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AN AUTOMATIC PROCEDURE FOR NUMERICAL MODELLING OF TOPOGRAPHY APPLIED TO TERRAIN CORRECTION IN GRAVIMETRY

Abstract. The application of terrain correction to gravimetric data using computers requires the definition of a numerical model for the survey area topography. In order to shorten this lengthy and intensive operation, an automatic procedure was implemented, based on the digitization of topographic map contour lines and on the processing of the data obtained with an efficient program for evaluating mean elevations in an user-selected grid of square cells. The efficiency of this procedure was tested in two cases with satisfactory results: considerable time saving can be obtained in data processing, and the parameters for the topography numerical modelling can be easily modified, thus verifying their influence on the final results before choosing the most appropriate values.

INTRODUCTION

Different strategies have been devised for automatic computation of terrain correction in gravimetry: some of them make use of numerical integration of the topography gravitational effects, discretized according to a distribution of prismatic elements (Nagy, 1966; Baldi et al., 1978) or polyhedral bodies (Götze and Lahmeyer, 1988); others work out the correction as a convolution integral to be solved by the Fast Fourier Transform (Manzino, 1988). In both cases a numerical representation of the topography is necessary, generally through a matrix of elevation values.

The traditional digitization method is based on a manual procedure consisting in placing a transparent template divided into square cells on the topographic map, and estimating the average elevations in each cell; this is a rather long and tedious operation which involves the estimate, recording and checking of thousands of numerical values, which can easily contain errors difficult to detect.

In some countries, a numerical model of the topography throughout the national territory has been made available in specific data banks, but generally their characteristics suit the requirements of large scale surveys. For instance, during the making of the gravimetric map of Italy (Carrozzo et al., 1986), a general archive of average elevations was generated for the Italian territory; this can conveniently be used for the terrain correction of regional surveys, but the digitization step (about 220 m: see Carrozzo et al., 1981) is too large for detailed local surveys. Therefore we considered it opportune to develop an automatic procedure for a quick and reliable numerical modelling of topographic maps having high scale ratios.

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The procedure we report here is aimed at a topographic representation according to the scheme expected as input by a specific program for the computation of terrain correction. We derived this program by modifying a previous one which E. Klingèlè (E.T.H., Zurich) provided us with personally in 1980, so the topographic representation is the same as that in the Klingélé program, but the general strategy of the procedure could easily be adapted to different programs as well.

A detailed description of how topography is represented in the Klingélé program can be found in Burki (1979). Summarizing its main characteristics, it adopts a decomposition of the topographic surface which has to be modelled over a distribution of small rectangular grids divided into square cells and completely covering the surface around the survey area. The input data are arranged in blocks of records, one for each grid: the first record of a block contains the coordinates of the grid top left corner and its number of rows and columns; the following records give the topographic average elevations estimated in each of the grid cells. In order to compute the terrain correction at a gravity station, the contribution of the topographic mass inside each cell is equated to that of a right prism whose top is at the height of the average elevation and bottom at the height of the station altitude. However, the contribution is taken into account only if the cell falls within a range of distances from the station specified by means of two parameters (RMIN and RMAX, respectively minimum and maximum distance) preceding the topographic description, together with some other parameters like cell size and various options. This allows the computation of terrain correction through the use of multiple topographic models, each having a different detail degree, for different distance intervals, so that a more accurate representation of the topography can be employed at a short distance from stations, and a quicker computation can be applied to distant zones.

PROCEDURE DESCRIPTION

Basic data acquisition

The basic data necessary for the topographic modelling are "altitude points", i.e., topographic surface points whose elevation and planimetric coordinates (in a kilometric reference) are known. Obviously a large number of these altitude points are needed for a good numerical representation of the topography.

A quicker way to obtain a sufficiently large number of points consists in using automatic tools, like digitizers or scanners, to sample the contour lines of topographic maps. For instance, we used a digitizer controlled by a P.C.; and a function like SKETCH of the AUTOCAD package allows a curve to be sampled simply by tracing it with the tablet pointer. Point coordinates are recorded with a user selected sampling step, and data can then be extracted in a vectorial format (DXF) and reorganized into a list of X, Y, Z coordinates. The database is completed by the acquisition of single points corresponding to relative elevation minima and maxima; the coordinates of measurement stations, if accurately levelled, can be conveniently inserted as well.

The sampling interval must be chosen with some care in order to avoid both too gross a topographic representation and unnecessary input data for the following processing stages. As empirical criterion, the sampling step should be comparable to the size fixed for the cells of the final topographic model (from one to one fifth, according to the roughness of the topography).

The collection of altitude points so obtained does not generally show an areally homogeneous distribution, and hence cannot be used directly for a reliable computation of average elevations without some interpolation operations (which are done subsequently with a more efficient, specific program). This must be taken into account when choosing which contour lines have to be sampled in a map: generally, in fact, it is not necessary to sample all of them, but only those marking a slope change; and where several constantly spaced contour lines run parallel, only the two extreme curves need be acquired, thus shortening considerably the operation, especially in cases of high gradient surfaces. Furthermore, to obtain reliable interpolations also in the marginal zones of the topographic model, the sampling area should be extended, if map coverage allows it, to include at least a couple of contour lines outside the area for which modelling was planned.

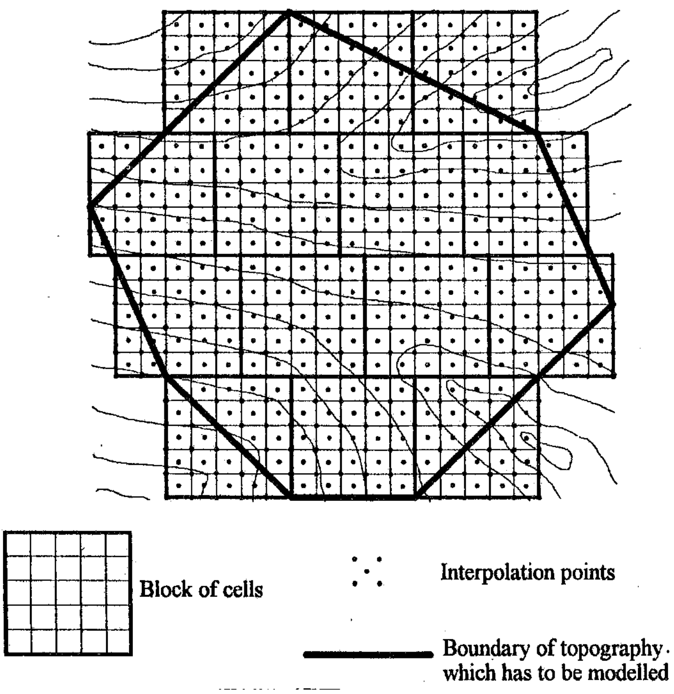

Fig. 1 — Sketch of the framework used by the program QUOTAMED to organize the numerical modelling of topography starting from a contour line map.

A final quality check of the sampling operations is obtained by plotting the sampled countour lines on a transparent sheet and placing this upon the original map, in order to detect possible sampling mistakes, omissions and data coverage incompleteness.

Numerical modelling

Assembling the altitude point database is by far the longest part of the work, even though it is much quicker than traditional manual digitization. The database is then used as input to a purposely written FORTRAN program, named QUOTAMED (Del Gaudio, 1992), which can generate a numerical topographic representation according to detail and extension characteristics choosen by the user.

The program starts processing by fixing the limits of the area whose topography has to be modelled. For this purpose it gives two possible alternative options: to specify the coordinates of the vertices of a polygonal boundary, or to have boundaries fixed by an automatic routine which takes into account the location of the stations and the value of RMAX (the maximum distance at which the topographic model has to be employed in terrain correction); on the resulting polygonal area a division into square cells is set up, which is arranged in rectangular blocks corresponding to the block organization of the terrain correction input data (Fig. 1).

The main part of the program is devoted to the interpolation of topographic altitude at the vertices and at the centre of all digitization cells (see again Fig. 1) on the basis of the altitude point database. This processing stage can be rather heavy, on account of the large amount of data involved (frequently some tens of thousands in both input and output), so a strategy was adopted to save CPU time, particularly for the data selection: since altitude points located near any interpolation point need to be extracted from the data base, the program makes this operation quicker by applying an auxiliary grid of 100×100 square elements to the sampling area, and using a matrix to record the vectorial addresses of the altitude points located in each grid element, so the search for points closest to any position can be restricted to a small sub-set of data.

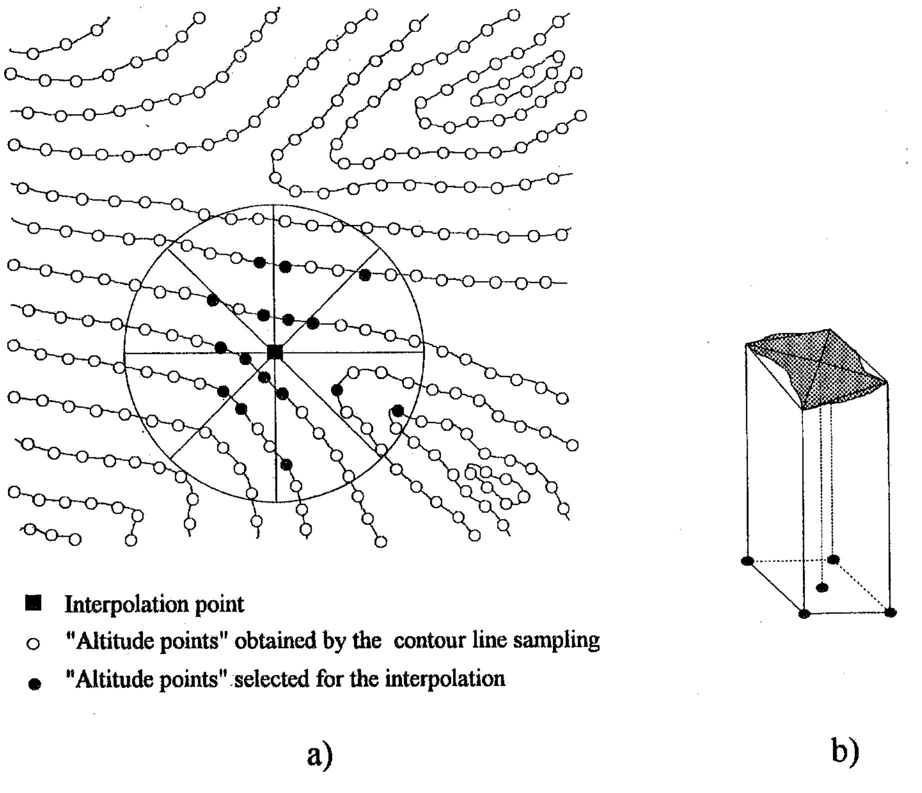

Fig. 2 — Criteria adopted by QUOTAMED to select, from the "altitude points" derived by the sampling of contour lines, those which will be employed in elevation interpolation (a); and simplification assumed to compute the mean elevation inside a digitization cell (b): the topographic surface (in grey) is approximated by a polyhedral surface whose vertices coincide with points on the topographic surface obtained by altitude interpolation at the vertices and at the central points of the digitization cells.

The data used for any altitude interpolation are selected using a criterion which guarantees good azimuthal coverage: the space around any interpolation point is divided into eight azimuth intervals; for each of them the two closest altitude points of different height are extracted from the database (see Fig. 2a); if such a couple of points is not found in one or more azimuthal sectors (e.g., in marginal zones of the sampling area) supplementary points are taken from the contiguous ones. This kind of selection eliminates the danger that all or most of the data employed by the interpolation belong to the same contour line (for instance inside a loop), causing a "flattening" of the result.

Any elevation interpolation is carried out using the 16 altitude points selected: the second degree polynomial surface is calculated which best fits these data according to a least squares criterion, which assigns weights inversely proportional to the squared distance from the interpolation point; then, from the resulting polynomial surface, the elevation at the selected location is obtained. Some checking is also carried out on the data, so that warnings are printed on output when situations requiring verification occur (e.g., linear distribution of the 16 points selected, interpolated altitudes outside the elevation range of input data, and solving system singularities which force the adoption of a different simplified interpolation criterion).

After the altitude is evaluated at all the vertices and central points of the digitization cells, the mean elevations are calculated by equating the topographic surface to a polyhedron having the interpolated points as vertices (Fig. 2b): in each cell the mean height of this surface is expressed as a weighted average of the polyhedron vertex heights (points at the cell centres having a weight twice as large as the others), and this is assumed as mean topographic elevation.

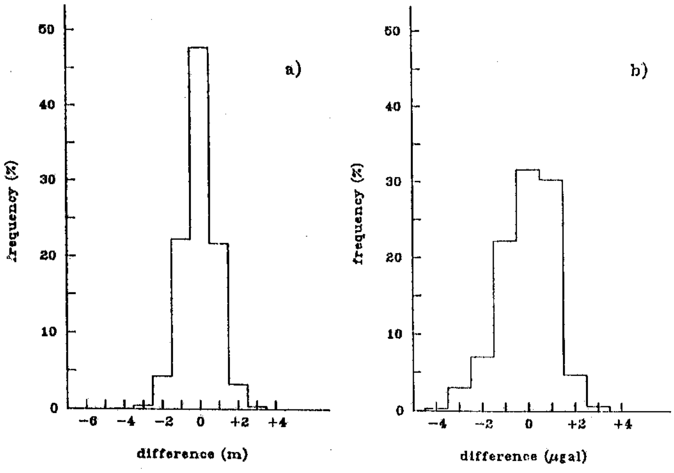

Fig. 3 — Castellana test: distribution of differences (a) between mean elevations obtained by manual and automatic procedures, and (b) between the corresponding terrain correction values.

TESTS

The efficiency and reliability of this procedure were verified by applying it to two areas for which a manually digitized topography was available from previous studies. The program QUOTAMED was originally implemented on an IBM 6150, a microcomputer supplied with a 16 Mb RAM, for which a 0.3 Mflops computation velocity is estimated. Comparisons between topography models obtained manually and automatically, and between the terrain correction values respectively obtained were carried out with a supplementary routine.

Castellana Grottoes

The first test was performed in the Castellana Grottoes area (Bari), where two recent surveys (Bruno et al., 1991; Bruno et al., 1992) had been carried out for the detection of possible continuations of the well-known grottoes. From a geological point of view, this area is characterized by karstic limestones producing small topographic irregularities. Gravity measurements had been carried out at 297 stations with a precision estimated at about 0.01 mGal.

As far as terrain correction is concerned, the topography of a 4×4 km area (with altitude ranging from 275 to 380 m) had been manually digitized from a map to the scale 1:5000. 180 blocks of 10×10 cells with a 25 m step had been defined: this model had been employed for a distance interval between minimum value (RMIN) of 50 m and a maximum (RMAX) of 1 km from each measurement station, a local supplementary correction having been considered for the zone to 50 m.

In the automatic topographic modelling, the contour lines were sampled with a 25 m step and a 5 m contour spacing (save local integrations where topography was more complex). A database of 6924 elevation points was obtained, from which QUOTAMED computed 18000 mean elevation values in about 30 minutes CPU time.

Fig. 3a) shows the statistical distribution of differences between elevations obtained by automatic and manual digitization. These differences range from —6 and +4 m, with a 0 m mean and a 1 m standard deviation (σ); they show a strong concentration in the central classes: almost half of the cells have the same altitude in both models, and only in 14 cases out of 18000, are differences greater than 3 m. These discordances do not show any systematic relation

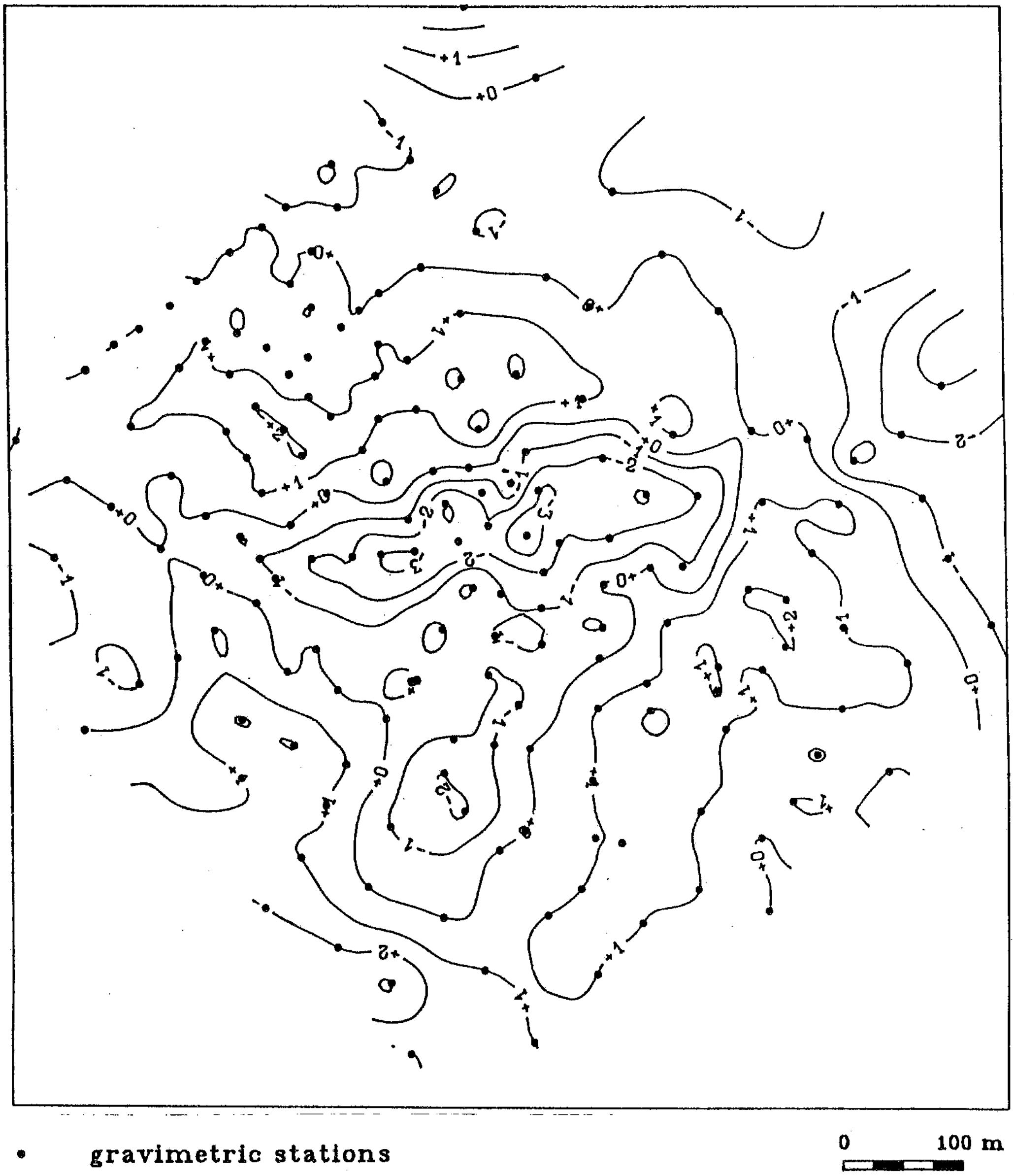

Fig. 4 — Castellana test: map of differences between the terrain correction values obtained by the manual and the automatic procedures (contour spacing 1 μGal).

to elevation (the cross correlation is only —0.1).

The terrain correction was then computed with both topographic numerical representations: values between 0.010 and 0.196 mGal resulted. Fig. 3b) shows the statistical distribution of the correction differences, and Fig. 4 gives a map of their spatial distribution. Differences are distributed between —4 and +3 μGal (mean value=0 and σ=1 μGal) without any correlation with the correction size (cross-correlation=—0.3). The shape of this distribution differs from that observed for the elevations, which is sharper: this reflects the different influences that different parts of the topographic model have on correction in the presence of a non-purely random spatial distribution of discrepancies.

It is noteworthy that the differences observed are all much lower than the experimental

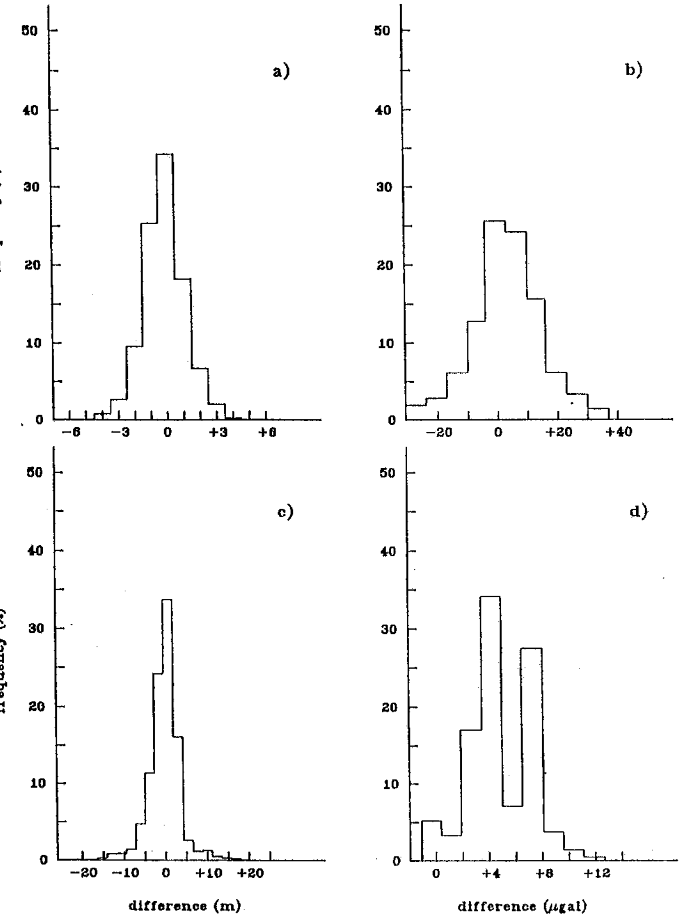

Fig. 5 — Pisticci test: distribution of differences between mean elevations (a - first model, c - second model) obtained by the manual and automatic procedures, and between the corresponding terrain correction values (b - first model, d - second model).

measurement precision, so that in this case the reliability of the procedure proved particularly good.

Pisticci

Between 1979 and 1980, two gravimetric surveys, using 211 measurement stations, had been carried out south of Pisticci (Matera) to study the characteristics of some landslide bodies. A mean measurement precision of respectively 0.05 and 0.01 mGal had been obtained.

This area is characterized by a "badlands" morphology: Pisticci is on the top of a clayey hill modelled by a branched network of sharp erosional incisions, so the terrain correction had been rather complex and the use of three topographic models, covering progressively outer zones, had been necessary. However the most external model was not obtained from a direct estimate of mean elevations from the map but from a simplified procedure using single, regularly distributed elevation values. Thus for comparison between manual and automatic procedures only the two inner models were considered, which are respectively characterized by cell sizes of 20 and 50 m, and were applied to distance ranges of 20-150 and 150-2500 m.

For the first model, main contour lines at 10 m altitude intervals and, locally, secondary ones (having a 2 m contour spacing) were sampled from a map to the scale 1:2000 with a step equivalent to 5 m. A data base of 20647 points (with altitude ranging from 140 and 395 m) was obtained, and 3600 mean elevations were calculated in 10 minutes of CPU time.

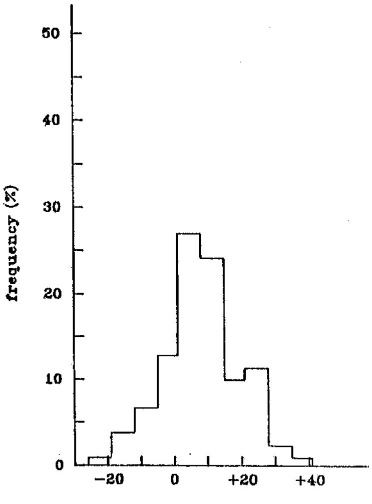

Fig. 6 — Pisticci test: distribution of differences between the terrain correction total values obtained by the manual and automatic procedures.

Differences beetween elevations obtained manually and automatically do not exceed ±6 m, the mean being 0 m and $\sigma = 1$ m; their distribution was totally uncorrelated to altitude (cross correlation=0.03). The frequency histogram is reported in Fig. 5a). The distribution of differences is a bit less concentrated in the central classes than in the previous test, but is still noticeable: in 2804 digitization cells, differences do not exceed 1 m, and in just 4 cases are they greater than 4 m.

The corresponding terrain correction ranges from 0.064 to 0.665 mGal: the two procedures give results differing by values ranging from —0.027 to +0.035 mGal, with a 0.003 mGal average and a 0.011 mGal standard deviation. Fig. 5b) shows the distribution of these differences: even though the elevation difference distribution is not very unlike that of the previous test, terrain correction discrepancies are one order larger. This can be attributed to the minor value of parameter RMIN (20 versus 50 m in the Castellana case): similar elevation discordances cause larger discrepancies when the minimum radius of correction is smaller. This is confirmed by the fact that, extending RMIN to 50 m in this test also, discrepancies between manual and automatic correction are about 3 times smaller. So it is evident that the topography within a few tens of meters has a critical role in producing a different correction evaluation.

For the zone from 150 to 2500 m, terrain correction was based on maps to the scale 1:25000. Both the altitude interval of the contour lines selected, and the sampling step were 25 m; from the data base obtained, including 25408 points, a grid of blocks consisting of cells with a 50 m side was fixed, and 19200 mean elevation values were computed by QUOTAMED in 50 minutes. Figs. 5c) and d) show the distribution of differences resulting from the two kinds of digitization, relative to mean elevation values and terrain correction respectively. Altitude discrepancies are obviously greater (mean=—0.3 m, $\sigma=4$ m, extreme values=—25, +23 m) than in the inner correction zone, but terrain correction (which ranges from 1.254 to 2.162 mGal) shows smaller differences (mean=0.005 mGal, $\sigma=0.003$ mGal, extreme values=—0.001, +0.012 mGal). The correction difference distribution is clearly bimodal notwithstanding the fact that elevation differences have a single sharp maximum, as in the Castellana test; this discordance is clearly due to the inhomogeneous influence that different parts of the topography have on the correction values, according to their proximity to the survey area. Elevation discrepancies are spatially clustered, so the closeness of different "clusters" to the measurement station zone can determine irregularities in the correction difference statistical distribution.

The contribution of the two correction zones (20-150 and 150-2500 m) were then summed,

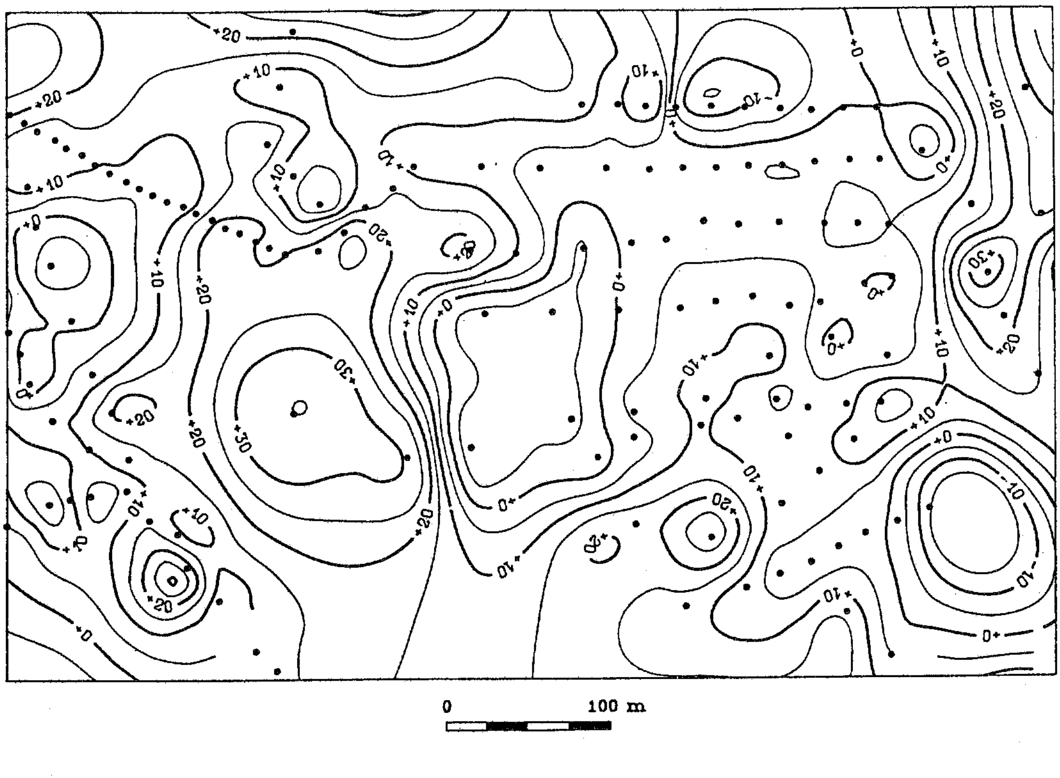

Fig. 7 — Pisticci test: map of differences between the terrain correction values obtained by the manual and automatic procedures (contour spacing 5 μGal).

and total correction values between 1.401 and 2.707 mGal obtained. Fig. 6 shows the distribution frequencies of the differences between results obtained from the manual and automatic procedures, and Fig. 7 the corresponding spatial distribution map: differences range from —0.020 to 0.040 mGal, the mean value being 0.018 mGal and $\sigma=0.011$ mGal. On the whole, discrepancies do not exceed the approximation expected for terrain correction in the survey considered, so the performance of the automatic procedure can be considered satisfactory in this test as well.

General comments

The tests pointed out some potential discordancies between manual and automatic procedure results:

a) If sampling is not extended beyond the strict limits of the digitization area, unreliable altitude extrapolations may occur in marginal zones.

b) Significant inaccuracy may result in the altitude interpolation as a consequence of omission of a relative minimum or maximum altitude point which has a very different elevation from the closest contour line sampled around it.

c) In high gradient zones, even a slightly different positioning of the digitization cells (due to discordance between manual and automatic coordinate system tracing) can generate considerable discrepancies in the evaluation of mean elevations: this proved to be the most common cause of major discordancies when maps have significant distortions.

d) Where contour lines form loops, interpretation of the topographic curvature may differ between manual and automatic procedures, particularly if other elevation data inside the loop are not available.

e) On a topographic saddle, the biquadratic interpolation adopted by QUOTAMED can give anomalous results if the spacing between contour lines sampled is too large: in this case an increase in sampling is advisable.

f) Sharp changes in topographic gradient (i.e., a cliff at the border of a flat surface) can produce aberrant interpolations in the automatic procedure (which, however, cause a warning message in the program output): in these cases the sampling of two close contour lines of slightly different elevations along the edge of the slope discontinuity can improve the results.

All these cases should be dealt with carefully, particularly e) and f): if necessary the density of the altitude points should be locally increased, and successive checks of the program evaluations could be very useful for detecting anomalous estimates.

As far as computation time is concerned, tests show that it is affected more by the number of digitization cells (N_c) than by the number of database points (N_p): a linear regression of CPU time gives for N_c a coefficient two and half times as much as for N_p. So it is convenient to generate databases containing many points to obtain a more accurate digitization, because this does not excessively increase the processing time.

An interesting aspect of the automatic procedure described is that it allows the possibility of a more meditated choice of topographic modelling parameters: in fact, starting from a database of altitude points, it is very easy and quick to generate different topographic models having different extensions and digitization steps, so a direct verification of the effects that representation differences have on the final results can be obtained, thus allowing evaluation of the uncertainty introduced into the data by terrain correction as well.

CONCLUSIONS

On the whole, the test results are quite encouraging: the automatic procedure allowed a time saving of at least a factor of 5 (with further improvement possible), and an accuracy adequate to the specific survey requirements.

Naturally the number of cases examined is still limited, and refinements could be necessary under other situations, but this does not seem a particularly hard task: just a decomposition of digitization cells into minor sub-cells to which the same procedure described would be applied could be sufficient; the cell mean elevation could then be obtained by averaging those of the sub-cells. The increase in computation does not seem a problem, considering the increasing availability of computation power: for instance, a recent implementation of QUOTAMED on a IBM RISC System/6000 mod. 340 reduced the execution time of the Castellana test from 30 to just 2 and a half minutes.

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