BOLLETTINO DI GEOFISICA TEORICA ED APPLICATA VOL. 40, N. 3-4, pp. 295-302; SEP.-DEC. 1999

# GOCE mission concept, error derivation and performances

M. AGUIRRE-MARTINEZ[(1)] and S. CESARE[(2)]

[(1)] *European Space Agency Noordwijk, The Netherlands*
[(2)] *Alenia Aerospazio, Torino, Italy*

(Received October 4, 1998; accepted August 5, 1999)

**Abstract.** GOCE is a gravity-measuring satellite under consideration by the European Space Agency. The satellite carries a gravity gradiometer, a GPS receiver and a system to compensate external - non gravitational - accelerations. The detailed study of the errors affecting the mission needs complex computer simulations, however a simplified logic for error evaluation and for the allocation of derived requirements has been developed. This logic is based in the analysis of the error sources and their classification as either probabilistic or deterministic. A classification of the error sources has been performed. This analysis has been used to derive “worst case” and “most likely” performances for the mission. This simple approach avoids over-designing while still providing excellent scientific performances.

## 1. GOCE status and mission definition

A scientific justification and detailed description of the mission and can be seen in Balmino et al. (1996).

### *1.1. Programmatic background*

The Earth Explorer Gravity Field and Steady State Ocean Circulation Mission (GOCE) is one of the four candidates proposed for implementation in the ESA Earth Explorer programme of research-oriented missions. GOCE is intended to determine, with high accuracy and spatial resolution, the Earth’s gravity field and its geoid. After undergoing a selection process during the second half of 1999, two of the four candidates will become satellites. If it is chosen, the expected launch date for GOCE weel be March 2004.

Corresponding author: M. Aguirre Martinez; European Space Agency, Keplerlaan 1, 2200 AG Noordwijk, Netherlands; phone: +31 71 565 3828; fax: +31 71 565 4696; e-mail: maguirre@estec.es.nl

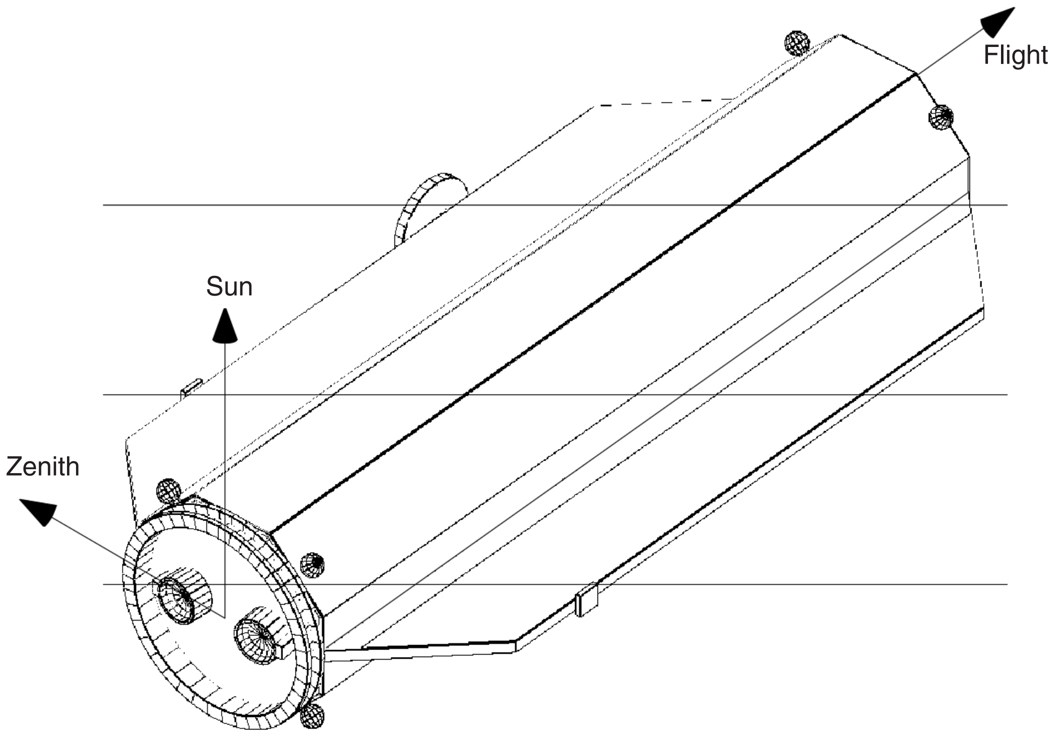


**Fig. 1** - GOCE configuration.

*1.2. Mission concept*

GOCE is a satellite carrying two instruments: a 3-axes gradiometer that could be either of ambient temperature or cryogenic and a GPS/GLONASS receiver called GRAS. The reference orbit is a dawn-dusk, sun-synchronous orbit at an altitude in the 240-250 km range. The baselined mission duration is 8 months. For the capacitive instrument an alternative mission of several years is under consideration.

*1.3. Satellite*

To minimise aerodynamic perturbations and to position the gradiometer near the centre of masses of the satellite, the satellite is slender and fully symmetric. The external satellite dimensions are approximately 0.9 by 0.9 by 4 m. Its mass will be around 700 kg. The satellite is flying with an Earth-pointing attitude. In this situation: the Zenith, Sun and Velocity directions are fixed on a satellite reference frame. These directions can be seen in Fig. 1. The satellite has a octagonal transversal section and two solar array wings fixed to the satellite. Ion-thrusters are used for orbit maintenance and aerodynamic perturbations compensation. They can be seen at the wake face of the satellite.

The key instrument is a three-axial gradiometer. It will provide not only the gravity gradient but also the angular and linear acceleration vectors. The gradiometer bandwidth is from 0.005 to 0.1 Hz. There are two gradiometer alternatives: capacitive and inductive. One of them will be chosen after a trade-off to be performed at the end of 1998. The capacitive gradiometer operates at ambient temperature and is developed by Onera (F). It started development with the cancelled Aristoteles mission. The mass of the instrument is 70 kg, its volume is 0.8 m Ø and 0.9 m long. The inductive gradiometer is under development by Oxford Instruments (U.K.). It is kept below 4° K by a cryostat. The mass of the instrument is 375 kg, its volume is 0.8 m Ø and it is 1.3 m long.

### *1.4. External disturbing drag and accurate measurement*

The gravity field outside the Earth's surface is usually expressed in an (theoretically) infinite series of spherical harmonics (Lambeck, 1988). The observable of the gradiometer will be the second derivative of the gravity potential. This observable will be attenuated by a ratio $(R/r)$ to the power $n+3$. $R$ is the Earth's radius, $r$ is the distance from the centre of the Earth to the satellite and n is the degree of the corresponding spherical harmonic. So, to enhance the scientific return it will be necessary to fly the satellite as low as possible, but the lower the altitude the larger the atmospheric drag forces. The atmospheric drag is compensated by the Ion-thrusters already mentioned. They use electrical energy to accelerate an ionised gas to very high speeds. This allows the compensation of high drag forces, using small amounts of gas.

## 2. Mission performances and error handling approach

### *2.1. Background*

To specify a gravity measuring mission it is necessary to provide geoid and gravity anomaly accuracies for a given resolution. These figures will define the acceptable "commission errors" down to the degree and order corresponding to the resolution. In the other hand, at satellite level, a gradiometry mission has an error budgets on Eotvos. This error budget will be used to derive requirements for all the key elements of the satellite. Tools able to provide the mission performance (in mGal) of a given satellite concept are already available and a description can be seen in Rummel et al. (1993). These tools are used regularly by the GOCE to study the scientific output of the mission. With their help it is possible to transform the scientific requirements on mgal's into satellite level gradiometric error budgets in Eotvos.

A complex "end to end" simulation can be established to provide a sophisticated determination of the total error in Eotvos of the satellite. Such a simulation is already under development and preliminary results are already available, a description of this work done can be seen in Koop 98. In the meantime, an easy to use approach for error budgeting has been implemented. The rest of the paper is the description of this simplified method.

*2.2. Error derivation for GOCE*

GOCE is Earth pointing; in this case the acceleration on each accelerometer will be:

$$\vec{\gamma}_i = [T]\overrightarrow{O_oO_i} + \ddot{\overrightarrow{O_oO_i}} + \dot{\vec{\Omega}} \times \overrightarrow{O_oO_i} + \vec{\Omega} \times (\vec{\Omega} \times \overrightarrow{O_oO_i}) + 2\vec{\Omega} \times \dot{\overrightarrow{O_oO_i}} + \vec{\gamma}_{common} + \vec{\gamma}_{selfgrav}$$

$T$ is the gravity tensor in the gradiometer reference frame. $\Omega$ is the angular velocity. $O_oO_j$ is the vector joining the centre of mass of the satellite with the accelerometer. are the non gravitational acceleration - mainly drag - acting upon the satellite. $\vec{\gamma}_{selfgrav}$ . are the accelerations produced in the accelerometer by the self-gravity of the satellite. The terms with $\Omega$ are generated by the rotation of the gradiometer with respect to the inertial space. They include the terms of: relative, centrifugal and Coriolis accelerations. The above expression can be expanded and the differential accelerations for the different accelerometer pairs can be derived. If each term is made equal to a nominal value plus an error, a list of errors terms can be derived. This type of analysis can be find in Aguirre (1997). All the error terms obtained can be listed and added to provide a "Gradiometric Error Budget". Four types of errors can be derived:

1. Instrument produced: intrinsic noise, i.e. resolution, and thermal stability;
2. satellite produced: space and time tagging, self-gravity changes and errors on satellite pointing;
3. couplings between satellite and instrument: angular and lineal accelerations of the satellite will couple with errors on the misalignment and scale factor of the accelerometers;
4. post processing: error on the recovery of the centrifugal term.

*2.3. The error sources as probabilistic events*

To use the simplified approach described above as a tool for error budgeting, it is necessary to add the different error terms, but many error sources can be treated as uncorrelated stochastic events of known distribution. In this case the standard deviation of the total error will be obtained adding root sum square, the standard deviation of the uncorrelated errors on the list. This approach to error budgeting was proposed by Weinberger 94 for pointing errors of satellites. This approach is a more realistic alternative to the derivation of total error values than the "straight" addition of all the worst cases of all errors listed. The reference above proposed a strategy for the treatment of the different types of errors. They are divided there in the following types:

- constant, i.e. not stochastic: the intrinsic noise of the accelerometers, the reference atmospheric density;
- bounded Unimodal (Gaussian-like). This category includes all satellite pointing errors. Pointing errors distribution would have a moda located at the centre of the specified pointing interval, but 'true' (unbounded) Gaussian distribution shall not be used because they allocate a finite probability to extremely large errors and this is not physical;

- bounded with flat distribution. Here we can include all the mounting and alignment errors. The flat distribution is used because it is realistic to assume that all mounting positions within the specified accuracy interval are equally probable;
- bounded (harmonic bi-modal). All the thermal distortions-induced errors can be included here. Temperature dependent alignments will change along the orbit. These changes will be (harmonic-like), i.e.: two modas at both extremes of the specified interval and a minimum at the centre.

GOCE is using this approach for the overall error budgeting of the satellite in Eotvos. With this probabilistic approach it is possible to have two budgets: a "worst case" budget that is being used for the dimensioning of all the satellite elements and a "most likely" budget to be used for the determination of the scientific performances. In the "worst case" budget the extreme 100 % confidence values are used. In the "most likely" the 1 s values are used.

### *2.4. Main GOCE error sources*

INSTRUMENT ERRORS. - They include: the resolution and quantization errors of each accelerometer and the stability in the distances and relative alignments between accelerometers.

The resolution has been analysed, and expected values have been derived. Tests to support the analysis are in progress. Quantization errors are produced by the "limited accuracy" analogue to digital conversion needed at the output of the accelerometers. This error can be reduced by using converters with a large dynamic range but there are limits on what it is available. The gradiometric stability errors are produced by distortions - mainly thermal - between accelerometers. They are only significant for the capacitive instrument. To reduce them, a "double thermal domain" for the capacitive instrument, is proposed. An inner domain contains the gradiometer and it is thermally isolated, but for a high thermal resistance conductive path to the outer domain. Inside this inner domain there is a very small power dissipation and a very high thermal capacity; then, high stability of the temperature is ensured. This approach is used successfully on the thermal control of H-Masers clocks. All the high power dissipating electronics of the gradiometer will be located in the outer domain. In case an active thermal control of the order of $\pm 1^{\circ}$ were implemented in the outer domain, the analysis will forecast a frequency-dependent stability of the temperature of the order of $mK$ to $\mu K$ in the inner domain. This shall provide a performance compatible with the requirements.

COUPLED ERRORS. - They include the coupling of the accelerometers alignment and scale factor errors with the external angular and lineal accelerations. Common errors on both accelerometers will couple with external differential, i.e. angular, accelerations and differential errors will couple with external common, i.e. lineal, accelerations. The lack of linearity of the accelerometers will also generate errors of this type. There are two ways to reduce these errors:
- the calibration in orbit of the misalignment and scale factor errors. This will be done by shaking the satellite with controlled sinusoidal accelerations of known directions;
- the active control of the external angular and lineal accelerations. The satellite will have an

active drag control system that will keep the external angular and lineal accelerations below specified values.

The calibration will be done using cold gas proportional thrusters, the drag control will be done using Ion thruster whose thrust will be continuously modulated so that it can compensate the external drag. The capacitive instrument will provide external acceleration rejection ratios of ≈ 10-5 (already verified by a test) and the inductive will be in the $10^{-6} \div 10^{-7}$ range (predicted). The expected performance of the drag control system, will be specified so that this error source has the same contribution to the overall error budget as the instrument. This requires a drag control of 5 $10^{-8}$ m/($s^2\sqrt{Hz}$) within the measurement bandwidth of the instrument: 0.005-0.1 Hz. These target values are compatible with the proposed drag control architecture of the missions.

SATELLITE ERRORS. - These errors are produced by the transfer of the gradiometer output from a satellite to an Earth reference frame. They include: instrument and satellite attitude and time and space tagging errors. GPS provides accurate time and position information. With a reasonable pointing requirement of ≈ 4 mrad, the contribution of the satellite errors to the overall budget is very small. A special case is self-gravity, the constant selfgravity pull of the satellite on the accelerometers will not produce an errors but changes on this pull will. To reduce this effect, the satellite will be build with high accuracy materials and all movements of masses in the satellite are minimised, e.g. there are no fluids or mechanisms and the solar array wings are not deployable but fixed.

CENTRIFUGAL ACCELERATIONS RECOVERY ERRORS. - The gravity tensor is symmetric. The angular acceleration field is antisymmetric and can be separated from the gravity tensor, but this cannot be done with the also symmetric centrifugal term. The centrifugal accelerations along the axis perpendicular to the orbit, are very small but along the vertical and along the flight direction, the resulting error term is not acceptable. Nevertheless, this term can be estimated because the gravity tensor shall be traceless; so, the determination of the trace will allow to recover it. It can be considered that all the errors mentioned up to now are uncorrelated; so, the recovery of this term will have an error equal to the root sum square addition of all the instrument and coupled errors already considered. The satellite errors are not included because the centrifugal accelerations can be recovered on the satellite reference frame.

It is clear that this last error is perfectly correlated with the instrument and coupling errors that contribute to it; then the centrifugal acceleration recovery term will be added directly - not the root sum square - to the other error sources. This will determine the total error budget in Eotvos for the satellite. Due to this effect, the performance along ZZ and XX will be roughly twice worst than along YY.

## *2.5. Error budgets*

The Table 1 provides the key error values:
- instrument errors are : I.1 (resolution and quatization) and I.2 (stability);

**Table 1** – The "most likely" error budget.

| Values below in $\frac{m}{s^2\sqrt{Hz}}$ | | 0.005 (Hz) Capacitive | | |
|---|---|---|---|---|
| | | XX | YY | ZZ |
| Number | Name | | | |
| I.1 | Accelerometers | 5.8E-13 | 5.8E-13 | 5.8E-13 |
| C.1 | Coupled with lineal acc. | 6.2E-13 | 6.2E-13 | 4.1E-13 |
| C.2 | Coupled with angular acc | 1.6E-13 | 1.1E-13 | 1.1E-13 |
| I.2 | Gradiometric stability | 4.8E-13 | 4.6E-13 | 5.1E-13 |
| S.1 | Self gravity | 2.12E-13 | 2.12E-13 | 2.12E-13 |
| R | Angular rate knowledge | 1E-12 | 7.3E-14 | 9.1E-13 |
| S.2 | Gradiometer attitude | 9E-14 | 5.2E-14 | 7.5E-14 |
| S.3 | Time and Space errors | 3E-14 | 3E-14 | 6E-14 |
| | Total in m/s^2√Hz | 2E-12 | 1E-12 | 1.8E-12 |
| | **Total in mEotvos** | **3.378** | **1.668** | **3.04** |

- couple errors are divided in two: C.1, that is related to the lineal acceleration and C.2 related to the angular accelerations;
- satellite errors are divided in S.1 (Self-gravity), S.2 (attitude) and S.3 (time and space errors);
- there is only one line for Recovery error *R*.

The total "worst case" error budget for the capacitive instrument along the axis YY is 2.5 mE. This value will be compared with the 1.6 mE "most likely" value indicated in the table of the next page. As said before, the worst case value will be used for the dimensioning of the satellite and will guarantee the performance, nevertheless the not-so conservative "most likely" figure will be used for the determination of scientific performances.

## 3. Conclusions

Stochastic approaches for the handling of satellite pointing errors have been usedsauccesfully in the past. The application of a similar philosophy to the GOCE satellite allows to use better figures for the determination of the scientific performances. This avoids overdimensioning of the satellite while still providing a "fair and honest" view of the most likely scientific output of the mission.

## References

Balmino et al.; 1996: *Gravity Field and Steady-State Ocean Circulation Mission.* Report for Assessment. European Space Agency SP-1196 (1).

Lambeck; 1988: *Geophysical Geodesy.* Clarendon Press Oxford.

Aguirre; 1997: *Goce Error Analysis.* Goce Project ESA Internal Document JP/97-9-1571/MA.

Rummel R., van Gelderen M., Koop R., Schrama E., Sansó F., Brovelli M., Migliaccio F. and Sacerdote F.; 1993:

*Spherical Harmonic Analysis of SatelliteGradiometry.* Netherlands Geodetic Commission, New Series No.39, Delft.

Koop et al.; 1998: *End to End Simulation and Analysis for the GOCE mission*. IGC and IGeC 2nd Joint meeting Trieste, session 3: Dedicated gravity field missions.

Weinberger; 1994. *Statistics for Spacecraft Pointing and Measurement Error Budgets*. Journal of Guidance, Control, and Dynamics; **17**, 55-61.