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A preliminary geoid model for the North Sea from gravimetry and altimetry, GPS and levelling

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Abstract. The main goal of the study is to obtain a consistent height and depth reference system in the form of a geoid for the Dutch mainland and marine area. For this purpose a procedure has been developed and tested to combine available gravity data and external data from satellite altimetry, GPS and levelling in an optimal manner. The initial, purely gravimetric geoid is adjusted in longer wavelengths by means of the external geoid data. The preliminary North Sea geoid GEONZ97 has a precision of better than 4 cm at sea and along the Dutch coast. As soon as instantaneous GPS height components in off-shore applications reach a comparable accuracy, tides and meteorological response can be eliminated in an efficient and effective way simply by subtracting the geoid.

1. Introduction

The main objective of this study (de Bruijne et al., 1997) was to develop and implement a procedure for determining a consistent height and depth reference system for the Netherlands in the form of a unified land and marine geoid. This geoid is mainly needed for coastal zone management by Rijkswaterstaat. For instance for maintenance of important sea lanes, or maintenance of coastal areas depending on sediment transport related to (surface) currents. An accurate geoid and permanent sea surface topography model (PST) are necessary tools to achieve this. It also allows us to convert depths related to mean sea level (MSL) into depths related to Lowest Astronomical Tide for use in nautical charts.

The setting for the geoid is:

$$h - H = N, \tag{1}$$

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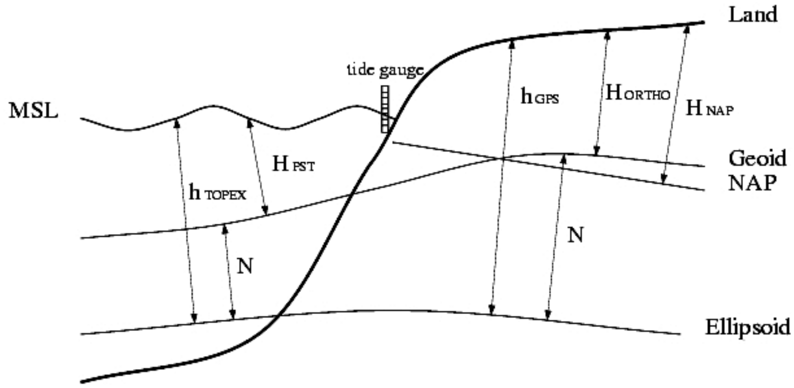

Fig. 1 - The land and sea geoid concept.

where h is the measured geometrical height, H is the known orthometric height, and N is the unknown geoid height that can be approximated with a gravimetric one. Specifically, at sea H_{PST} and h_{TOPEX}, and on land H_{ORTHO} from levelling and h_{GPS} are needed. In Fig. 1, the relations between all components are visible together with MSL and Normaal Amsterdam's Peil (NAP). The hypothesis used in the preliminary setup is that the PST is negligibly small compared to the required accuracy of 10 cm. This level can hopefully be achieved in the near future for instantaneous GPS heights off-shore. The neglect implies that the MSL from altimetry should coincide with the marine geoid, and consequently the equipotential for height reference should coincide with current MSL at tide gauges. The land height datum NAP, refers to the MSL at the time of definition, and in a strict sense the NAP heights do not refer to an equipotential anymore, due to land subsidence and sea Level change. Therefore, the NAP heights are transformed to orthometric heights referring to an equipotential at current MSL in the tide gauges. At sea, the spatial resolution and quality of the altimeter measurements is not homogeneous. Therefore, only good quality TOPEX altimetry is used as external geoid data, rather than a direct altimetric result. This allows the optimal exploitation of details in the gravimetric geoid.

2. Procedure

The procedure we followed for determining the North Sea geoid can be summarized as follows:

- compute detailed gravimetric geoid N_g from gravity data;
- determine possible empirical models to correct the expected error in the gravimetric geoid;
- include external altimetry, GPS and levelling data;
- correct the gravimetric geoid N_g with the external data, yielding MSL.

The following sections explain this procedure. We refer to (de Bruijne et al, 1997; Haagmans and de Min, 1998) for a detailed explanation on the second item.

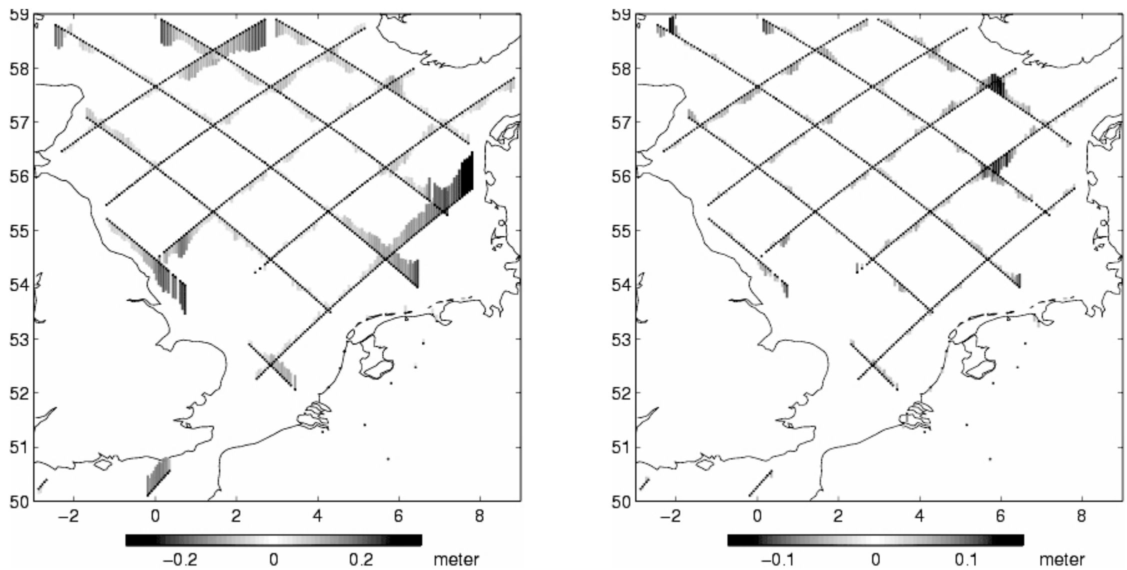

Fig. 2 - External data minus gravimetric (left) and corrected (right) geoid at external data locations.

3. Gravimetric computation

To determine the gravimetric geoid N_g, we refer to (de Min, 1996). Here, to suffices the main formula

$$N_g = \frac{GM}{\gamma r} \sum_{n=2}^{360} \left(\frac{a}{r} \right)^n w_n \sum_{m=0}^{n} \Delta C_{nm} Y_{nm}(\varphi, \lambda) + \frac{R}{4\pi\gamma} \int_{\sigma_0} \left\{ \sum_{n=2}^{\infty} \frac{2n+1}{n-1} (1 = w_n) P_n(\cos\Psi) \right\} \Delta g \;\, d\sigma. \quad (2)$$

The first part of Eq. (2) is a weighted, spherical, harmonic expansion based on the EGM96 global geopotential coefficients ΔC_{nm}. The second part involves numerical Stokes' integration up to a capsize, based on a weighted difference between local gravity data and EGM96 (Δg), in order to resolve all details. The weights w_n that take care of an optimal combination of the longer and shorter wavelength contents, belong to the Meissll/Wong&Gore modification (Heck and Grüninger, 1987; de Min, 1996; de Bruijne et al., 1997). The gravity data involved are:

- the EGM96 global geopotential model referring to GRS8O (Lemoine et al., 1996) and (Rapp, 1996);
- 6'x 10' block mean free-air gravity values in Europe (Weber, 1984), and 3'x5' block mean free-air gravity values in parts of the computation area, predicted from various data sets (de Min, 1996).

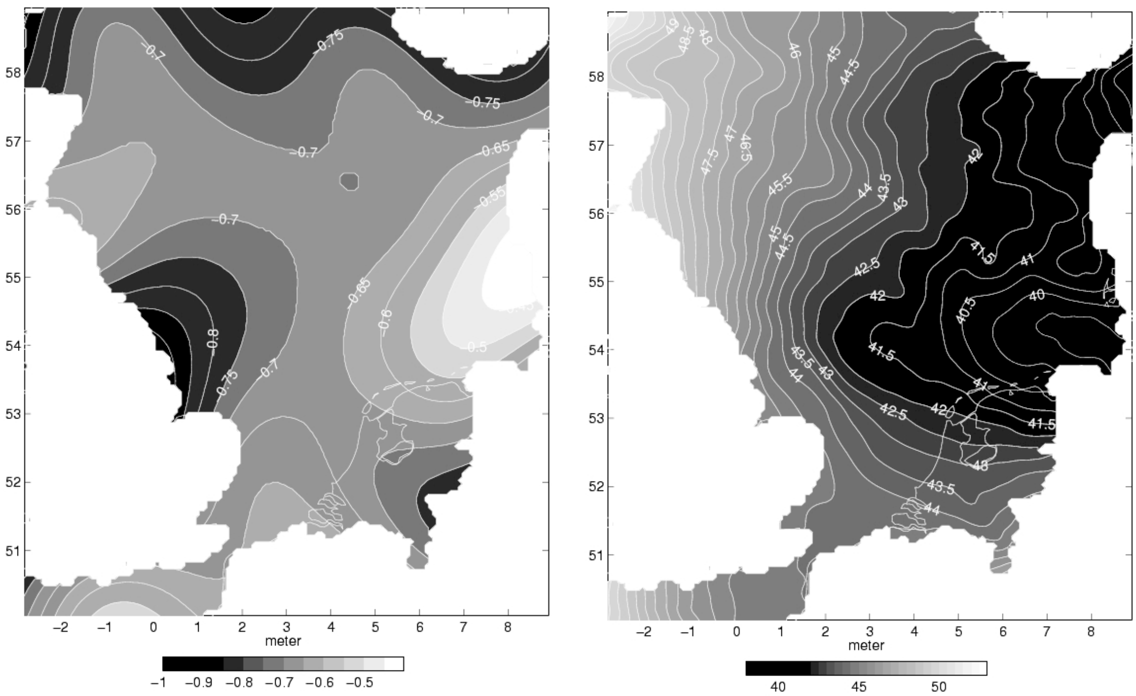

Fig. 3 - Correction surface for the gravimetric geoid (left) and preliminary geoid GEONZ97 (right).

4. Connecting the gravimetric geoid with external data

In (de Min, 1996; de Bruijne et al., 1997) the gravimetric geoid N_g has to be corrected for its longer wavelengths, based on external data:

$$N = N_g + F_{c.} \tag{3}$$

The correction function F_c depends on the error in the gravimetric geoid and the control data (altimetry, GPS and levelling). Tests resulted in the choice of a sum of a bilinear and a trigonometric surface, expressed in 12 or at most 28 parameters, cf. (Haagmans and de Min, 1998).

Fig. 2 (on the left side) contains the residuals with an rms of 8.5 cm between the external geoid and the gravimetric geoid values, after removal of the mean of 69 cm with respect to two year averaged TOPEX and GPS/levelling heights. A least squares adjustment procedure, including iterative data-snooping, resulted in the acceptance of the 28-parameter model for the required accuracy. Fig. 2 (on the right side) shows the residuals after the correction; the rms of all accepted points is 3.2 cm, and the mean of all points is -0.1 cm and the rms 4.2 cm.

The 28-parameter correction surface is shown on the left of Fig. 3. Combining the gravimetric geoid and the correction surface, yields the preliminary North Sea geoid GEONZ97 shown to the right of Fig. 3.

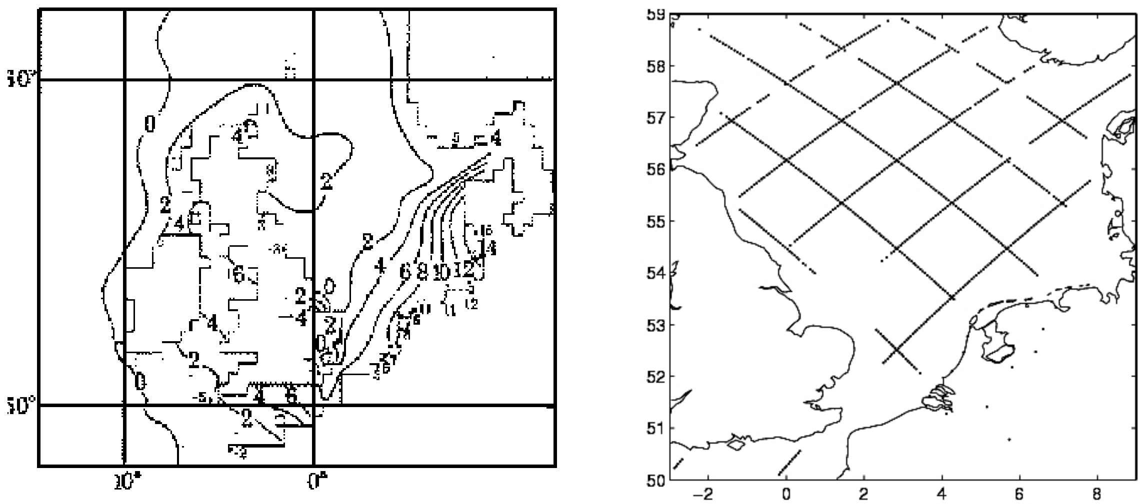

Fig. 4 - Model PST in cm (Prandle, 1984) (left) and accepted points in the final adjustment (right).

5. Conclusions

A first successful attempt to determine a consistent geoid for land and sea in the Dutch region, within a ± 10 cm level at sea and a few cm's on land was made. However, some aspects can be improved in future computations. The statistical testing procedures lead to the choice of a 28-parameter model, but the overall model test was not fully accepted. A further extension of the model could lead to a better fit, but is not expected to be realistic (Haagmans and de Min, 1998). One reason could be that the assumption that H_{PST} us small and of random nature is not valid, since a model based upon the major tidal component and wind, predicts a PST with a trend pattern as shown in Fig. 4 (Prandle, 1984). Comparing the left plots of Fig. 3 and Fig. 4 leads to the suggestion that the correction surface absorbs part of the systematic effect of the PST. So, inclusion of a state of the art PST model may very well lead to acceptance of the 12-parameter model and improvements near the coast. From the right plot of Fig. 2 and Fig. 4 and the original data distribution, it appears that data rejection took place at locations where residuals are large due to poor marine gravity coverage, and poor tidal modelling of altimetry close to the coast. Improving these aspects and including GPS and levelling of other countries can lead to an accurate unified geoid as a height and depth reference for the whole region.

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