BOLLETTINO DI GEOFISICA TEORICA ED APPLICATA VOL. 39, N. 2, pp. 99-111; JUNE 1998

Influence of systematic errors in density determination from gravimetric data

V. DEL GAUDIO[1,2], C. MAGRÌ[1], G. RUINA[1] and R. CANZIANI[1]

[1] *Dipartimento di Geologia e Geofisica, University of Bari, Italy*
[2] *Osservatorio di Geofisica e Fisica Cosmica, University of Bari, Italy*

(Received July 7, 1994; accepted April 1, 1998)

Abstract. Density values recently obtained from bidimensional versions of the Nettleton and Parasnis methods showed some discrepancies from those commonly employed for the reduction of gravity data in the Murgian area (Apulia, Southern Italy). This suggested the need to re-examine the procedures adopted and test the influence of data processing methods on density determinations. Particular attention was given to the fictitious correlations that systematic errors can generate between topography and Bouguer anomalies in small scale surveys. The tests showed that the main problem is the presence of errors correlated with elevation or topographic correction, particularly if the range of these errors is a significant fraction of the latter (5% or more). However normal care in data processing should ensure reliable results, and in the case of surveys specifically devoted to density determination (for example in technical geological problems) the choice of correction parameters can even be less crucial than in normal surveys.

1. Introduction

The determination of rock in-situ gross density is required in some technical problems (e.g. microgravimetric surveys, estimation of slope stability, seismic microzonation, and evaluation of rock gross porosity). Estimate based on laboratory measurements present several problems since a scattered collection of rock samples may not be sufficiently representative of the vertical and horizontal lithology distribution. Moreover the extraction and transport of samples to the laboratory generally alter some of their characteristics, giving significant differences from those of the undisturbed in-situ rock. Finally, the gross density depends also on elements which cannot be evaluated at the rock sample scale (fractures, faults, karstic structures) and which must be taken into account with calculations introducing further uncertainties.

For all these reasons the use of independent density estimates based on gravimetric measu-

Corresponding author: V. Del Gaudio; Dipartimento di Geologia e Geofisica, Università degli Studi di Bari, Via E. Orabona 4, 70125 Bari, Italy

rements can be extremely useful. The two methods most commonly employed were proposed by Nettleton (1939) and Parasnis (1962). In their original formulation, they were applied to gravity observations distributed along profiles responding to particular requirements, with obvious limitations to their applicability and representativeness, but generalisations for set of areally distributed measurements were also described (Legge, 1944; Grant and Elsaharty, 1962).
Some "bidimensional" versions of these methods were recently applied by us (Canziani et al., 1989) in detailed studies carried out in the Murgian area, a carbonatic plateau characterised by a relatively homogeneous lithology. The density values we obtained for calcareous formations (2.5 g/cm^3) were slightly greater than those reported in previous studies (2.2 ÷ 2.4 g/cm^3) and derived with other methodologies, such as observation of differences between surface and underground gravity measurements (Boaga et al., 1950), laboratory measurements on rock samples (Carrozzo, 1964) or single Nettleton profiles (Mongelli and Ruina, 1977). All these estimates refer to the same area (Castellana Grotte, BA) and were aimed at obtaining density for topographic correction of detailed gravimetric surveying. The observed discordance could be due to different systematic errors in each procedure, so we decided to choose a test area where independent "geological" and "geophysical" determinations could be carried out and compared under controlled conditions. We were particularly interested in evaluating the reliability of density values computed from gravity surveys of small extension and for this purpose we examined some possible sources of systematic errors.

2. Theoretical aspects

Bot the Nettleton and Parasnis methods give an average estimate of the rock density within the elevation interval of the measurement stations. We adopted a bidimensional variant of the Nettleton method based on preliminary bivariate linear regressions applied to the topographic elevations and to the Bouguer anomalies computed for different trial correction densities. The cross-correlation ϕ between topographic and gravimetric residuals are calculated and the density value for which ϕ is zero is assumed as the correct one. The use of non-linear components of topography and gravity prevents the possible introduction into ϕ of a correlation amount due to the presence of dominant linear components.

As far as the Parasnis method is concerned, we proposed a bidimensional modification (Canziani et al., 1989) based on a multivariate regression of the Faye anomalies F according to the relation

$$F = a + b\,X + c\,Y + \rho\,T, \tag{1}$$

where X and Y are the co-ordinates of the measurement stations and T the topographic correction (Bouguer minus terrain) for unit density. In Eq. (1) the first three terms can be considered representative of a first degree regional field, so the regression residuals can be assimilated to the corresponding residual anomalies. The value obtained for the coefficient ρ gives the density sought, on condition that residuals are not correlated with T.

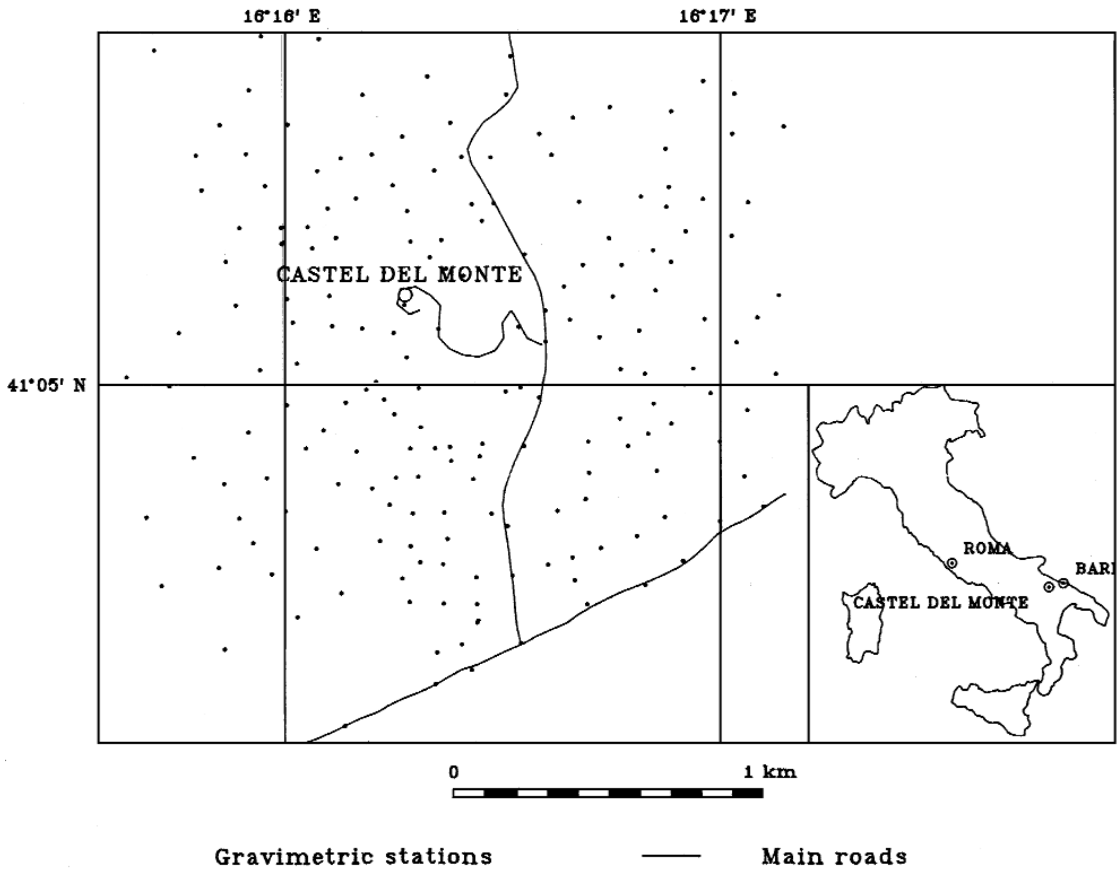

Fig. 1 - Location of the test survey area.

These two methods present an evident conceptual analogy. The Nettleton method fixes the annulling of correlation as a target, the Parasnis as a pre-condition. The kind of correlation considered is different in that it involves topographic elevations in the first method and topographic corrections in the second one. However elevations and corrections are generally strongly correlated with each other, so these two methods give very similar results, particularly if the terrain correction is not very large in comparison with the Bouguer reduction.

The reliability of both these methods is conditioned by the implicit assumption that the correction density is the only factor correlating gravity with topography. Any other correlation source, if present, would alter the density evaluation, moving the result in the sense producing an opposite correlation change: positive or negative contribution to ϕ would give a density, over- or under-estimated respectively.

These considerations point out the importance of recognising all the possible causes of correlation between topography and gravity. Both the Nettleton and Parasnis methods assume implicitly that factors modelling topographic surface and factors controlling the underground mass distribution are totally independent, but this may not be true: for instance, in a tabular style region, erosion leaves topographic highs where rock is more compact and accumulates incoherent sediments in depressions; vice-versa in a region of sandy or clayey lithology, material at the base of highs can be more compressed and dense due to loading than at the top. In such cases a gradual systematic variation of rock density would occur at different elevations and an analogous change in gravimetric observations would be expected. Another possible cause of similarity between topography and gravity is isostatic adjustment (Thorarinsson and Magnusson, 1990): because of isostasy, Bouguer anomalies should mirror the topography, but this effect should be significant only at a large scale, and only in tectonic situations where static equilibrium has at least been approximately reached.

Potential sources of correlation between gravimetric anomalies and topography may also derive from data processing methods. It should be remembered that reductions and corrections of gravity data are done in a simplified way which can create non random effects (LaFehr 1991): for instance, approximations adopted in the Faye reduction and inaccuracy of terrain correction produce systematic errors, so their possible dependence on topographic elevation should be taken into consideration.

The influence of such systematic errors depends on their variation range $\Delta\varepsilon$ with reference to Eq. (1), if errors ε affecting residual anomalies show an approximate proportionality to T by a factor k, the density estimate will result in a value $\rho+k$. Having the possibility of evaluating $\Delta\varepsilon$, the ratio $\Delta\varepsilon/\Delta T$ (with ΔT expressing the range of topographic corrections for unit density) affords an estimate of the density error, or rather, its upper limit corresponding to a 100% correlation between ε and T.

3. Experimental tests

The aforementioned aspects of data processing were the object of an experimental test aimed at studying the influence they can have on the determination of density, particularly when the Nettleton and Parasnis methods are applied to gravity surveys of small extent. For this purpose we carried out a gravimetric survey using 189 measurement stations distributed in the area around Castel del Monte (Fig. 1), an octagonal castle built in the XIII century on top of a calcareous hill about 50 km east of Bari (Southern Italy).

The first target of this test was to calculate a rock density value with a high confidence level. Therefore we applied the gravimetric methods with the maximum possible accuracy in data processing (second order approximation for Faye reduction, maximum extension and detail in terrain correction) and we verified the density determination by collecting and processing geological data as well. For this reason we choose a test area where the most favourable conditions would occur from the point of view of both geophysical and geological methodology (i.e. significant elevation range, lithological homogeneity, minimum presence of anomaly sources, and presence of extended rock outcrops). After obtaining a reliable reference value, we examined how the employment of less accurate processing of gravity data could affect the results.

3.1. Choice of the test area and reference density determination

Morphologically, the area chosen (Fig. 2) is characterised by a not very steep hill about 150 m

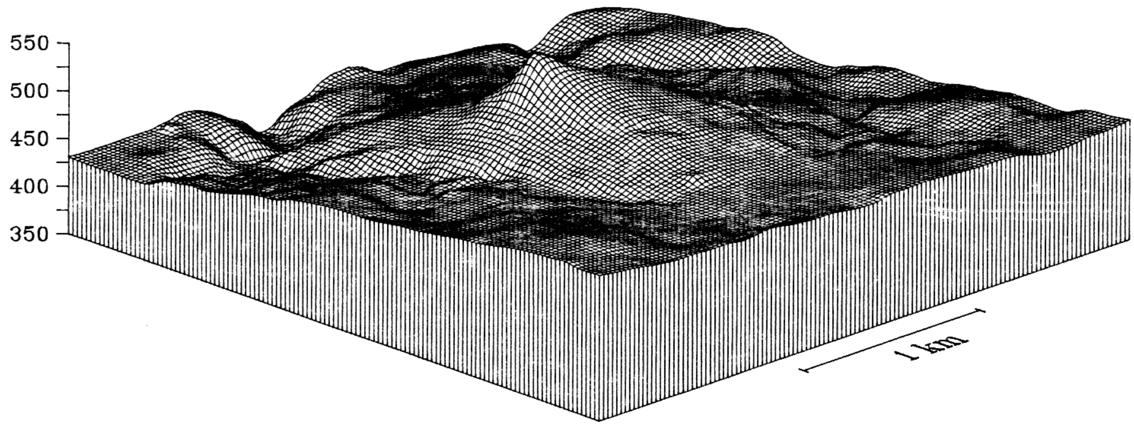

Fig. 2 - Topography of the survey area (the vertical scale reports elevations in meters from sea level).

high, rising on a plateau of Mesozoic limestones. This kind of morphology allows a significant deviation of topography from linearity (which is necessary for the Nettleton and Parasnis methods to work) but without excessive morphological complications which would make an accurate computation of terrain correction problematic. Layers are only weakly deformed from their original horizontality, no important tectonic structure is recognisable, and karstic phenomena are not particularly developed, so the Bouguer anomalies do not show sharp variations. Owing to the poor vegetation cover and the presence of quarries, rocky exposures are fairly frequent: this facilitated the collection of limestone samples and the observations of discontinuity characteristics (dimension, frequency and fillings). On the whole, the characteristics of the chosen test area ensured very favourable operative conditions, so that a high signal/noise ratio can be expected in the data, and systematic effects are better recognisable above the limit of accidental errors.

The density of the samples collected was measured in the laboratory and ranged from 2.59 to 2.71 g/cm^3. The void percentage was evaluated by processing outcrop observations on the basis of three-dimensional numerical models (Broili, 1971), and values from 1 to 10 % were obtained, so a rock in-situ average density of 2.58 ± 0.08 g/cm^3 was estimated.

Gravimetric measurements were carried out with a LaCoste & Romberg G gravity meter and a mean precision of ±0.02 mGal was estimated for them. These data were processed using computer programmes requiring a numerical topography representation consisting of a grid of average elevation values, so the terrain correction is calculated by decomposing topographic relief into prismatic elements. Two numerical models were employed, digitising the maps at the scales 1:10000 and 1:50000: the first model, digitised with a 25 m step, was employed over a distance range of 50-1000 m from each station; the second one was extended up to 10 km with a digitisation step of 100 m. A supplementary contribution to correction was computed for the topography within 50 m from each station on the basis of direct field observations. The results of the density determinations were the same for the Nettleton and Parasnis methods, that is 2.61±0.01 g/cm.^{3}: this value is in excellent agreement with that obtained from the geological measurements.

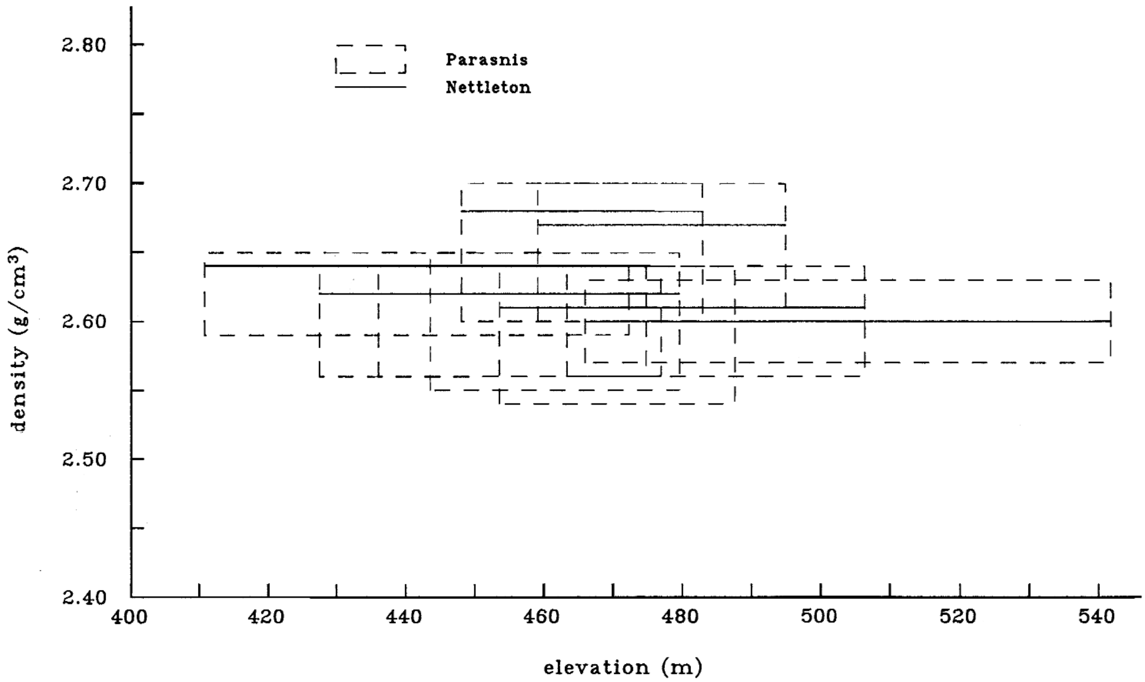

Fig. 3- Densities obtained from the Nettleton and Parasnis methods applied to subsets containing a fixed number (109) of gravimetric observations versus their elevation range. For the results of Parasnis method the confidence band is reported.

3.2. Test on density-elevation correlation

To evaluate the possible presence of density variations depending on altitude, both the Nettleton and Parasnis methods were applied to subsets of gravimetric observations included in different elevation intervals. Since the result reliability is conditioned by the extent of the elevation range and by the number of data, two series of calculations were carried out using subsets of measurements containing a fixed number of observations (about one hundred) and subsets distributed within equally large altitude intervals (about 50 m). These subsets partially overlap each other, however a systematic trend in vertical density distribution, if present, should be recognisable.

Figs. 3 and 4 report the density values obtained against the elevation interval of observations employed (for the Parasnis method the confidence band defined by standard deviation is also represented). The results of the Nettleton and Parasnis methods are fairly consistent: density shows irregular oscillations around the reference value in the interval 2.57÷2.68 g/cm3, and the maximum deviations are observed for values having major uncertainty, so no evidence of a significant systematic variation of rock density can be recognised.

This conclusion is expected, if considering the criteria adopted for the choice of test area; however this kind of test is very easy to perform and so it is recommendable whenever the number of observations available is large enough to provide multiple density estimates with a satisfactory confidence level.

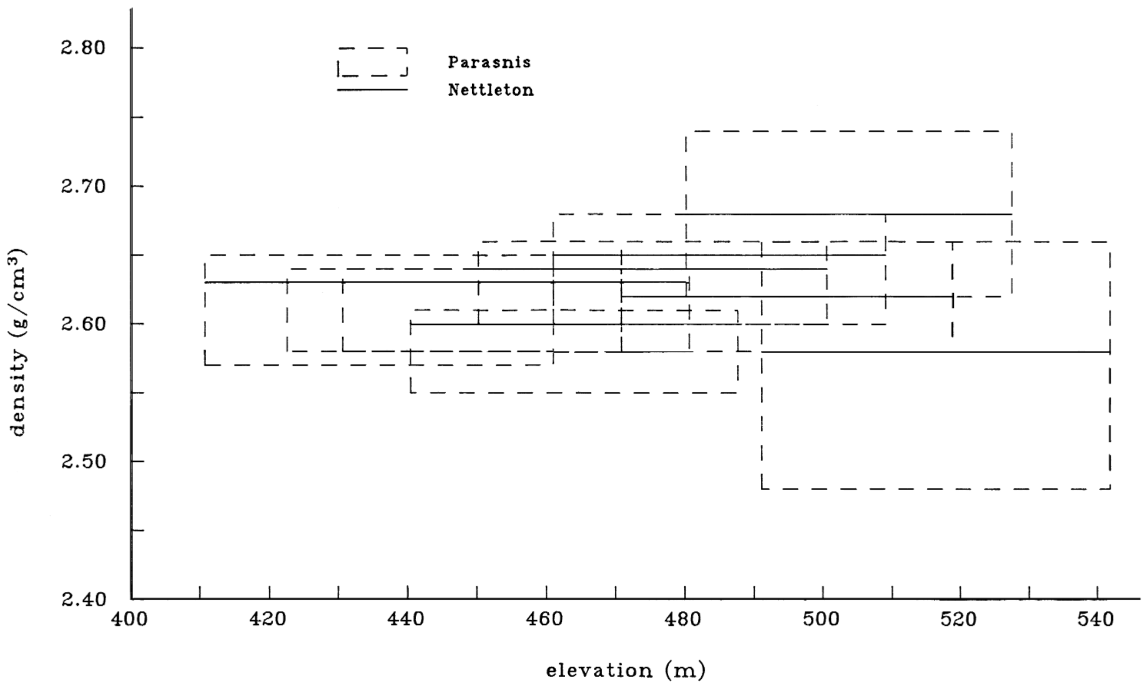

Fig. 4 - Densities obtained from the Nettleton and Parasnis methods applied to subsets of measurements distributed within equally large (about 50 m) altitude intervals. For the results of Parasnis method the confidence band is reported.

3.3. Influence of correction accuracy

In surveys of limited extent the "free air" correction is frequently applied using a linear approximation adopting a mean coefficient valid for the mean latitudes: this practice introduces errors which are certainly correlated to the altitude and so the influence on density determination should be examined.

In order to evaluate this influence we considered the difference between the linear approximation and a second order one defined according to the expression (Morelli, 1968)

$$C_F = 0.30877\, h - 0.44 \cdot 10^{-3} \sin^2\phi\, h - 0.72 \cdot 10^{-7}\, h^2 \qquad (2)$$

where h is the elevation expressed in meters and ϕ is the latitude, with C_F being the correction in mGal.

The differences between these two approximations can be assumed approximately proportional to the elevation h by a coefficient which, in the worst case, is less than 10^{-3}. Indicatively (neglecting the terrain correction contribution) the T value in eq. (1) is $0.04 \cdot h$, so the error introduced in density determination by the adoption of the linear «free air» reduction can be estimated as less (and generally much less) than 0.02 g/cm^3. In the test area, for instance, the errors implied in the linear approximation have a variation range $\Delta\varepsilon$, which is 400 times smaller than

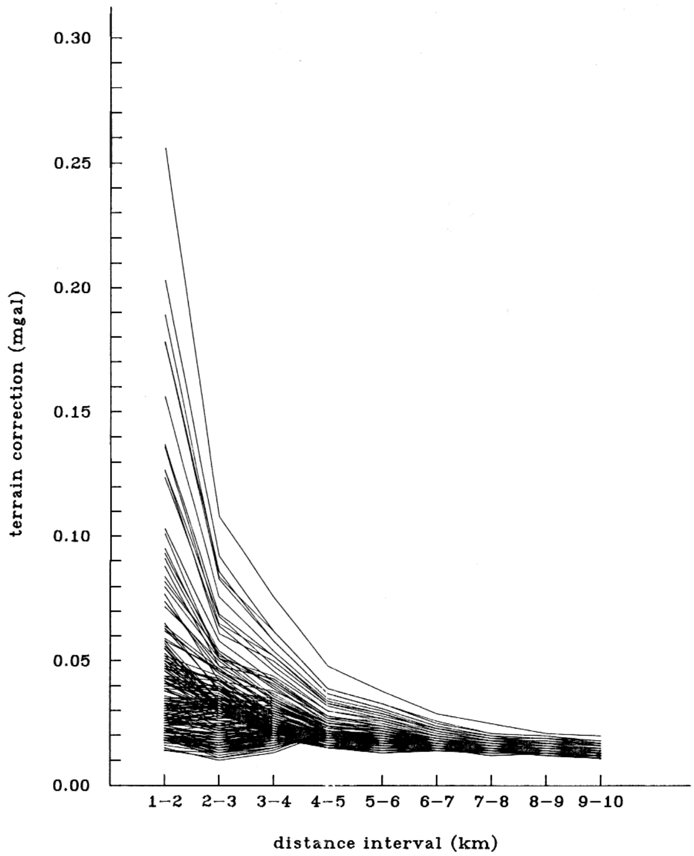

Fig. 5 - Variation of terrain correction contribution through topographic zones located at different distance intervals. Each curve refers to a different measurement station.

ΔT, so the error in density calculation turns out to be inappreciable.

The possible introduction of systematic errors through the simplifications involved in terrain correction was then examined, taking into account particularly the size and representation detail. For this purpose we calculated the terrain correction using different topography models and then compared the density values obtained with the correct one. This part of the test was made easier by the use of an automatic procedure for topographic map digitisation (Del Gaudio and Ruina, 1993).

Different standards have been proposed in the literature for the extension of terrain correction (Hammer, 1939; Kane, 1962; Krohn, 1976; Cogbill, 1990; LaFehr, 1991); however for very small surveys a good economical criterion consists in extending correction up to such a distance that further contributions to correction show differences minor than the survey precision. In the test area such a condition is obtained when the maximum radius of correction (RMAX) reaches the value of 8-9 km, as can be seen in Fig. 5.

Considering the more extended topography model (RMAX=10 km) as a reference, the decrease in terrain correction obtained by reducing the correction extension can be assumed to

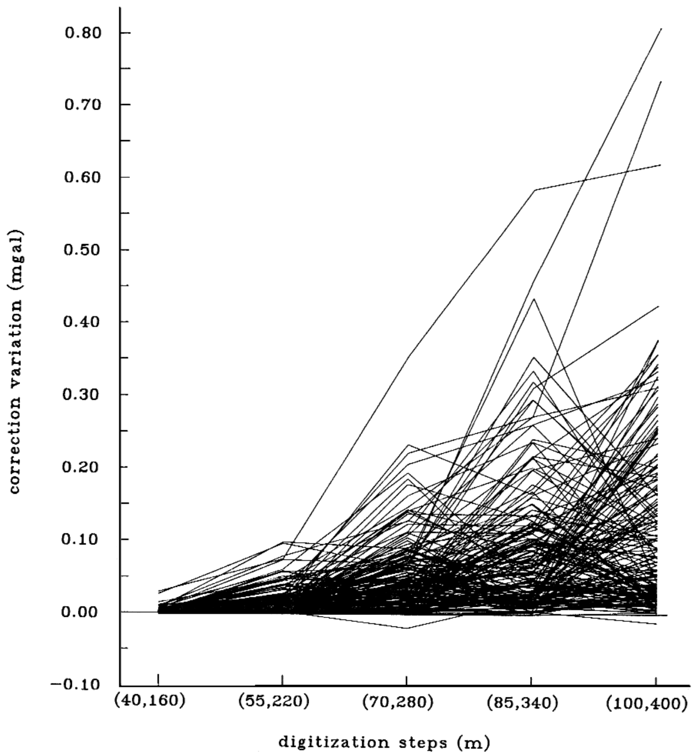

Fig. 6 - Variations of terrain correction with the increase of digitisation step. In abscissa each couple of values represents the steps adopted within and beyond 1 km from the measurement stations. Each curve refers to a different station.

be representative of errors caused by an excessive limitation of RMAX. In the case examined, these negative errors show a strong inverse correlation with both elevation *h* (between -0.905 and -0.954 for different RMAX) and topographic correction *T* (between -0.884 and -0.938). This trend is explained by the fact that, since the distant topography is prevalently at a lower level than the gravity stations, the contribution to the correction of more external zones is greater for higher stations. However the variation interval $\Delta\varepsilon$ of these errors decreases quite rapidly (from 0.132÷0.621 mGal for RMAX=1 km to 0.011÷0.020 for RMAX=9 km): since ΔT is about 5 mGal, the negative error in the density calculation can be expected to affect the first decimal figure only in the case of the minimum RMAX.

Table 1 summarises the results obtained for different RMAX: it is seen that a good approximation to the correct density value is already obtained for RMAX equal to 2-3 km, which is much less than the extension required using the usual prospecting criteria.

To examine the influence of detail in topographic representation, five new numerical topography models were generated using larger digitisation steps, i.e. from 40 to 100 m for the correction zone up to 1 km, and from 160 to 400 m for more distant areas. The differences of terrain correction obtained are shown in Fig.6. This diagram is representative of the error trend: even though there are some oscillations, errors are practically always positive and increasing as

Table 1 - Density values obtained from Nettleton and Parasnis methods for different terrain correction extensions (RMAX).

RMAX(km)	Nettleton (g/cm^3)	Parasnis (g/cm^3)
1	2.53	2.52 ± 0.01
2	2.56	2.56 ± 0.01
3	2.58	2.58 ± 0.01
4	2.59	2.59 ± 0.01
5	2.59	2.59 ± 0.01
6	2.60	2.60 ± 0.01
7	2.60	2.60 ± 0.01
8	2.60	2.60 ± 0.01
9	2.61	2.60 ± 0.01
10	2.61	2.61 ± 0.01

topography model grossness increases. Such a trend is explained by the fact that topographic representation with flat, top prismatic elements causes mass displacement in the model from its exact position in real topography. The amount of this displacement increases with the digitisation step and generates positive or negative errors according to whether the average slope from station to a topographic element has or has not the same direction as the real topographic gradient inside the element (LaFehr, 1991). A minor cause of negative errors occurs when the topography inside a digitisation cell is partially above and partially below the station level, but this is relatively rare: in the area close to the station the situation producing positive errors is necessarily the most frequent and, since this zone gives the main contribution to correction, positive errors prevail over negative ones.

Comparing these correction errors with the elevation h and the topographic correction T, a weak positive correlation is observed (respectively from 0.333 to 0.614 for h, and from 0.309 to 0.578 for T). This can be related to the morphology of the survey area, which is characterised by a hill whose top is at a higher level than most of the surrounding topography: the occurrence of situations generating negative errors is more frequent for lower stations which thus show an attenuation in the positive error dominance.

The range of errors in this case reaches remarkable values: for the grossest model $\Delta\varepsilon$ is 0.8 mgal, so the ratio $\Delta\varepsilon/\Delta T$ is about 0.15 g/cm^3. However the actual error in density determination is attenuated by the low correlation between $\Delta\varepsilon$ and ΔT, which in the case of the grossest model is only 0.309. Table 2 shows the density values obtained for different models: as the digitisation steps increase, the density values show an ascending trend, but in the worst case the error is only 30% of the upper limit estimated from the ratio $\Delta\varepsilon/\Delta T$.

The last parameter whose influence on terrain correction was examined is the correction intermediate radius (RINT) separating the close correction zone (characterised by a finer digitisation step) from the distant one. This parameter was varied from 100 m to 1 km: if we assume the model adopting the largest value as the most accurate one, the differences of correction from this model can be considered representative of the error trend. Fig. 7 shows the results obtained.

To vary the intermediate radius means to vary the representation detail degree over a distance

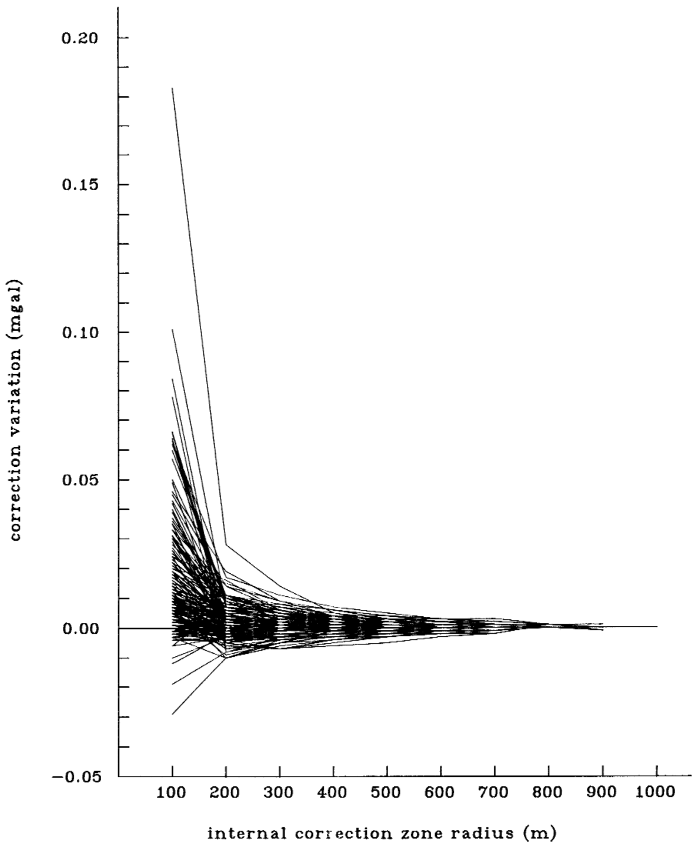

Fig.7 - Variations of the terrain correction with the increase of the radius delimiting the closest correction zone. Each curve refers to a different measurement station.

range between the minimum and maximum values of RINT. When this variation involves an area close to the measurement stations (e.g. for RINT=100 or 200 m) the correction errors tend to be prevalently positive, but at greater distances, situations generating positive and negative error have almost the same probability of occuring and the error distribution becomes symmetrical.

Table 2 - Densities obtained from the Nettleton and Parasnis methods for different digitisation steps employed in the modelling of topography.

Digitisation steps (m)		Nettleton (g/cm^3)	Parasnis (g/cm^3)
Up to 1 km	Beyond 1 km		
25	100	2.61	2.61 ± 0.01
40	160	2.61	2.61 ± 0.01
55	220	2.62	2.61 ± 0.01
70	280	2.63	2.63 ± 0.02
85	340	2.65	2.64 ± 0.02
100	400	2.66	2.65 ± 0.02

In this case the correlation between errors and topography is low (from -0.076 to +0.487 for h, and from -0.082 to +0.474 for *T*) and also the range $\Delta\varepsilon$ is limited (0.2 mGal in the worst case), so this parameter is not relevant to the accuracy of the density determination. Indeed the Nettleton and the Parasnis methods give the correct result for all the RINT values, with a difference of only 0.01 g/cm^3 when RINT=100 m.

4. Conclusions

Application of the Nettleton and Parasnis methods to the determination of density in technical problems requires some care to identify possible sources of correlation between residual anomalies and "residual" elevations or topographic correction for density unit. Any amount of correlation not due to correction density can cause an error that has the same sign as the "fictitious" correlation.

One possible correlation source occurs when rock average density varies gradually with the elevation: this should be verified from time to time according to the local geological situation; possible indications can come from tests using a data subset arranged in order of increasing elevation.

In processing data of small scale surveys, some attention should be given to the choice of extension and digitisation steps adopted in the numerical topography modelling, particularly when situations occur which are potential sources of correlation between errors and topography. This is, for instance, the case in survey areas whose surrounding topography is prevalently at a lower (or higher) level. If the variation range $\Delta\varepsilon$ of the errors implied in topographic correction can be estimated, the ratio $\Delta\varepsilon/\Delta T$ (ΔT being the topographic correction for unit density) gives an upper limit to the error in density measurement. So if the total topographic correction is affected by a relative error lower than 5%, the associated error in density is less than 0.1 g/cm^3.

The test results suggest that a normal care in the choice of parameters for numerical topography modelling allows reliable density determinations to be obtained. On the other hand, in the case of a survey specifically devoted to density measurement (e.g. in technical geology problems), these choices can be less crucial than those required in ordinary surveys.

Finally, the results of the tests substantially confirmed the density values obtained for Murgian limestones in previous applications of gravimetric methods, so the possibility should be taken into consideration that values commonly reported in the literature are underestimated.

Aknowledgements. Work carried out with the financial support of the Italian Ministry of University and Scientific and Technological Research (40% and 60%).

References

Boaga G., Tribalto G. and Zaccara G.; 1950: *Misure gravimetriche nelle Grotte di Castellana*. Annali di Geofisica, ottobre 1950, 439-449.

Broili L.; 1971: *Meccanica delle rocce*. C.N.R. Padova. 73-89.

Canziani R., Del Gaudio V. and Ruina G.; 1989: *Study of gravimetric data in the area of the Castellana Grottoes (Bari)*. Boll. Geof. Teor. e Appl., **31**, 259-267.

Carrozzo M. T.; 1964: *Su un metodo per il calcolo delle anomalie di gravità del gradiente in quota e verifiche sperimentali.* La Ricerca Scientifica, **34**, (II-A), 107-154.

Cogbill A. H.; 1990: *Gravity terrain corrections calculated using digital elevation models.* Geophysics, **55**, 102-106.

Del Gaudio, V. and Ruina, G.; 1995: *Digitization of topographic map for terrain correction in gravimetry: an automatic procedure*. Boll. Geof. Teor. Appl., **37**, 119-129.

Grant F. S. and Elsaharty, A. F.; 1962: *Bouguer gravity corrections using a variable density.* Geophysics, **27**, 616-626.

Hammer S.; 1939: *Terrain correction for gravimeter stations.* Geophysics, **4**,184-194.

Kane M. F.; 1962: *A comprehensive system of terrain correction using a digital computer.* Geophysics, **27**, 445-462.

Krohn D. H.; 1976: *Gravity terrain correction using multiquadric equations.* Geophysics, **41**, 266-275.

Mongelli F. and Ruina G.; 1977: *Sulla ricerca di cavità carsiche in presenza di copertura argillosa*. Geol. Appl. e Idrogeol., **12**, I, 27-35.

Legge J. A.; 1944: *A proposed least square method for the determination of the elevation factor.* Geophysics, 9, 175-179.

La Fehr T. R.; 1991: *Standardization in gravity reduction.* Geophysics, **56**, 1170-1178.

Morelli C.; 1968: *Gravimetria.* Del Bianco Udine. p. 169-172.

Nettleton L. L.; 1939: *Determination of density for reduction of gravimeter observations.* Geophysics, **4**, 176-183.

Parasnis D. S.; 1962: *Principles of Applied geophysics.* Methuen London. p. 40.

Thorarinsson F. and Magnusson S. G.; 1990: *Bouguer density determination by fractal analysis.* Geophysics, **55**, 932-935.