Estimation of geoid and sea surface topography from satellite altimetry by the adjoint method

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Abstract. In the North Atlantic fully reprocessed ERS-1 altimeter data of the year 1993 are analysed in a simultaneous estimation of the geoid undulations and the sea surface topography. The estimation is based on an assimilation of so-called sea surface heights maps into a dynamic model by the adjoint method. The preliminary result for the geoid of the North Atlantic corresponds remarkably well with the geoid of the EGM96 gravity model.

1. Introduction

A satellite altimeter measures the range A from the satellite to the sea surface over large areas of the oceans. With the height H_S of the satellite above a reference ellipsoid being determined independently by precise orbit determination techniques, the height H_M of the sea surface above the reference ellipsoid is obtained by (Fig. 1)

$$H_M = H_S - A. \tag{1}$$

If the sea surface were only affected by the earth gravity force, it would be at rest and coincide with the geoid, the equipotential surface of the earth gravity field at mean sea level. However, because of oceanic currents the actual sea surface deviates from the geoid. The geoid may be described at any given point by the geoid undulation N, the height of the geoid above the reference ellipsoid, while the height of the sea surface above the geoid is known as the sea surface topography h. Thus, the sea surface height H_M is related to the sum of the geoid undulation N and the sea surface topography h by

$$H_M = N + h. \tag{2}$$

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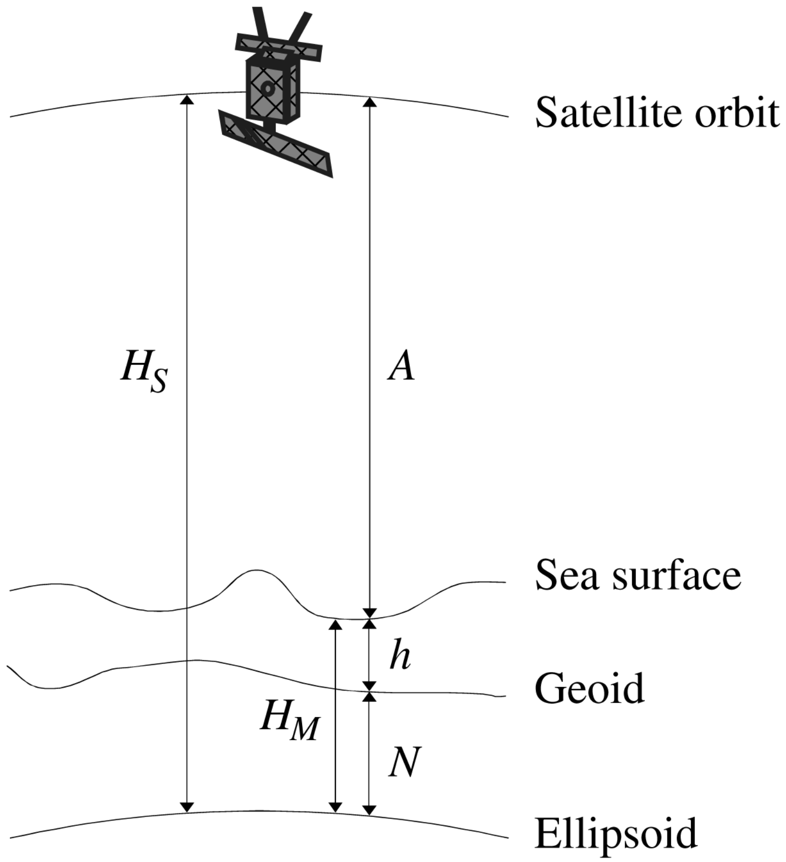

Fig. 1 - Geometry of satellite altimetry

2. Adjoint data assimilation

The satellite altimetry is used to improve estimations of both the geoid undulations N and the sea surface topography h simultaneously. The approach is based on an assimilation of the altimeter data into a dynamic model of the sea surface height H_M. The model consists of stationary geoid undulations and a wind-driven quasi-geostrophic model (Holland, 1978) for the sea surface topography. The quasi-geostrophic model is formed by the quasi-geostrophic vorticity equations for a rotating hydrostatic fluid with two immiscible layers of different densities

$$\begin{bmatrix} \partial_\alpha \partial_\alpha - \mu_1 & \mu_1 \\ \mu_2 & \partial_\alpha \partial_\alpha - \mu_2 \end{bmatrix} \begin{bmatrix} \partial \psi_1 / \partial t \\ \partial \psi_2 / \partial t \end{bmatrix} = \begin{bmatrix} J_1 + R_1 + F \\ J_2 + R_2 + B \end{bmatrix} \quad (3)$$

with

$$J_1 = -\varepsilon_{\alpha\beta}\partial_\alpha\psi_1\partial_\beta\left(\partial_\gamma\partial_\gamma\psi_1 + \mu_1\psi_2 + f\right)$$
$$J_1 = -\varepsilon_{\alpha\beta}\partial_\alpha\psi_2\partial_\beta\left(\partial_\gamma\partial_\gamma\psi_2 + \mu_2\psi_1 + f + \left(f_0 / D_2\right)\eta_B\right)$$
$$R_\gamma = A_H\partial_\alpha\partial_\alpha\partial_\beta\partial_\beta\psi_\gamma$$
$$F = 1/\left(D_1\rho_0\right)\varepsilon_{\alpha\beta}\partial_\alpha\tau_\beta$$
$$B = -e\partial_\alpha\partial_\alpha\psi_2$$
$$\partial_\alpha = \partial(...)/\partial x_\alpha$$
$$\mu_\alpha = f_0^2/\left(g' D_\alpha\right)$$
$$\alpha,\beta,\gamma \in \{1,2\}$$

where ψ_1 is the streamfunction of the upper layer, ψ_2 the streamfunction of the lower layer, f the Coriolis parameter linearized around the midlatitude, f_0 the Coriolis parameter at midlatitude, g' the reduced gravity, ρ_0 the mean density, D_1 the depth of the upper layer, D_2 the depth of the lower layer, τ_β the windstress at the sea surface, A_H the Laplacian friction coefficient, e the coefficient for the bottom friction, η_B the bottom topography, x_α the surface coordinates of an ellipsoidal local system, t the time and $\varepsilon_{\alpha\beta}$ the ε-tensor of the second rank.

In this investigation the quasi-geostrophic model is forced by an annual averaged wind, derived from the results of Hellerman and Rosenstein (1983), which are commonly used in oceanographic assimilation studies. Since the model does not take account of circulations which are due to changes of temperature or salinity, it is only capable of an approximate modelling of the sea surface topography. But its approximation is quite well if one considers the additional computational effort for modelling the thermohaline circulations.

The sea surface topography $h(t)$ at a time t follows from the streamfunction $\psi_1(t)$ of the upper layer by (Arent et al., 1992)

$$h(t) = \frac{f_0}{g}\psi_1(t) + c, \tag{4}$$

where g is the gravity and c a constant, which relates the sea surface topography of the quasi-geostrophic model to the geoid undulations of the area of investigation.

The unknown parameters of the model of the sea surface height H_M are the the geoid undulations N, the streamfunctions ψ_β of the two layers at the initial time and at the open boundaries, the vorticities $\partial_\alpha\partial_\alpha\psi_\beta$ at the open boundaries and the constant c. These parameters are simultaneously estimated by minimizing a weighted least-squares difference between the model and the data. Prior information on the unknown parameters is introduced as additional observations. The Hessian-matrix of the optimization problem is therefore regular and always well-conditioned. For minimizing the least-squares cost function the adjoint method is used. The principle of this method is shown in Fig. 2.

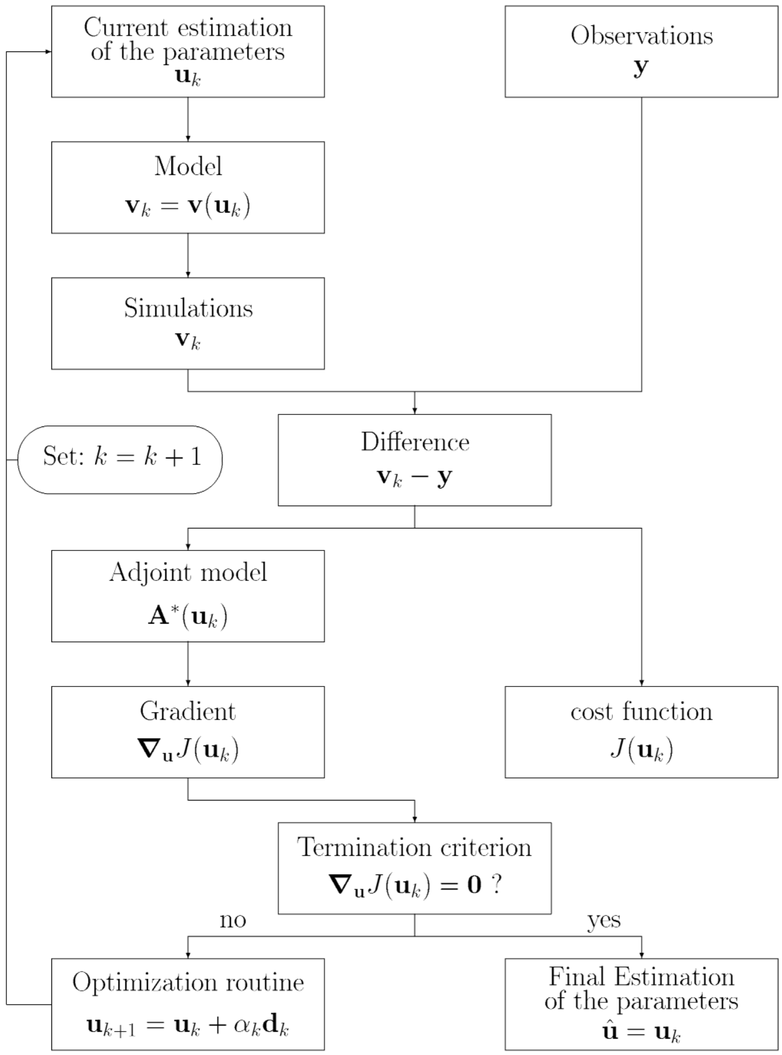

Fig. 2 - Principle of the adjoint method.

3. Results

The simultaneous estimation of the geoid undulations *N* and the sea surface topography *h* from satellite altimetry by the adjoint method is applied in an area of the North Atlantic, bordered in the south by the parallel at 20° latitude, in the north by the parallel at 60° latitude, in the west by a line following the 200 m depth contour line of the ocean along the coast of North America and in the east by the meridien at 350° longitude. The discretisation of the model of the sea surface height is performed on an uniform grid with a spatial resolution of 0.25° resulting in approximately 35000 mesh points. Maps of sea surface heights of fully reprocessed and corrected ERS-1 data (Anzenhofer and Gruber, 1998) from the 5th to the 180th day of the year

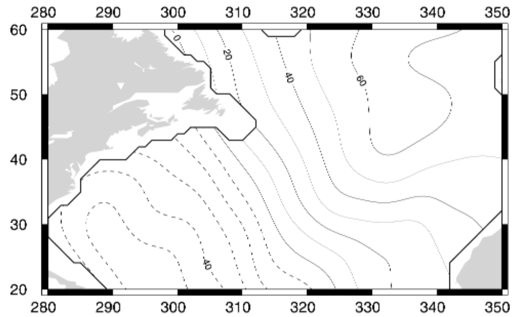

Fig. 3 - Geoid undulations of the PGM055 gravity model in meters.

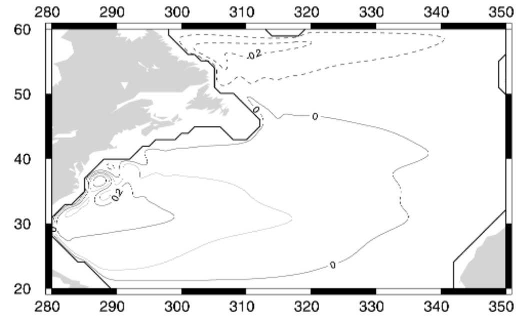

Fig. 4 - Sea surface topography after 20 years of integration in meters.

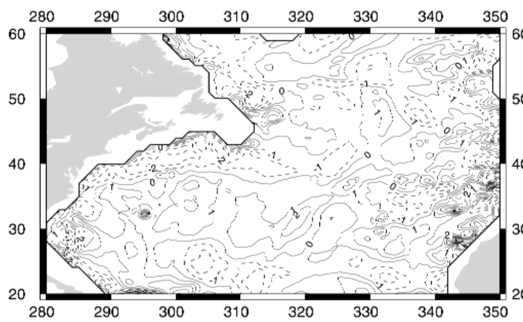

Fig. 5 - Estimated corrections of the PGM055 geoid in meters.

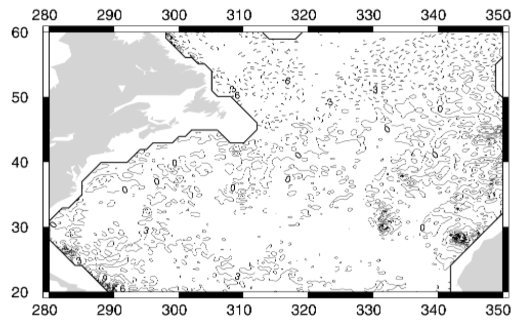

Fig. 6 - Differences between the corrected PGM055 geoid and the EGM96 geoid in decimeters.

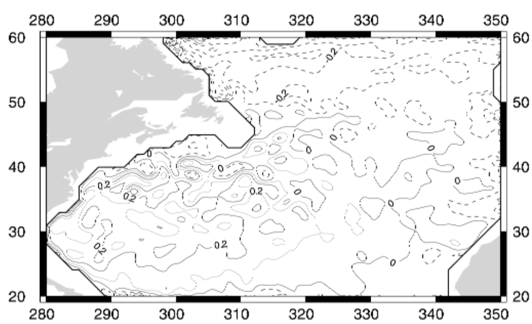

Fig. 7 - Estimated sea surface topography at the end of the assimilation period in meters.

1993 have been analyzed. The maps, which contain a temporal shift of 7 days and spatial resolution of 0.25° were obtained from the German Processing and Archiving Facility (D-PAF) for ERS. Because of its high resolution the data is well suited for this investigation. More data will be processed, so that the results have to be regarded as preliminary. The results and a detailed presentation of the dynamic model and the adjoint method is published in Blinken (1999).

The geoid undulations *N* and the sea surface topography *h* before the assimilation are shown in Figs. 3 and 4. The geoid undulations are obtained from the PGM055 gravity model, which is an upgraded GRIM4-S4 gravity field and is used by the D-PAF for the computations of the ERS-1 orbits. The PGM055 model is represented by spherical harmonics up to degree and order 60. The sea surface topography is obtained by the integration of the quasi-geostrophic model from an initial state of rest over a time of 20 years up to a state of quasi-stationarity. The heights of the simulated sea surface topography are smaller than those of the real sea surface topography. This is a typical feature of the sea surface topography derived from numerical ocean models.

The estimated corrections of the PGM055 geoid are shown in Fig. 5. The large corrections in the order of several meters are due to the fact that the PGM055 geoid contains only the long wavelength undulations, while the altimeter data are capable of resolving the short wavelength undulations. The quality of the estimated geoid follows from Fig. 6, which represents the differences between the corrected PGM055 geoid and the geoid of the well-known EGM96 gravity model. The rms-difference is 32 cm. The deviations are mainly due to the simple quasi-qeostrophic model. But comparing the effort to correct the PGM055 geoid as described in this paper to the one of generating the EGM96 gravity model, the results are remarkably good. Fig. 7 shows the estimated sea surface topography at the end of the assimilation period. Due to the influence of the altimeter data a more realistic sea surface topography is obtained, which corresponds quite well with the results of oceanographic investigations.

4. Concluding remarks

The analysis has shown that the assimilation of altimeter data into a dynamic ocean model makes a signicant contribution to the improvement of the marine geoid, although a simple model has been chosen. The incorporation of a more realistic ocean model with high spatial resolution is being planned. This will result in further improvements of the marine geoid.

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