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Time series of low degree zonals obtained analyzing different geodetic satellites

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Abstract. The geodetic satellite orbit is used as a gravity probe to sense the time varying field. We analysed ten years of Satellite Laser Ranging (SLR) observations of the LAGEOS-I, LAGEOS-II, Starlette and Stella satellites and estimated the time series of the low degree zonal coefficients in the Earth gravity field up to degree six. The corresponding secular drifts have been derived and compared with other published results.

1. Introduction

Geodetic SLR satellites, customarily used for precise geodetic measurements, have proven to be useful in measuring the time evolution of the long wavelength part of the gravity field (Cheng et al. 1989; Nerem et al. 1993; Nerem and Klosko, 1996; Eanes, 1995; Cazenave et al., 1996). In particular, the long observation history of Starlette and LAGEOS-I satellites is important to properly extract the secular drift from the higher frequency variations in the zonal harmonic coefficients of the gravity field. On the other hand further observations from their twin satellites Stella and LAGEOS-II are important to de-correlate the low degree coefficients from the higher degree terms. In this study we are interested in measuring the secular drift of the zonal part in the gravity field optimizing the experimental design of the estimation scheme.

2. Experimental design

It has now been proven that the orbital motion of both LAGEOS satellites is strongly affected by thermal forces that are highly correlated with the odd zonal terms of the gravity field (Eanes, 1995; Ries et al., 1997; Ries et al., 1998). Therefore, because of the unreliability of the current non-gravitational models, we do not use the LAGEOS for providing the odd degree zonal variations. The

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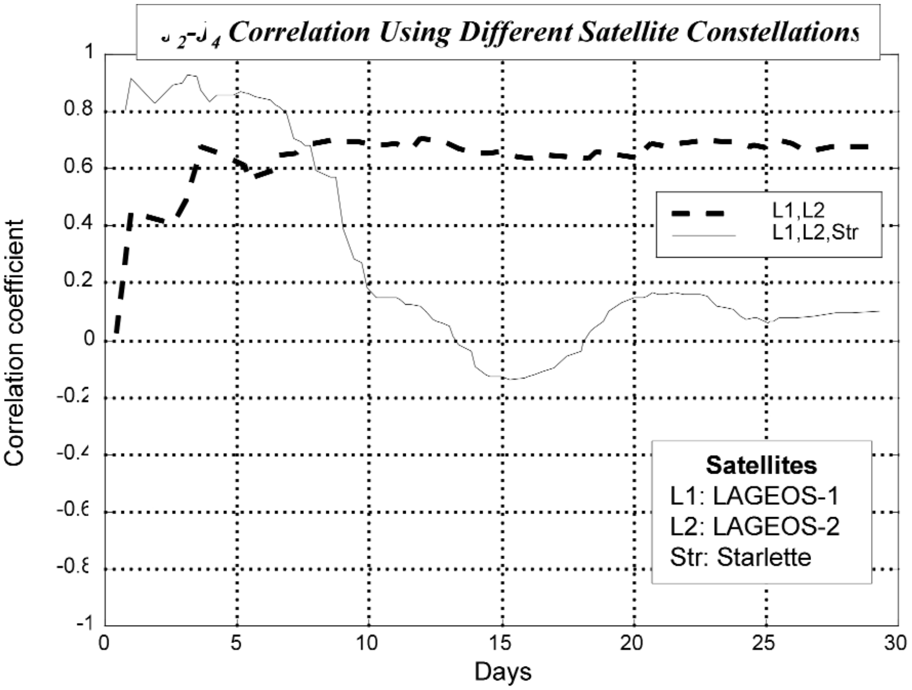

Fig. 1 - The correlation coefficient between J_2 and J_4 zonals as a function of time. The two curves show the different correlation functions using only the two LAGEOS observations (dashed line) and adding also Starlette observations (solid line).

even zonals could be regarded as independent from the thermal perturbation because they mostly excite the ascending node of the orbit which is not corrupted by thermal forces. The low orbiting satellites Starlette and Stella are not affected by strong unmodeled thermal perturbations because of their different orbital and thermal characteristics and thus can be suitable candidates to recover the odd degree zonal harmonics.

In order to set up the optimal estimation scheme for the zonal rate, using the best satellite constellation capable to de-correlate the zonal coefficients, we studied the normal matrix product $(A^TA)^{-1}$, where A is the matrix of the partial derivatives of the real observations (design matrix). We define the Dilution of Precision (DOP) factor of each estimated parameter being the square root of the corresponding diagonal element of the normal matrix. Given the required precision, the DOP factors computed as a function of time define the optimal estimation approach for the rate solution, and the off-diagonal factors represent the statistical correlation between the estimates. Therefore it is important to choose the satellite constellation in such a way that the correlation coefficients are close to zero, otherwise the estimates can be biased. Fig. 1 shows how we can make use of the three satellites LAGEOS-I, LAGEOS-II and Starlette to obtain uncorrelated even zonals, in particular J_2 and J_4 as a function of time. The dashed line indicates that these two zonals cannot be statistically de-correlated on the basis of only LAGEOS-I and LAGEOS-II observations. The SLR data reveal a high correlation between the estimates in almost all the arc solutions. One can overcome this problem by adding Starlette observations thus allowing a more robust estimate of J_4.

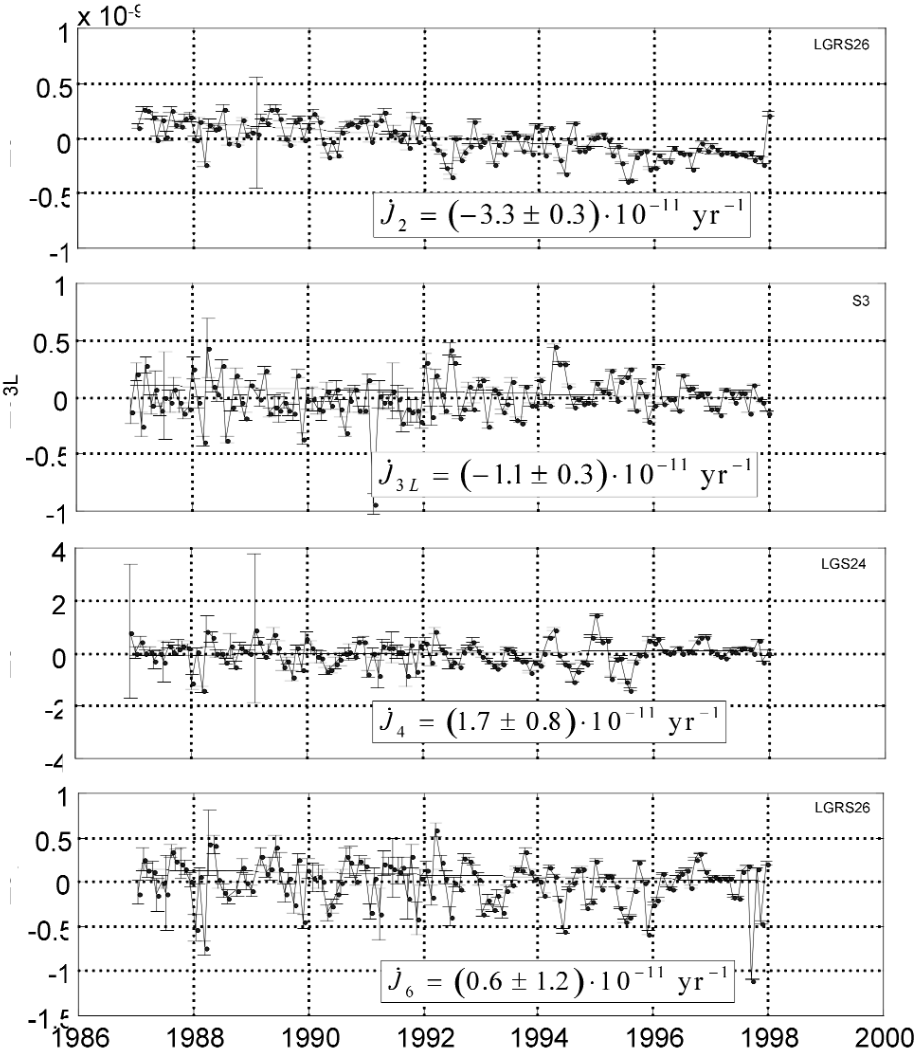

Fig. 2 - Time series of the estimated zonal coefficients from degree two up to degree six. The quoted secular drifts are obtained from a weighted mean of three different solutions using a different combination of the observed satellites. The odd degree zonals were not well separated in our Starlette solution and only the lumped odd zonal estimate is shown.

3. Analysis setup

The analysis procedure foresees three sequential steps: the performing of many arc solutions, their combination in an unique global solution estimating the zonal time series, and the secular drift estimation. The SLR data reduction has been performed using the Geodyn II and Solve software developed at the NASA Goddard Space Flight Center.

During the first phase, the complete orbit and force model is defined together with the analysis approach (i.e. arc length, type of estimated parameters) but only the arc dependent parameters are estimated, namely those related to the orbit (state vector and non-gravitational forces) and to the observations (measurement bias). We adopted the JGM3 gravity model and its own tide model,

Table 1 - Comparison of SLR zonal rates computed from different analysis groups.

(Units: 10-11 /year)	$\dot{J}_2$	$\dot{J}_3$	$\dot{J}_4$	$\dot{J}_5$	$\dot{J}_6$
Yoder et al. (1983) L1 (1976-1981)	-3				
Rubincam (1984) L1	-2.6±0.6				
Cheng et al. (1989) Str	-2.5±0.3	-0.1±0.3	0.3±0.6		
Gegout & Cazenave (1993) L1 (1985-1989)	-2.8±0.4				
Nerem & Klosko (1996) L1,L2,Aj,Str (1986-1994)	-2.8±0.3	0.2±1.5			
Cazenave et al. (1996) L1,L2 (1984-1994)	-3.0±0.5	-1.7±0.1	-0.8±1.5		
Eanes (1995) L1 (1977-1995)	-2.4±0.2				
Cheng et al. (1997) L1,L2,Stl,Str,Aj,Et1,Et-2, BE-C (1975-1996)	-2.7±0.4	-1.3±0.5	-1.4±1.0	2.1±0.6	0.3±0.7
This study L1,L2, Str, Stl (1987-1997)	-3.3±0.3		1.7±0.8		0.6±1.2
This study Str (1987-1997)					

considered the secular variation and the dynamic polar motion of the C_{21} and S_{21} coefficients, realized the terrestrial reference frame using the *a priori* ITRF93 site coordinates and velocities, and constrained the Earth Orientation Parameters to the IERS homogeneous series. The compressed SLR observations (normal points) have been analyzed using the worldwide network of tracking stations that differ both in terms of quality and quantity of acquired data. To take into account these two aspects, each normal point has been weighted by the quantity *n/bin_variance* where *n* is the number of single range data that has been used to construct the normal point and *bin_variance* is the quadratic bin standard error.

All the different arc solutions (normal matrices) are then combined to estimate the global and arc parameters together. In a typical solution 1 500 000 normal points were processed and up to 50 000 parameters were estimated.

In principle we can expect that the zonal coefficients will be affected by seasonal oscillations originating principally in the ocean and atmosphere. Therefore in every zonal time series we filtered

Table 2 - Comparison of odd zonal rates.

Author (Units: 10-11 /year)	$\dot{J}_{odd}$ Using Constraint from this solution $\dot{J}_{odd}=\dot{J}_3+0.9\dot{J}_5$	$\dot{J}_{odd}$ Using Constraint from Nerem and Klosko (1996) $\dot{J}_{odd}=\dot{J}_3+0.837\dot{J}_5$
Nerem & Klosko (1996)		1.6±0.4
Cheng et al. (1997)	0.6	0.5
This study	-1.1±0.3	

out the residual seasonal signal fitting an annual and semestrial sinusoid and at the same time we estimated also a secular rate. The frequency dependent correction to the degree two zonal (J_2) has been implemented through the classical relationship and tide amplitudes indicated in the IERS conventions (1996). We correct the J_2 for all tide terms that contribute more than 2 10^{-13} and these include periods of 1 month up to 18.6 years.

4. Results and discussion

The estimated zonal rates up to degree six are shown in Table 1. The even zonal rates are averaged values obtained from a number of different solutions. Each solution has a common modelization but differ in the adopted satellite constellation and in the number of estimated zonal coefficients. Figure 2 shows as an example, a typical solution of zonal time series. As far as the odd zonals are concerned we analyzed only Starlette observations getting the lumped value $\dot{J}_{3L}= \dot{J}_3 +0.9 \dot{J}_5$ that is a linear combination of the odd degree zonals. In the following we will outline the estimated zonal drifts comparing them with the findings of other analysis groups.

J_2 - Anelasticity of the mantle causes a phase lag in the deformational response of the Earth to the tidal forces and also gives rise to a further variation with frequency particularly pronounced for long period tides. The corrections of the second degree zonal due to the frequency dependent anelastic response can reach the significant amount of a few times 10^{-11} especially for 18.6 year and semestrial tides (IERS Conventions, 1996). Since the anelastic tidal response is not modeled in our data reduction, we expect the J_2 time series to be affected by residual long period oscillations. Therefore in order to separate these contributions from the secular drift we subtract the anelastic tidal correction from the time series using amplitudes and phase lags as specified in the IERS Conventions (1996). This J_2 drift is significantly greater than the values reported by other groups, however, if the time series is not corrected for the mantle anelasticity effects we get a value of $\dot{J}_2$ =-2.6 that is in agreement with the published values. We notice a systematic increase of the J_2 drift in the order of $0.5 \cdot 10^{-11}$ yr^{-1} in all our J_2 solutions. Thus the J_2 secular drift estimation cannot be disentangled from a correct modeling of the anelastic tide response of the Earth. Ignoring this effect could severely bias the estimates especially if the observation history does not cover an integer number of tidal cycles.

J_3, J_5 - Using only Starlette's observations we are not currently able to separate the odd zonal drifts and we estimate only a linear combination of those zonals $\dot{J}_{3L} = \dot{J}_3 + 0.9\,\dot{J}_5$ which is inferred from the observations themselves. Since other authors use different satellite constellations the linear combination of the odd zonal drift solution may differ slightly from group to group. In Table 2 different odd zonal solutions have been compared and the agreement between them is rather poor.

J_4 - The fourth degree zonal has a slightly positive drift (positive in three different solutions) that could be in agreement with the other published values with perhaps the exception of those obtained by Cheng et al. (1997) showing a slightly negative drift. However, in our solutions the J_4 estimates are highly correlated with the J_2 zonals and hence the observed differences may be ascribed to a partially unresolved lumped estimation of J_4.

J_6 - The sixth degree zonal drift is less stable in our solutions and is therefore not yet a robust estimate; nevertheless our mean value agrees fairly well with the only published value (Cheng et al., 1997).

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