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SST estimation using collocation: method refinements and seasonal variability

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Abstract. The collocation method can be fruitfully applied in SST estimation on repeated mission tracks. Some applications have been carried out in the western Mediterranean Sea using ERS1-ERM and TOPEX/POSEIDON data. In this work some refinements of the method are analyzed to obtain improvements in the estimates of the stationary component of the SST in the area mentioned above. Moreover, seasonal variability is investigated, on the basis of the previous stationary SST estimate, using ERS1 repeats. Some comparisons are made with the circulation model obtained in the whole Mediterranean Sea from a different elaboration of the TOPEX/POSEIDON data. They prove that altimetric-derived seasonal variability is in quite good agreement with such circulation patterns, although they are extremely weak and display irregular features.

1. The use of collocation in stacking procedure

1.1. The methodology

The difference between the fully corrected *SSH*, and the geoid *N* along one track, can be modeled as follows (Barrile et al., 1994):

$$\Delta(P^k,t^k) = SSH(P^k,t^k) - N(P^k) = \\ = SST^0(P^k) + SST^t(P^k,t^k) + \delta N(P^k) + \varepsilon(P^k,t^k) + noise \tag{1}$$

where $SST^0(P^k)$ is the stationary Sea Surface Topography; $SST^t(P^k, t^k)$ is the time dependent Sea Surface Topography; $\delta N(P^k)$ is the residual geoid; $\varepsilon(P^k, t^k)$ is the residual radial orbit error plus the residual error of tide; *k* labels the epoch of the different repetitions of the same track. In order to stack the data referring to the same ground track by a collocation filtering, after the

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determination of the common origin in each block of repeated tracks, and of the along-track abscissa of the observations of each track, a reduction of each track observations to zero average is needed. We then calculate, trackwise, a bias (for TOPEX) or a bias and a tilt (for ERS1) (Barzaghi et al., 1997), and remove this linear trend with respect to a reference surface which, in this case, is the Geomed geoid of the Mediterranean. This trackwise estimate of bias and tilt will be refined by a crossover adjustment (Brovelli and Sansò, 1993). Note that in the case of short tracks, like the ones we work with in the Mediterranean area, this linear trend is an adequate model for the residual radial orbit error (Schrama, 1989). We obtain:

$$\delta(P^k, t^k) = SST^0(P^k) + SST^t(P^k, t^k) + \delta N(P^k) + noise \quad (2)$$

In order to understand the idea which stands behind the use of collocation filtering, to stack the repeat mission data, one has to think of the nature of the signal we are analysing. The along-track heights of the instantaneous sea surface, obtained from the altimeter observations during one of the repeated missions, contains the geoid and the stationary part of the *SST*, which are the same in all the repeat missions, and a time dependent part that is connected to phenomena which act in different ways during the different passes. If one considers the whole group of repeated tracks, the stationary part of the signal (geoid and stationary *SST*) persists from one pass to the other; on the contrary, the time dependent part is not correlated and can be regarded as “noise” (i.e. an uncorrelated component) in a collocation approach. So we split δ values into a signal component equal to $SST^0(P^k) + \delta N(P^k)$, and a noise component $SST^t(P^k, t^k)$ + noise. Having made this assumption, we compute the empirical covariance function of the signal using $\delta(P^k, t^k)$ values for each group of repeated tracks, then we fit it with a proper covariance model and estimate $SST^0(P^k, t^k) + \delta N(P^k)$ by means of the collocation formula. We analysed the contribution of the two terms to the estimated stationary signal and convinced ourselves that the contribution to δN, at least some 30 km away from the coasts, is negligible; so that, finally, our interpretation, at least off coasts, is summarized by the formula:

$$SST^0(P_i^k) = \sum_{nmlj} C_{s\delta}(P_i^k, P_n^l) C_{\delta\delta}^{-l}(P_n^l, P_m^j) \delta(P_m^j, t_m^j) \quad (3)$$

Fig. 1 shows the result obtained filtering one of the ERS1-ERM tracks.

1.2. Analysis for a possible improvement in the methodology

In the filtering phase of the repeat tracks the signal has to be derived from residuals of a previously performed detrending. The linear regression on each track is performed by assuming uncorrelated residuals; this gives a non-optimal estimate, inconsistent with the fact that data contain a correlated signal. This drawback can be overcome in the context of a kriging approach (Wackernagel, 1995). Let us consider a stationary signal having a non zero-mean; the difference between the process at two different points has a mean value equal to zero, in fact

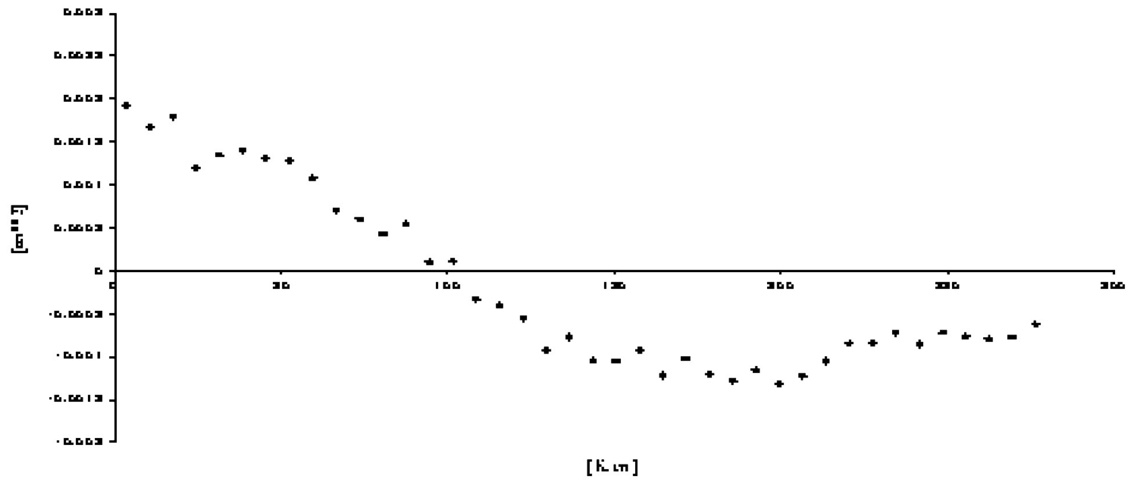

Fig. 1 - Result of the collocation filtering applied to one of the ERS1-ERM tracks.

$$E\{Y(t_2)-Y(t_1)\}=E\{S(t_2)+a+\upsilon_1-S(t_1)-a-\upsilon_2\}=0. \tag{4}$$

We define the variogram 2γ of the process $S(t)$ as

$$2\gamma(t_1,t_2)=E\{[Y(t_2)-Y(t_1)]^2\}. \tag{5}$$

It is possible to determine the relationship between the variogram and the covariance function of the process and use it to estimate the covariance itself without determining the mean a of $S(t)$. In the case of a stationary signal the relationship between the covariance function and the variogram is

$$\gamma(\tau)=\gamma(|t_2-t_1|)=C(0)+\sigma_\upsilon^2-C(|t_2-t_1|)=C(0)+\sigma_\upsilon^2-C(\tau) \tag{6}$$

In the case of a non-zero mean signal, the idea is to derive the empirical covariance function using the relationship in Eq. (6). We start from Eq. (5), deriving an empirical estimator for $\gamma(\tau)$, namely for each $\tau_k = (2k+1)\Delta\tau^{1/2}$

$$\gamma(\tau_k)=\frac{1}{2N_k}\sum_{ij}[Y(t_i)-Y(t_j)]^2 \tag{7}$$

with i, and j such that τ_k - $\Delta\tau$<| t_i - t_j|≤ τ_k + $\Delta\tau$. This is in strict analogy with the empirical estimation of the covariance function. From Eqs. (7) and (6) an empirical estimate of the covariance function can be derived by exploiting the following remark: if the signal $S(t)$ is the realization of a process with fading memory, what we assume, for large values of τ Eq. (6) shows that $\gamma(\tau)$ has an horizontal asymptote equal to $C(0) + \sigma_v^2$. Therefore, by estimating $\hat{C}(0) + \sigma_v^2$ from the horizontal asymptote of $\gamma(\tau)$, Eq. (6) can be reversed to provide

$$\hat{C}(\tau_k) = \hat{C}(0) + \sigma_v^2 - \hat{\gamma}(\tau_k). \tag{8}$$

Once the empirical values of the covariance function are computed, we must interpolate them with a definite positive model, and use this to obtain the estimates of the mean of the process. An analogous reasoning can be made for the case of a process with a mean linear in time. The point here is to invent a generalization of the concept of a variogram by using a suitable combination of the values of the observations which annihilates the linear trend.

1.3. First experiments and results

We considered 12 ERM of TOPEX/POSEIDON fully corrected Sea Surface Heights over the same groundtrack. In this case, the residual radial orbit error can be regarded as a constant so we are in the case discussed in the previous section. For each block of repeated tracks we compute the empirical variogram in order to derive an estimate of the empirical covariance function from it. To get the horizontal asymptote of the variogram we had to consider high values of τ, but the empirical variogram of our data doesn't reach an horizontal asymptote as expected because of the non-stationarity of tha data set considered as a whole. So we proceeded with an iterative procedure. We estimated and removed a bias for each track of the block misregarding the correlation among data. Thus we computed the empirical covariance function of those reduced data and used it to estimate a second order bias (residual radial orbit error). No relevant second order residuals orbital error were found in any of TOPEX/POSEIDON block of tracks analysed in the Western Mediterranean area due to the good distribution of the data. Further investigation should be made on ERS1 data, for which ε is modeled as a linear trend.

2. The seasonal variability

The result of collocation filtering over repeated tracks is the separation of the stationary part of the SST from the time varying one. Keeping in mind that we have considered a time span of one year, the stationary part is that part of the *SST* which is correlated during this period, while what we have called noise is, of course, a part which has no correlation during the whole year, but which we can expect to be correlated during shorter time intervals, such as a season. The aim of our work is to find this part of the signal in the uncorrelated residual of the annual analysis and determine both the time interval in which a correlation exists and the position of the seasonal pattern eventually found. Fig. 2 shows that a correlation exists in the annual residuals considering a subset of passes over the same groundtrack which cover a period of three months only.

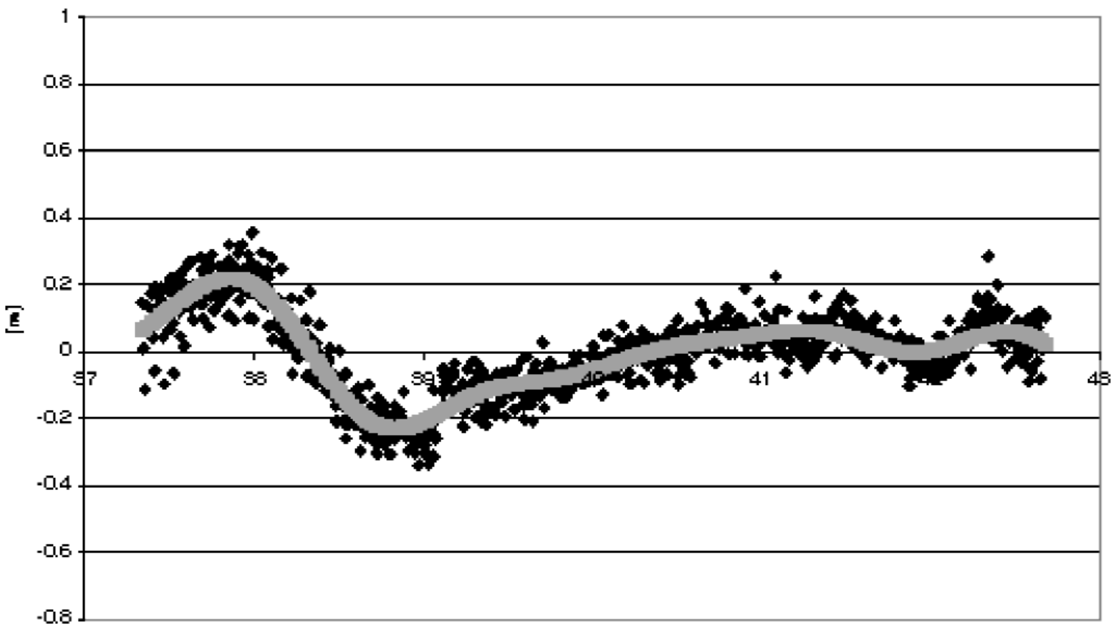

Fig. 2 - Empirical covariance function of SST^t + noise of 3 months' data.

2.1. The experiment

For our experiment we considered the ERS1 data from the Exact Repeat Mission (ERM) in the following time intervals: April 1992 - April 1993 (12 months), October 1992 - October 1993 (12 months), April 1992 - December 1993 (20 months). This has been done to check the stability of the stationary *SST* over different periods. Since no relevant differences were obtained in the three periods, we used the stationary *SST* derived from the third dataset, also to investigate some kind of repeatability of the seasonal components.

The following steps were finally performed on the time varying *SST*, result of the previous collocation altering:

1. selection of the residual $SST^{(t)}$ + noise with a moving window, three months long, translated each time period of 1 month;
2. estimate of the covariance function of each selected block of residuals;
3. selection of those covariance functions which reveal a relevant correlation among the residuals (that is selection of the block of repeated tracks which contains a signal);
4. determination of the zones in which the seasonally correlated *SST* is significantly present.

As for point 3 the choice was performed by means of a t-Student test. We computed the empirical correlation index of the signal $\hat{r}(\tau_i) = \dfrac{\hat{C}(\tau_i)}{\hat{C}(0)}$, for each empirical covariance evaluated at point 2. If more than 50% of the $\hat{r}$ values of a block of tracks satisfies the hypothesis $H_0 : \rho = 0$, we concluded that there is no appreciable correlation among the values of the block itself and vice versa. The crossing of at least two tracks, for which the correlation is considered appreciable, identifies the zones where the seasonal *SST* is present. A repeatability analysis was performed too. The residuals of a block of tracks of the same period and of the same area but of two different years were analyzed to discover the presence of seasonal phenomena. A comparison of the two data sets was made. The conclusion that can be drawn from our results is that where a seasonal phenomenon doesn't exist in one year it doesn't exist even in the following one. More detailed investigations should be made in this direction.

3. Conclusions

The collocation filtering proved to be an efficient method to separate the stationary part of the SST from the time dependent one. The use of a kriging procedure on TOPEX/POSEIDON tracks in order to refine the estimate of the bias of each track, although more correct from a theoretical point of view, doesn't change the result of the whole procedure, due to the homogeneous distribution of the altimetric data along track. The correlation analysis applied to the annual ERS1-ERM residuals, to discover seasonal phenomena, gave satisfactory results. The seasonal variability map that we obtained in the Western Mediterranean area, and the one obtained by Laenicol et al. (1995) in the same area using TOPEX/POSEIDON, are in good agreement; that is the zones with a significant seasonal signal and the power of the signal itself are almost the same, the small misalignment being due to the interpolation of data tracks with a different resolution in space as for TOPEX/POSEIDON and ERS1-ERM.

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