On the stability of the LaCoste-Romberg gravimeter scale factor

D. ARABELOS and D. ROSSIKOPOULOS

Department of Geodesy and Surveying, University of Thessaloniki, Greece

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Abstract. The stability of the scale factor of the LaCoste Romberg gravimeters LCR201 and LCR265 has been studied by analyzing repeated gravity measurements covering a period of five years in a “relative” calibration line. Further consideration of the previously used adjustment algorithm showed that it is not necessary to eliminate the additional parameters before the minimization of the trace of the cofactor matrix in order to overcome the singularity problem, since in our model these parameters do not depend on the reference level. Extended reliability tests showed that the network is satisfactory for calibration of the instruments. The results obtained show an almost linear change in the scale factor for both gravimeters, corresponding to $3x10^{-5}$ per year for the LCR201 and $1x10^{-5}$ per year for the LCR265. No clear trend was observed for the drift behaviour of the instruments.

1. Introduction

Our LCR201 and LCR265 gravimeters have been used for the last twenty-two years over a wide field of applications such as geophysical prospecting, high accuracy control networks for geodynamic purposes, monitoring of earth tides etc. (Makris et al., 1973; Torge et al., 1976, 1981; Mavridis et al., 1980a, 1980b; Arabelos et al., 1982; Arabelos, 1991). In the case of control networks for geodynamic purposes, the goal is the detection of gravity changes with the time, of the order of some μGal/year. Taking into account that the magnitude of the measured gravity differences varies from a few mGal to some hundred mGal, the importance of accurate knowledge of the linear calibration coefficient, the so-called “scale factor” of the gravimeters, is obvious. In Greece eight absolute sites were established in 1993 (Baker et al., 1995). Before 1993 no absolute gravity stations were included in the Greek national gravimetric network that could help the calibration of the gravimeters. In order to overcome the lack of an “absolute” calibration line, a “relative” calibration line was established in 1975 in north Greece.

Corresponding author: D. Arabelos; Dpt. Geodesy and Surveying, University, Univ. Box 474, 54006 Thessaloniki, Greece; tel. +30 31 996091; fax +30 31 996408; e-mail: arab@eng.auth.gr

Table 1 - Relative gravity values of the control gravity network. Unit is mGal.

Station	Coordinates Latitude	Longitude	Height (m)	Relative gravity value (mGal)
1	40° 37’ 00.00”	22° 57’ 00.00”	22	0.016±0.006
2	40° 16’ 21.00”	22° 29’ 43.80”	29	-80.451±0.004
3	40° 12’ 17.55”	22° 19’ 37.12”	368	-171.858±0.003
4	40° 08’ 22.01”	22° 18’ 16.27”	1037	-319.659±0.003
5	40° 06’ 03.38”	22° 15’ 21.73”	1263	-376.799±0.004
6	40° 02’ 22.79”	22° 20’ 16.72”	1909	-501.799±0.006

Our control network consists of 6 stations, starting from Thessaloniki and ending at mount Olympus. The sites of these stations were selected in geologically stable places. In Table 1 the coordinates, the height and the adjusted relative gravity values of the stations are listed. The gravity differences between successive stations cover a wide part of the range of measured differences in the country. The horizontal distance between stations No. 1 and No. 6 is about 85 km, and the corresponding latitude difference is only 35’. The large difference of 500 mGal between the stations 1 and 6 is due to change in gravity with the 1887 m height difference between these two stations. The disadvantages of this relative calibration line are the rapid changes in temperature and atmospheric pressure during the calibration measurements.

In the following sections, the adjustment model will be discussed in connection with the reliability of the data, and the results of the adjustment will be presented. Finally, conclusions will be drawn on the stability of the scale factors, and the drift of the instruments in the specific calibration network.

2. The adjustment model

The relation between observations and unknowns is given by the observation equation for a measured gravity difference between stations *i* and *j* (e.g., Torge et al., 1976; Arabelos, 1985; Torge, 1989):

$$\left(g'_j - g'_i\right) - \left(g_j^0 - g_j^0\right) = x_i + x_j - \left(g'_j - g'_i\right) y + \left(t_j - t_i\right) d + v_{ij}\,, \tag{1}$$

where

g_j'-g_i' is the measured gravity difference after the tidal correction,

g_i^o is an approximation of the actual value g_i of the gravity value at station i,

x_i is the corresponding correction,

d is the linear drift factor,

t_j-t_i is the time difference between the measurements at the stations i and j,

$Y=1+y$ is the linear scale factor of the gravimeter, and

v_{ij} is the residual which represents the unavoidable observation error.

It is easy to recognize that, with the present form of eq. (1), either the gravimeter readings (converted to values in mGal) or the gravity differences could be used as observations.

Table 2 - Measuring periods in the calibration line with the three gravimeters.

Period	LCR201	LCR265	LCR481
1	30.09.75-31.10.75	30.09.75-31.10.75	-
2	07.06.77-10.06.77	07.06.77-10.06.77	-
3	22.10.80-25.10.80	22.10.80-25.10.80	22.10.80-25.10.80

In matrix form, the system of observation equations could be written as

$$\mathbf{b} = \mathbf{Ax} + \mathbf{Bp} + \mathbf{v} \tag{2}$$

where

b is the vector of n reduced observations (observations g'-approximations g^o),

x is the vector of q corrections of approximated gravity values,

p is the vector for r_1 drift factors **d** and r_2 corrections **y** for scale factors,

A, B are the n×q and n×r design matrices, and

v is the vector of the residuals.

By the adjustment principle, $\mathbf{v^T P v}$=min, where **P** is the weight (in our case diagonal) matrix, the system of normal equations reads

$$\begin{bmatrix} \mathbf{N_x} & \mathbf{N_{xp}} \\ \mathbf{N_{xp}^T} & \mathbf{N_p} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{u_x} \\ \mathbf{u_p} \end{bmatrix}. \tag{3}$$

It is well known that, depending on the geodetic problem, the matrix $\mathbf{N_x}$ has a rank deficiency related to the definition of the reference frame. In our case the rank deficiency is equal to one. The problem could be easily solved by fixing the gravity value at a station, but in this case the actual error during the gravimeter reading at this station should be propagated to the other stations. To avoid this, in earlier papers (see e.g., Torge et al., 1976; Arabelos et. al., 1983) the inner constraint solution was adopted.

For defining the scale of the network, we use the condition y=0 for one scale factor.

The inner constraint solution ($\mathbf{e_q^T\, x=0, e_q=}$ $[1\ 1 \ldots 1]^T$) is given by the equations

$$\hat{\mathbf{x}} = \mathbf{R_x^{-1}}\left[\mathbf{u_x} - \mathbf{N_{xp}}\,\mathbf{N_p^{-1}}\,\mathbf{u_p}\right], \tag{4}$$

$$\hat{\mathbf{p}} = \mathbf{N_p^{-1}}\left[\mathbf{u_p} - \mathbf{N_{xp}^T}\,\hat{\mathbf{x}}\right], \tag{5}$$

where

$$\mathbf{R_x} = \left[\mathbf{N_x N_{xp}}\,\mathbf{N_p^{-1} N_{xp}^T}\right] + \mathbf{e_q e_q^T} \tag{6}$$

and

Table 3 - Individual weights used for the common adjustment.

Period	Instruments LCR201	LCR265	LCR481
1	0.450	0.681	-
2	0.386	0.902	-
3	1.000	0.500	0.351

$$\mathbf{e_q\,e_q^T} = \begin{bmatrix} 1 & 1 & . & . & . & 1 \\ 1 & 1 & . & . & . & 1 \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 1 & 1 & . & . & . & 1 \end{bmatrix}. \tag{7}$$

The estimates $\hat{\mathbf{v}}$ of random errors and the a posteriori variance $\hat{\sigma}^2$ are given by the equations

$$\hat{\mathbf{v}} = \mathbf{b} - \mathbf{A}\hat{\mathbf{x}} - \mathbf{B}\hat{\mathbf{p}}, \tag{8}$$

and

$$\hat{\sigma} = \frac{\hat{\mathbf{v}}^{\mathbf{T}}\mathbf{P}\hat{\mathbf{v}}}{\mathbf{n} + \mathbf{1} - (\mathbf{r} + \mathbf{q})}. \tag{9}$$

The relative matrices of covariances are given by

$$\hat{\mathbf{C}}_{\hat{\mathbf{x}}} = \hat{\sigma}^2\mathbf{Q}_{\hat{\mathbf{x}}} = \hat{\sigma}^2\left(\mathbf{R}_{\mathbf{x}}^{-1} - \frac{\mathbf{1}}{\mathbf{r}^2}\mathbf{e_q e_q^T}\right), \tag{10}$$

$$\hat{\mathbf{C}}_{\hat{\mathbf{p}}} = \hat{\sigma}^2\mathbf{Q}_{\hat{\mathbf{p}}} = \hat{\sigma}^2\left(\mathbf{N}_{\hat{\mathbf{p}}}^{-1} + \mathbf{N}_{\hat{\mathbf{p}}}^{-1}\mathbf{N}_{\mathbf{xp}}^{\mathbf{T}}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{N}_{\mathbf{xp}}\mathbf{N}_{\mathbf{p}}^{-1}\right), \tag{11}$$

$$\hat{\mathbf{C}}_{\hat{\mathbf{x}}\hat{\mathbf{p}}} = \hat{\sigma}^2\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{p}}} = -\hat{\sigma}^2\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{N}_{\mathbf{xp}}\mathbf{N}_{\mathbf{p}}^{-1}, \tag{12}$$

$$\hat{\mathbf{C}}_{\hat{\mathbf{V}}} = \hat{\sigma}^2\mathbf{Q}_{\hat{\mathbf{V}}} = \hat{\sigma}^2\left(\mathbf{P}^{-1} - \mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}}\mathbf{A}^{\mathbf{T}} - \mathbf{A}\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{p}}}\mathbf{B}^{\mathbf{T}} - \mathbf{B}\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{p}}}\mathbf{A}^{\mathbf{T}} - \mathbf{B}\mathbf{Q}_{\hat{\mathbf{p}}}\mathbf{B}^{\mathbf{T}}\right). \tag{13}$$

The test for outliers in the observations is carried out using the data snooping strategy

$$r_i = \frac{\hat{v}_i}{\hat{\sigma}(\hat{v}_i)},\ \hat{\sigma}(\hat{v}_i) = \hat{\sigma}\sqrt{\left[\mathbf{Q}_{\hat{\mathbf{V}}}\right]_{\mathbf{ii}}}, t_i = r_i\sqrt{\frac{f-1}{f-r_1^2}} \le t_{f-1}^{\alpha/2}. \tag{14}$$

The simultaneus testing of significance of the drift and scale factor (H_0:$\mathbf{p=0}$) is based on the relation

$$\frac{\hat{\mathbf{p}}^{\mathbf{T}}\mathbf{Q}_{\hat{\mathbf{p}}}^{-1}\hat{\mathbf{p}}}{q\hat{\sigma}^2} \le F_{q,f}^{\alpha}, \tag{15}$$

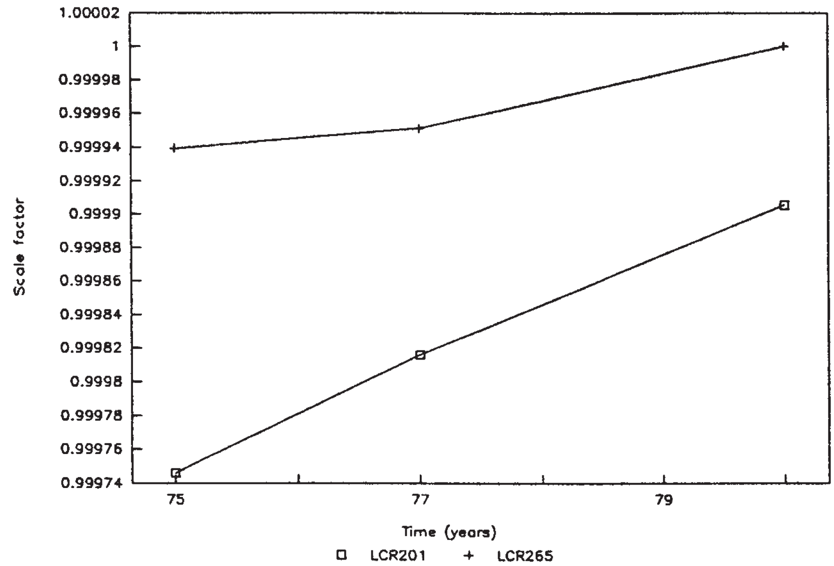

Fig. 1 - Linear scale factor of LCR201 and LCR265 versus time.

A more analytical test can be done separately, either for the drift factor or for the scale factor, according to the relation

$$\frac{\hat{\mathbf{d}}^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{d}}}^{-1}\hat{\mathbf{d}}}{q_1\hat{\sigma}^2} \le F_{q1,f}^{\alpha} \text{ or } \frac{\hat{\mathbf{y}}^{\mathrm{T}}\mathbf{Q}_{\hat{\mathbf{y}}}^{-1}\hat{\mathbf{y}}}{q_2\hat{\sigma}^2} \le F_{q2,f}^{\alpha}, \tag{16}$$

where

$$\mathbf{Q}_{\hat{\mathbf{p}}} = \begin{bmatrix} \mathbf{Q}_{\hat{\mathbf{d}}} & \mathbf{Q}_{\hat{\mathbf{d}}\hat{\mathbf{y}}} \\ \mathbf{Q}_{\hat{\mathbf{d}}\hat{\mathbf{y}}}^{\mathrm{T}} & \mathbf{Q}_{\hat{\mathbf{y}}} \end{bmatrix}. \tag{17}$$

When a single parameter p_i is to be tested (H_0:p=0), the statistics become

$$t_i = \frac{\hat{p}_i}{\hat{\sigma}(\hat{p}_i)} \le \sqrt{F_{1,f}^{\alpha}} = t_f^{\alpha/2}, \qquad \text{where } \hat{\sigma}(\hat{p}) = \hat{\sigma}\sqrt{\left[\mathbf{Q}_{\hat{\mathbf{p}}}\right]_{\mathbf{ii}}}. \tag{18}$$

A significant criterion for the assessment of the reliability of the data snooping and the other tests is introduced through the redundancy numbers, which can be computed for each observation from the relation

$$f_i = \frac{q^2(\hat{v}_i)}{\hat{\sigma}_1^2} = 1 - \frac{q^2(\hat{y}_i)}{\hat{\sigma}_1^2}, \qquad \text{where } q^2(\hat{v}_i) = \left[\mathbf{Q}_{\hat{\mathbf{v}}}\right]_{\mathbf{ii}} \text{ or } q^2(\hat{y}_i) = \left[\mathbf{Q}_{\hat{\mathbf{y}}}\right]_{\mathbf{ii}}. \tag{19}$$

The redundancy numbers serve us a measure of the controllability of each observation.

Table 4 - Linear drift factors for the various instruments and measuring periods. Unit for $\hat{d}$ and $\hat{\sigma}_d$ is μGal/h.

	LCR201			LCR265			LCR481		
Period	$\hat{d}$	$\hat{\sigma}_d$	t	$\hat{d}$	$\hat{\sigma}_d$	t	$\hat{d}$	$\hat{\sigma}_d$	t
1	0.45	0.44	1.02	0.69	0.49	1.41			
2	-1.13	0.50	-2.26	-0.60	0.39	-1.53			
3	-0.09	0.39	-0.23	-1.22	0.50	-2.44	-1.36	0.59	-2.31

3. Computations and results

Three periods of calibration measurements are available, performed with three G-type gravimeters LCR201, LCR265 and LCR481. The last gravimeter was used only during the last measuring period. Table 2 shows the measuring periods with the three gravimeters.

Although never published, the analysis of these measurements was done some years ago for the applications mentioned in the introduction. The reason for the readjustment of the network was that recently actual tidal parameters for the accurate tidal correction became available (Arabelos, 1991), periodical errors of the measuring screw were determined for LCR265, and a new idea was born for the treatment of the measurements. This idea was to introduce as a common adjustment the measurements of each gravimeter for each measuring period with individual drift factors, scale factors and measuring accuracy, i.e., like measurements performed by separate instruments each with its own behaviour. According to this, individual weight factors are needed to characterize the accuracy of each instrument, and each measuring period. To estimate such weight factors, separate adjustments of the observations per measuring period were performed, arbitrarily making the corresponding scale factors equal to one. Consequently, the required weight factors were computed using the formula

$$p^{(i,\alpha)} = \left(\frac{m_0^{(201,3)}}{m_0^{i,\alpha}} \right)^2 p^{(201,3)}, \tag{20}$$

where $m_0^{(i,\alpha)}$ is the r.m.s. error of one observed gravity difference, corresponding to instrument i and period α, and $p^{(201,3)}=1$. Table 3 shows the computed weight factors resulting from the previously described procedure the are used in the common adjustment.

The number of observations introduced in this adjustment was 162, and the number of unknowns was 19 (6 unknown corrections + 7 drift parameters + 6 scale factors) so that the number of degrees of freedom was 143. The mean square error of the unit weight was equal to 0.016 mGal. The redundancy number varies between 0.70 and 0.98. This means that the reliability of the network is satisfactory for the purpose of instrument calibration.

The results of the adjustment are shown in Tables 4 and 5.

From Table 4 it is evident that the linear drift coefficients are not significant. According to

Table 5 - Linear scale factors for the various instruments and measuring periods relative to the scale factor of LCR265 in 1980.

Period	LCR201			LCR265			LCR481		
	$\hat{Y}=1+\hat{y}$	$\hat{\sigma}_y\, 10^6$	t	$\hat{Y}=1+\hat{y}$	$\hat{\sigma}_y\, 10^6$	t	$\hat{Y}=1+\hat{y}$	$\hat{\sigma}_y\, 10^6$	t
1	0.99974598	3.11	-81.67	0.99993941	2.86	-21.19			
2	0.99981611	3.48	-52.84	0.99995079	2.73	-18.02			
3	0.99990518	2.68	-35.38	1.00000000			1.00008542	3.54	24.13

eq. (18), the value t_i is compared to the value $t_{144}^{0,025}$=1.98, for significance level α=0.05. On the other hand there is no clear trend characterizing the drift behaviour of the instruments. Although the gravimeters were always transported together in the same car, the poor results for the drift could be attributed to shocks suffered on the mountainous part of the road.

The same results for significance tests are shown in Table 5.

In Fig. 1 the scale factors of LCR201 and LCR265 are plotted. The almost linear change in the scale factor for both instruments is notable.

4. Conclusion

Repeated gravity measurements on a "relative calibration line" were used to study the stability of the scale factor of two LaCoste-Romberg gravity meters. The reliability tests showed that these measurements are satisfactory for calibration purposes. The changes in the calibration coefficient of the gravimeters used in this test are almost linear and of the order of $3.5\text{x}10^{-5}$/year for the LCR201, and $1.09\text{x}10^{-5}$/year for LCR265. Although these changes are small enough, they cannot be ignored, at least in the case of networks established for geodynamic applications, since they produce changes in the measured gravity differences of the order of 3.5 μGal/100 mGal for LCR201, and 1.1 μGal/100 mGal for LCR265.

Aknowledgement. The establishment of the control network as well as the repeated measurements were made in the framework of the former Department of Geodetic Astronomy of the University of Thessaloniki.

References

Arabelos D.; 1985: *Gravimetry-An Introduction.* Ziti Editions, Thessaloniki (in Greek).

Arabelos D., Karrinti I. and Mavridis L. N.; 1982: *Establishment of a First Order Gravity Network in Northern Greece*. In: Proceedings of the International Symposium on Geodetic Networks and Computations of the I. A. G., D. G. K. Vol. II, Reihe B, Heft Nr. 258, pp. 69-78.

Arabelos D., Karrinti I. and Mavridis L. N.; 1983: *Establishment of a high-precision gravity network in the area of Eastern Macedonia*. Presented IUGG XVIIIth General Assembly, IAG-Section III, Hamburg.

Arabelos D.; 1991: *Improvement of the accuracy of the LaCoste-Romberg Model G gravity meter with an electronic feedback*. In: Festschrift zum 60. Geburtstag Torge, Wissenschaftliche Arbeiten der Fachrichtung Vermessungswesen der Universität Hannover, Nr. 172, Hannover, pp. 7-13.

Baker T. F., Billiris H., Cerutti G., Corrado G., Kahle H. G., Marson I., Müller M., Paradissis D., Pezzoli L., Plag H.-P., Pomrehn W., Richter B., Romagnoli C., Spencer N. E., Tomasi P., Tsimplis N. M., Veis G., Verrone G., Wilson P. and Zerbini S.; 1993: *Sea-level fluctuations: geophysical interpretation and environmental impact (SELF)*. In: Proc. of the Symposium Global Change and Climate Change Impacts, Focusing on European Research, Copenhagen, Sept. 6-10, 1993, in press.

Makris J., Mavridis L. N., Menzel H., Stavrou A. and Veis G.; 1973: *The gravity field of Attika, the Peloponnese and Kithira, Greece*. ZfV, **39**, 929-936.

Mavridis L. N., Arabelos D. and Karrinti I.; 1980: *The Gravity Field of Northern Greece. I. Eastern Macedonia and Western Thrace*. In: Proceedings, Academy of Athens, **55**, pp. 372-381.

Mavridis L. N., Arabelos D. and Karrinti I.; 1980: *The Gravity Field of Northern Greece. II. Western Macedonia*. In: Proceedings, Academy of Athens, **55**, pp. 419-428.

Rossikopoulos D.; 1992: *Surveying networks and computations*. Editions Ziti, Thessaloniki (in Greek).

Torge W., Mavridis L. N., Drewes H. and Arabelos D.; 1976: *Anlage eines Schwerenetzes hoher Präzision im Bereich der Ägäischen Platte.* ZfV, **101**, 213-220.

Torge W., Mavridis L. N. and Arabelos D.; 1981: *High-Precision Gravity Network in the Aegean Region.* In: Proceedings of the International Symposium on the Hellenic Arc and Trench (H.E.A.T.), Vol. II, Athens, pp. 361-375.

Torge W.; 1989; *Gravimetry.* De Gruyter, Berlin, New York.