The previous simplifications are well supported by:

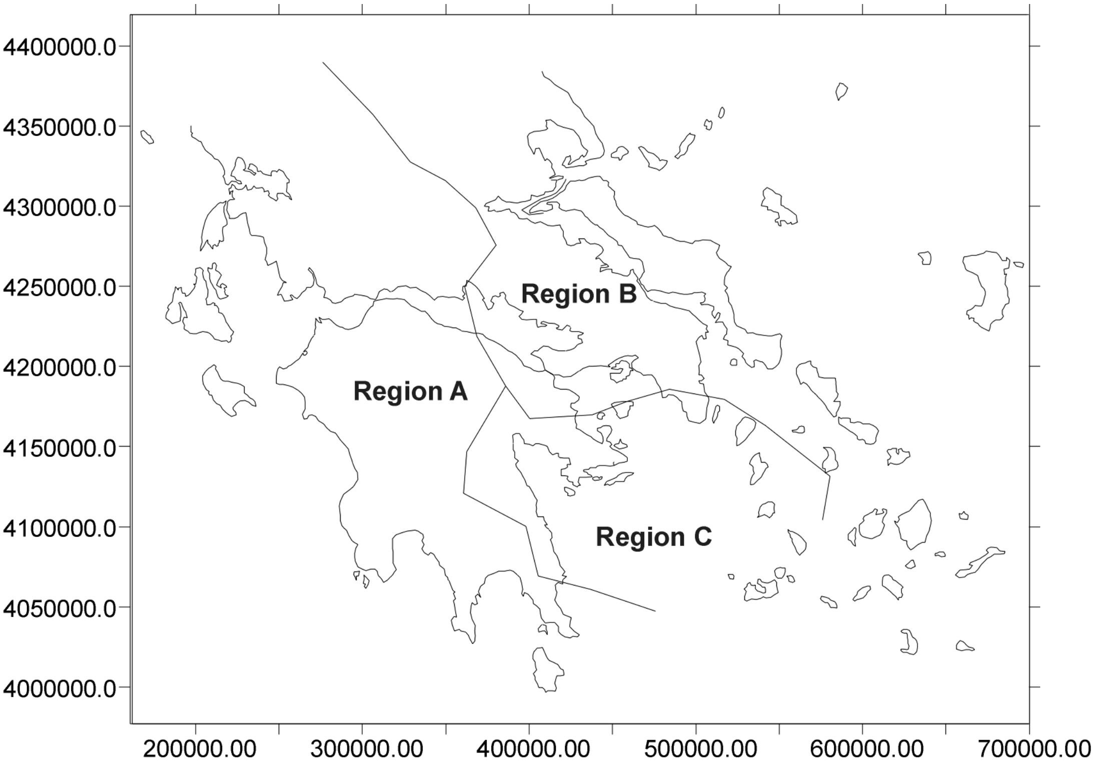

Fig. 1 - Tectonic Provinces A, B and C.

1. the limited geographical extention of the test region, that is not expected to introduce strong systematic errors of any source;
2. the differential character of all the numerical comparisons made so far in which the differential character tends to annul ate the contribution of any smooth systematic error;
3. the fact that no comparative results between the WGS84 datum and the geopotential datum exist for the test region that could possibly allow a detailed evaluation of point GPS undulations (Pagounis, 1999).

However, within larger areas, or in a more detailed approach, one must take note of possible datum tilts between the initial two types of undulations using e.g. a numerical comparison approach between E/W and N/S baselines at the two geoids (datum shift and tilt) (e.g. Merry, 1997).

3. Description of data - a raw consideration about the test region

The results are based on the signals from 35 GPS points within φ=36° 2' - 40° and λ=20° 3' - 23° 3' (Doufexopoulou and Pagounis, 1997). The orthometric heights of these points (mostly belonging to the National triangulation network) were released by the Hellenic Military

Geographic Service. At each point, from the GPS undulation (Eq. 1), the contributions of model undulation for OSU91A1F and EGM96 models were subtracted for the horizontal control each n=10 expansion degree lags and at each n=2 for the spectral investigations. The computations were carried with the Geocol program (Tcherning, 1996). At each GPS point, several amplitude “signals” δN per model were produced. These are used for the spectral tests.

The previous test results (Doufexopoulou and Pagounis, 1997,1998) the raw traces for 3 geographical “provinces” (Fig. 1). The 14 points that show a “capricious” behavior in signal performance were taken. Table 1 is an extract from a complete table in Doufexopoulou and Pagounis (1998) that presented the comparative results of maximal and the minimal signals δN and the harmonic degree n of their occurrence in both models. The table contains only the 14 points. The last column represents the difference between the two models in the minimal δN.

The 14 GPS point undulations (out of the total of 35) of Table 1 in which the minimal signal δN occurs below the expansion degree n=70 in both models are the main target signals in the present spectral investigation. In Table 1, the last point gives a very large difference between the same degree (n=120) in the two models (the number in the last column is –26.42 m).

The tectonic “provinces”, namely A, B, C (Fig. 1) traced according to the behavior of point “signal” groups in the lateral scale, are detected by both models. For most points that belong to group C (9 points) the maximal δN (i. e. possibility that the model does not “fit” to the local mass-density distribution) is received at a higher truncation degree than the degree that provides the smaller signal δN. The size of received maximal signals δN between the two tested models, varies per point from 1 cm until -0.39 cm. Differences in both models concerning to the truncation degree n at which the maximal signal occurs are unimportant.

The evaluation of point undulation spectra for the investigation of the depth of the main causal “source” within the regions A, B, C is done on several points within each regional group after the previous classification, without paying special attention on how the geopotential model used performs. This procedure is the main argumentation of this study. It wants that to trace local Newtonian effects by restricting the search to the usa of surface “signals”, and certainly does not evaluate how a certain geopotential model performs analytically in the very test region itself. From the geodetic point of view the interest on local tectonics is limited to investigating the expected numerical effect upon geodetic data used for other purposes (kinematical modeling, height control etc.)

4. The computation of signal undulation spectra

At each test point the harmonic “components” were used to compute:

1. variance of geoid undulations (3);
2. variability of geoid undulations (4);
3. relative variability spectra of geoid undulations (5).

Table 1 - Points with minimum and maximum signal δN in both models.

a degrees	OSU91A	EGM	a degrees	a degrees	OSU91	EGM	a degrees
10	5.51	5.37	10	20 30	0.08	0.08	20 30
130	10.63	11.82	30	20	0.63	0.63	20
10	18.04	17.89	10	40	7.97	7.97	40
40	10.25	10.63	20	200 210	0.00	0.00	200 210
20&30	8.92	8.99	20	70	0.09	0.08	70
200	2.26	2.22	20	50	0.39	0.39	50
10	9.77	9.66	10	90	0.22	0.22	90
10	3.24	3.40	40	140	0.48	0.48	140
10	6.08	5.96	10	60&70	0.06	0.06	60&70
20&30	6.51	6.54	20	60	0.19	0.19	60
20&30	9.10	9.13	20	80	0.50	0.50	80
10	3.26	3.14	10	60	0.00	0.00	60
40	1.21	1.11	70	20&30	0.06	0.06	20&30
10	5.16	31.58	10	120	0.09	0.09	120

$$1-\frac{N-N_m}{N} \tag{3}$$

The experimental values of raw *undulation degree variances* $\sigma_n(N)$ have been computed as the squared differences $\delta\delta N$ between the individual δNs for consecutive truncation intervals with lag 10 of expansion degrees (e.g. 10-20, 20-30, 30-40 etc.):

$$\sigma_n(N)=\delta\delta N=\left(\delta N_{n+10}-\delta N_n\right)^2 \tag{4}$$

The *variability spectra* are dimensionless raw numbers, received as the squared ratio between NGPS-Nm for model undulations at increasing 2 degree lags, in respect to the whole geoidal signal $\delta N=N_{GPS}-N_{360}$

$$\left(\frac{\delta N_n}{\delta N_{360}}\right)^2. \tag{5}$$

Finally, the relative variability spectra are the squared ratios at each two consecutive lag intervals:

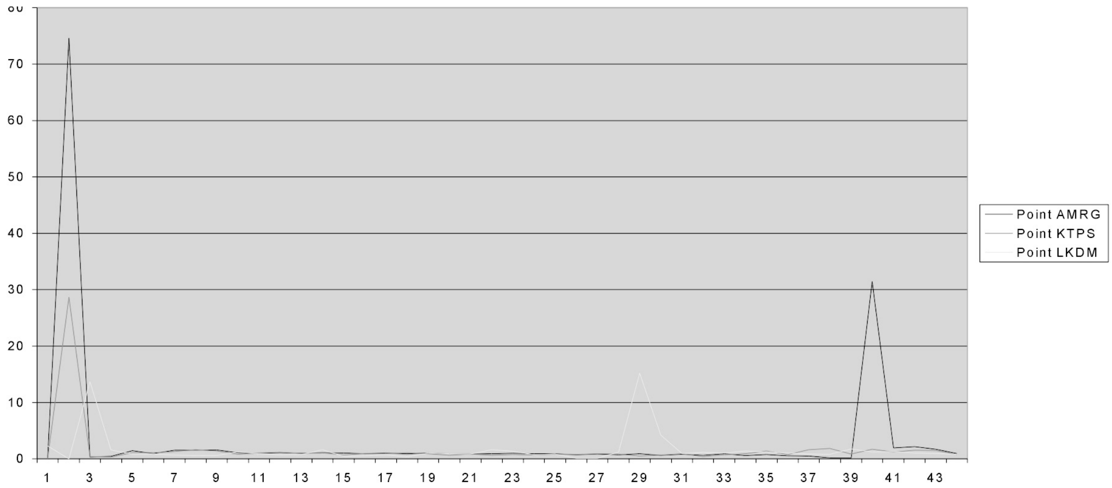

Fig. 2 - Relative Variability Spectra for OSU91A.

$$\left(\frac{\delta N_n}{\delta N_{n-2}}\right)^2. \tag{6}$$

These types of geoidal signal decompositions represent different forms of the information content, than the original signals δN carry:

1. variance spectra (Doufexopoulou, 1984) depict the harmonic degree of expansion beyond which no local scale mass-density information is added by the coefficients of the geopotential model;
2. amplitude undulation degree variances at degree intervals are experimental values of raw undulation degree variances at the point tested and may be comparable to model degree variances, which express an overall statistical behavior of the gravity field;
3. variability spectra represent the actual power distribution per lag (i.e the spectral power from local mass-density) in respect to the whole signal at the upper truncation degree of a certain geopotential model, at the tested point;
4. the relative variability spectra provide an insight in the detailed distribution of the power between the neighboring lags.

Figs. 2 and 3 depict the two types of raw point spectra for one representative point per region (A, B, C). All tested points show a quite similar spectral view in both models within the region they belong to. The spectra shown in Figs. 2 and 3 are in respect to OSU91A only. In Fig. 2 the relative variability spectra for 1 point per each region are presented for OSU91A. Slight and unimportant differences or shifts between the same types of spectra in respect to both models do not affect the results of the present study and are not discussed here.

The variance spectra of points among the 3 regions, have the “flat” part of spectrum (i.e. no information on the local mass exists in the model’s coefficients beyond a certain expansion

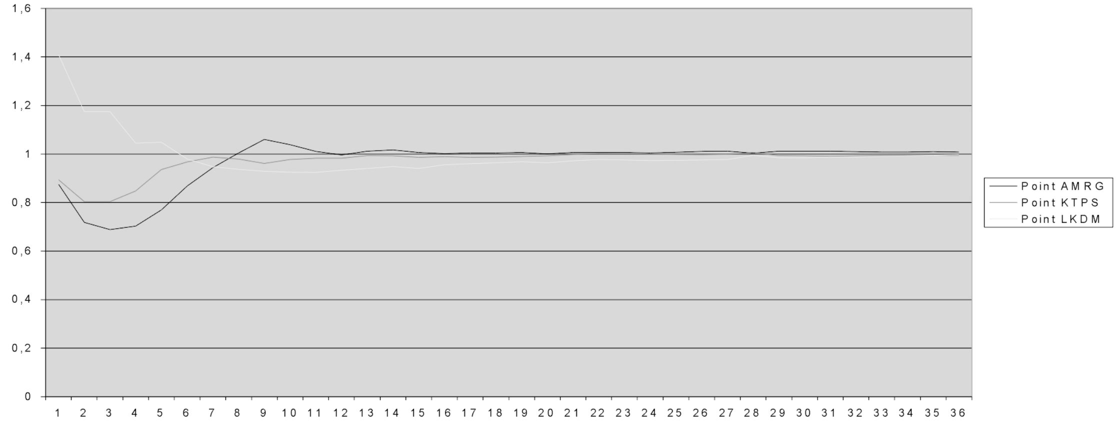

Fig. 3 - Variance Spectra for OSU91A.

degree n) starting at a different degree per region: For region A this is between *n*=60-70, for region B around *n*=70 and for region C beyond *n*=90. The variability spectra beneath the region A. It is worth to mentioning that the concentration of the effect of local "sources" around the harmonic *n*=45 within the overall region was received 12 years ago too (Doufexopoulou and Papafitsorou, 1986). This incidence reveals the possibility that the effect of the causal "source" beneath region A might dominate in longer wavelengths upon the local features that exist below regions B and C. The point group A is located along the region where the African tectonic plate and its boundary are drawn (McKenzie, 1972). The point groups B and C, roughly occur within the Eastern continental part and in the South Aegean Sea, respectively, beneath which different processes are undergone (Papadopoulos, 1982). Group C occurs particularly above the middle Aegean Sea, where the expected effect of the sub-ducting slab of the African plate is apparent on the geoid. This tectonic and geophysical information on the test region is discussed as external argumentation only that may allow the evaluation of results from a feasibility point of view.

Beyond the conclusions depicted from these spectral investigations, the use of time-dependent geodetic observable in the geodynamical evaluation within the overall tested region, suggests that at least two independent tectonic "provinces" (A, B and C or West-East) must be considered.

One may conclude that:

1. the geoid "signals" represent useful physical information on the mass-density, which is not biased by previous physical information;
2. the effect of the overall regional mass-density sources is strong before the harmonic degree *n*=80;
3. the raw spectra do indeed clearly shed light on the existence of various at depth dominating "sources" beneath each of the 3 regions;
4. the detailed investigation of the undulation signals within regions B and C should take into consideration the possible effect upon them of the mass-density source beneath the region A, if indeed its causal source is deeper.

References

Bjerhammar, A; 1981: *Longwave heterogeneities as seen by satellites in the upper mantle*. Presented at the VII EGS Meeting, Uppsala, Sweden.

Denker H.; 1998: *Evaluation and Improvement of the EGG97 Quasigeoid Model for Europe by GPS and Levelling Data*. Second Continental Workshop on the geoid in Europe, Budapest.

Doufexopoulou M.; 1984: *A tectonical consideration of the geoid in Greek region (in Greek)*. Technika Chronika, A3

Doufexopoulou M. and Papafitsorou, A.; 1986: *Inversibility of geopotential models in local Gravity field Approximation*. In: Proc. of IAG, Intern. Conf. “Figure of the Earth, Moon and other Planets”, Publ. of Geod.& Chart. Soc. , VUGKT.

Doufexopoulou M. and Paradissis D.; 1989: *Doppler undulations and Local Gravity Field in Greece*. In: Proc. of IAG Symp. “Instrumentation and Analysis for Integrated Geodesy”. Publ. GGKI, 1989, Sopron, Hungary.

Doufexopoulou M., Milas P., Nakos B., and Papafitsorou, A.; 1989: *On the relation between the gravity field and lithospheric features in the Greek area*. AGGM-ASH, Bp.21 (1989/3-4) 1989.

Doufexopoulou M.; 1992: *On the inverse character of the geoid determination using GPS data.* Proc. Of the 1rst Cont. Workshop on the Geoid in Europe, VUGKT, Prague 1992.

Doufexopoulou M. and Pagounis V.; 1997: *First numerical results in detecting heterogeneities in regional gravity field using GPS undulations.* EGS Meeting, Vienna 1997.

Doufexopoulou M. and Pagounis V.; 1988: *Investigation of tectonic traces upon the regional geoid without the use of gravity, 2d Cont.* Workshop on the Geoid in Europe, Budapest 1998.

Hahn A., Kind E. G. and Mishra D. C.; 1976: *Depth calculation of magnetic sources by means of Fourier amplitude spectra.* Geophysical Prospect., **24**, 28-43.

Lambeck K.; 1976: *Lateral density anomalies in the upper mantle.* JGR, 81, **35,** 6333-6340.

McKenzie D.; 1972: *Active tectonics in the Mediterranean region.* Geoph. J. Astr. Soc. **30**, 109-135.

Merry C. L.; 1997: *Comparison of geoid models for the SouthWestern cape Province.* South Africa (http://www.acs.ucalgary.ca/~sideris/SWG/marry.html).

Pagounis V.; 1999: *Processing methods of geometrical and physical geodata for signal detection. Application to the vertical datum (in Greek - tentative title).* Doctorial thesis to be submitted at NTU, Athens (to appear 1999).

Papadopoulos G. A.; 1982: *Contribution to the study of active tectonics at depth within the Aegean sea (in Greek).* Doctorial thesis, Aristote University, Thessaloniki 1982.

Pavlis E. C.; 1998: *On the Reference Frames Inherent in the Recent Geopotential Models.* Invited lecture at the Second Continental Workshop on the geoid in Europe, Budapest, March 1998.

Rapp R.; 1977: *Geoid undulation by wavelength.* Bulletin geodesique, **44**.

Souriau A. and Souriau M.; 1982: *Global tectonics and the geoid.* Phys. Earth Planet. Int.

Sansó F., Barzaghi R. and Tscherning C. C.; 1986: *Choice of norm for the density distribution of the Earth.* Geoph. J. R. Astr. Soc., **87**, 123-141.

Schwarz K. P.; 1985: *Data types and their spectral properties.* In: “Proc. of Beijing Intern. School on Local Gravity Field Approximation”, China 1985.