# Difference between geoid undulation and quasigeoid height in Hungary

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**Abstract.** The difference between geoid and quasigeoid in Hungary was estimated by an approximate formula given in Heiskanen and Moritz (1967) using a data set of mean 1.5' × 2.5' gridded Bouguer anomalies and elevations. The results obtained are discussed from the point of view of the precision GPS heighting.

## 1. Introduction

Nowadays the attention for geoid or quasigeoid determination has had a big push with the GPS technique which has great potential in high-precision height determination. According to whether the type of height is orthometric or normal, the basic equations for the GPS heighting and levelling are:

$$H = h - N, \tag{1}$$

$$\Delta H = \Delta h - \Delta N, \tag{2}$$

or

$$H^* = h - \zeta, \tag{3}$$

$$\Delta H^* = \Delta h - \Delta \zeta, \tag{4}$$

where $H$ is the orthometric height, $H^*$ the normal height, $h$ the ellipsoidal height, $N$ and $\zeta$ the geoid undulation and height anomaly. Since the ellipsoidal heights *(h)* or ellipsoidal height

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differences *($\Delta h$)* are known from high-precision field GPS measurements, the main problem is to calculate $N$ or $\zeta$ and their differences *($\Delta N$, $\Delta \zeta$)* with the necessary precision. Methods used extensively by surveyors consist in interpolation of $N$ or $\zeta$ on non-levelled GPS points between some well-distributed levelled GPS points. The precision of these methods is limited by the precision on the ellipsoidal height coordinates *($h$)* determined from GPS measurements, by the precision on the heights of the levelled GPS points, and by the precision on the undulations of the geoid or the quasigeoid. The ideal reference surface for orthometric heights is the geoid, while the one for normal heights is the quasigeoid. If the height reference system uses normal heights, and geoid undulations are derived from gravity, the difference in height types has to be taken into account, i.e., with a high-precision geoid, where the precision is of the order of centimeters; the difference between geoid and quasigeoid must be considered. Therefore, it is instructive to compare the geoid and the quasigeoid (see e.g., Dahl and Forsberg, 1998).

## 2. Basic relations

Consider a point $P$ located on or above the surface of the Earth. The height of this point above the reference ellipsoid is $h$. Associated with the point $P$ is the normal height $H^*$, the orthometric height $H$, the height anomaly $\zeta$, and the geoid undulation $N$. These quantities are related (Heiskanen and Moritz, 1967, p.325-327) by:

$$h = H + N = H^* + \zeta, \tag{5}$$

$$N - \zeta = \frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H = H^* - H = \delta H, \tag{6}$$

where $\bar{g}$ and $\bar{\gamma}$ are average values of actual gravity and normal gravity, respectively, between the geoid and point $P$ (for $\bar{g}$ depending on the density model) and between the ellipsoid and the equipotential surface corresponding to $U_P$ (for $\bar{\gamma}$). This means that the difference between the geoidal undulation $N$ and the height anomaly $\zeta$ is equal to the difference *($\delta H$)* between the normal height $H^*$ and the orthometric height $H$. Since $\zeta$ is also the undulation of the quasigeoid, this difference $\delta H$ is also the distance between geoid and quasigeoid. Heiskanen and Moritz (1967, p.327) show that:

$$\frac{\bar{g} - \bar{\gamma}}{\bar{\gamma}} H \approx \frac{\Delta g_B}{\bar{\gamma}} H, \tag{7}$$

where $\Delta g_B$ is the Bouguer anomaly. A quantitative estimate of the difference $N - \zeta$ is given in (Heiskanen and Moritz, 1967, eq. 8-103) by an approximate formula:

$$(N - \zeta)_{in\ meters} = \Delta g_{B\ in\ Gals} H_{in\ km}. \tag{8}$$

Since $N-\zeta = H^*-H$, this formula may also be used to estimate the differences between the

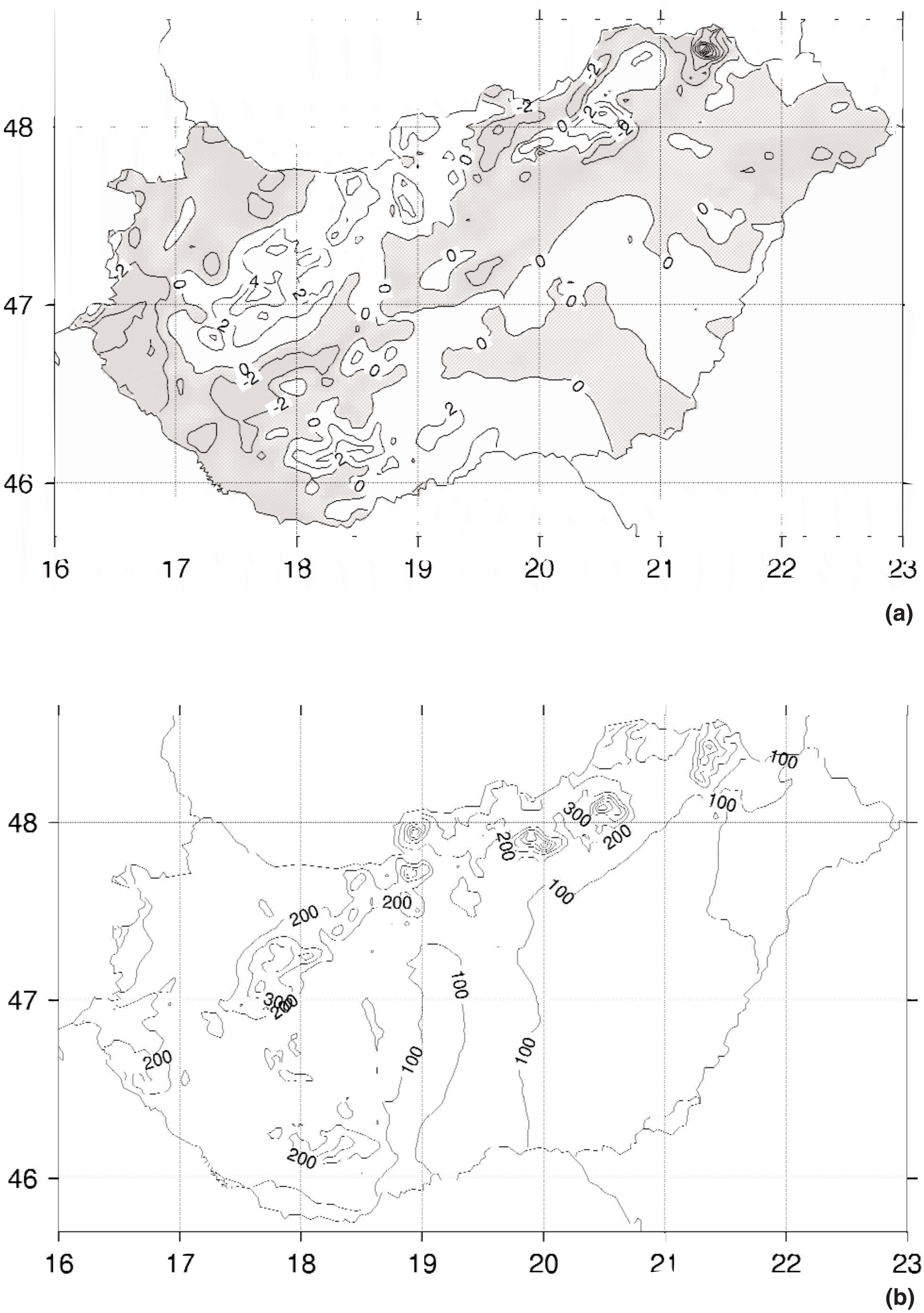


**Fig. 1** - Plot of (a) Bouguer anomalies (contour interval 10 mGal) and (b) elevations (contour interval 100 m) in Hungary.

**Table 1** - Characteristics of data sets.

| Data types | min | max | mean | std |
|---|---|---|---|---|
| Bouguer anomalies [mGal] | -31.8 | 25.2 | -1.5 | ±8.9 |
| Elevations [m] | 77 | 757 | 144 | ±69.1 |

normal height $H^*$ and the orthometric height $H$.

## 3. Computational results and discussion

The data needed in Eq. (8) consist of 13 089 mean 1.5' × 2.5' Bouguer anomalies and elevations (see Fig. 1 and Table 1) which were derived from the Eötvös Loránd Geophysical Institute's (ELGI) database. This data set is the same one Hungary provided for the determination of the European gravimetric geoid EGG97 (Ádám et al., 1995).

The difference between geoidal heights and height anomalies computed by Eq. (8) from the Bouguer map and heights ranges from -17.4 mm to + 7.4 mm (Table 1). Fig. 2 shows a contour plot of these differences, and provides an impression of the strong dependence of the difference between geoidal height and height anomaly on the topography.

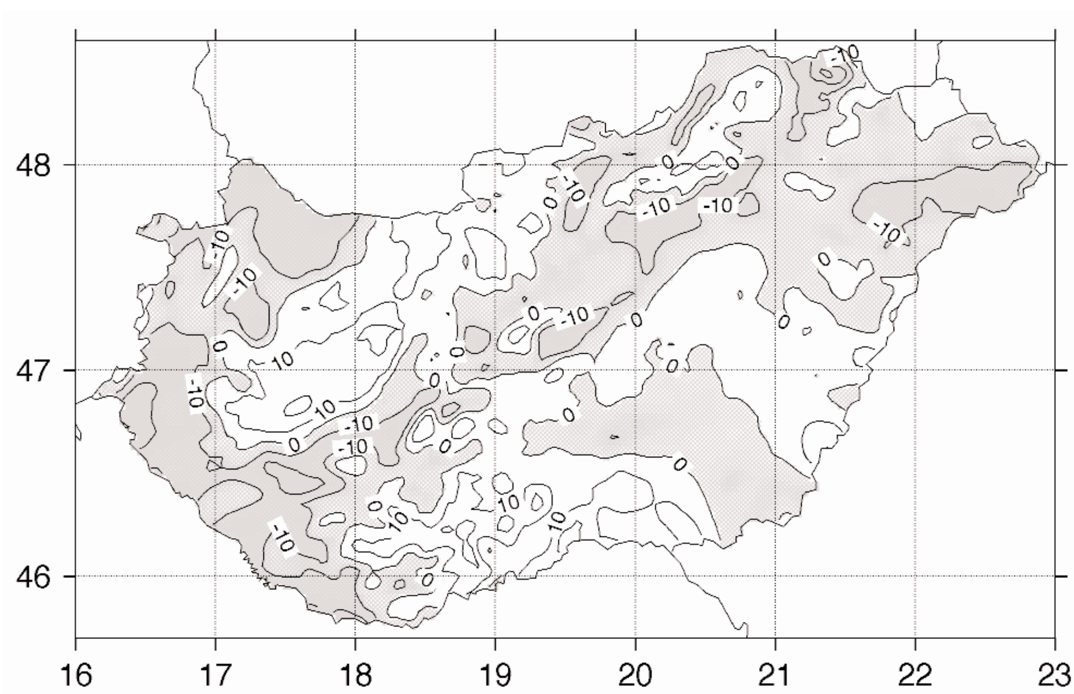


**Fig. 2** - The difference between geoid and quasigeoid in Hungary. Contour interval 2 mm.

Since the Hungarian levelling heights are normal heights, they refer to the quasigeoid. The conversion to orthometric heights for the latest models of the geoid of Hungary (Kenyeres 1997, 1998) is established by consistent modelling of the difference $\delta H$, see Eq. (6). For large parts of Hungary, and due to a moderate topography, the absolute value of the correction $\delta H$ is below 1 cm. Thus, it is negligible in GPS heighting. However, for some local areas it reaches 1 cm - 3 cm, which is obviously not negligible in high-precision GPS levelling. Following Sünkel (1987), and for the territory of Hungary, it would be interesting to compute both the geoid and the quasigeoid to arrive straight at the direct model of the difference between geoidal heights and height anomalies. This difference should be in good agreement with the values shown on Fig. 2 derived from the Bouguer map and topographical heights for Hungary (Fig. 1), see Eq. (8). In this way the procedures for the determination of the geoid and the quasigeoid in Hungary (databases, reductions, software, ...etc.) could be tested independently (Tóth et al., 1998).

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