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MEAN FREE PATH FOR S-WAVE SCATTERING UNDER CENTRAL GREECE

Abstract. The mean free path characterizing the scale of inhomogeneity of the earth medium was measured from coda waves of local earthquakes. Coda waves are interpreted as scattered S-waves due to random inhomogeneities in the earth medium. We analysed short period WWSSN seismograms of 55 local earthquakes in the magnitude range 3.8-5.2 recorded at the ATH station in Athens during the period 1982-1985. Applying the single isotropic scattering model to the data, we estimated the mean free path of S-waves under central Greece to be 89.1 km, which corresponds to a scattering coefficient of 0.01 km^{-1}.

INTRODUCTION

The scattering and attenuation characteristics of the lithosphere became evident from analyses of the coda excitation of local earthquakes conducted during the early 1970's, (Aki 1969; Aki and Chouet, 1975). Several attempts have been made to quantify the scattering of elastic waves by small inhomogeneities such as cracks, folds, and fluctuation of the velocity and density in the earth medium. Recent developments in coda analyses are reviewed by Herraiz and Espinosa (1987), and studies of the seismogram envelope due to forward scattering are reviewed by Sato (1991).

In coda models, scattering and attenuation are usually described by two parameters, namely the mean free path l and coda attenuation Q_C^{-1}. The reciprocal of the mean free path is the scattering coefficient g, which characterizes the energy transfer from the primary waves to scattered waves, and this quantitatively defines the absolute level of coda excitation. But measurements of g are few because of the difficulty of evaluating the total radiated elastic energy. The parameter Q_C^{-1} is interpreted to represent S-wave attenuation and its frequency dependence has been measured in various areas of the world, (Aki and Chouet, 1975; Caputo, 1984; Pulli, 1984; Baskoutas and Sato, 1989). The estimation of Q_C^{-1} is easily done from single station data without information about the primary energy source. Both parameters provide useful information for the evaluation of the scale of inhomogeneities and tectonic characteristics of the region under study.

In coda analyses, single scattering models (Aki and Chouet 1975; Sato, 1977a, 1977b) have often been used for the measurement of scattering strength, where coda waves of local earthquakes are assumed to be singly scattered S-waves from numerous heterogeneities distributed randomly and homogeneously in the earth medium. For the present study, we adopt the single

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isotropic scattering model of Sato (1977a), which defines the space-time distribution of seismic energy density of S to S scattered waves.

In this study we estimate the mean free path under central Greece based on analyses of short period seismograms available at the WWSSN station ATH of the Greek Seismological Institute in Athens. We obtain quantitative estimates of the seismic energy scattered by inhomogeneities using the displacement amplitudes recorded on ATH's paper charts for local earthquakes occurring in Central Greece.

METHOD

The fundamental assumption underlying the coda wave analysis theory is that the seismic coda waves from local earthquakes are single scattered waves from heterogeneities distributed homogenously and randomly in the earth medium, (Aki and Chouet, 1975). It is assumed that coda waves are spherically radiated from the source, and that the scattering process is isotropic (Sato, 1977a, 1977b). Following Sato (1978), the energy density, E_S (t), of the scattered S-waves is given by

$$E_S(t) = \frac{W(M_S)}{4\pi l R^2} K\left(\frac{t}{t_S}\right) \qquad \text{for } t>t_s, \tag{1}$$

where W is the total seismic energy in ergs estimated from the Gutenberg-Richter formula, $\log_{10} W = 11.8 + 1.5$ Ms, in which Ms is the earthquake surface wave magnitude, R is the hypocentral distance in km, and l is the mean free path of S-waves in the earth in units of km. The function K is defined by

$$K(\alpha) = \frac{1}{\alpha} \log \frac{\alpha+1}{\alpha-1} \qquad \text{for } \alpha>1, \tag{2}$$

where $\alpha = t/t_S$ and t_S is the S-wave travel time measured from the earthquake origin time. The asymptotic behaviour is the same as the single back-scattering model of Aki and Chouet (1975). Taking into account the empirical attenuation in the functional form 10^{-Bt}, eqn. (1) becomes

$$E_S(t) = \frac{W(M_S)\, 10^{-Bt}}{4\pi l R^2} K\left(\frac{t}{t_s}\right) \qquad \text{for } t>t_s, \tag{3}$$

where we related B (S^{-1}) and Q_C^{-1} by $B = \omega Q_C^{-1} \log_{10} e$. On the other hand, assuming that the kinetic energy density of plane waves is equally partitioned into three components, the total (kinetic+elastic) energy density can be written as a sum of particle velocities

$$E_{obs} = \left[\frac{\varrho}{2}(Velocity_x^2 + Velocity_y^2 + Velocity_z^2)\right]_{Time\ av.}$$

$$= 3\varrho\,[Velocity_x^2]_{Time\ av.} = 3\varrho\left[\frac{1}{2}\left(\frac{A_V(t)}{2}\right)^2\right] = \frac{3}{8}\varrho\, A_V(t)^2, \tag{4}$$

where A_V (t) is the mean double amplitude of the particle velocity, which is calculated from the displacement amplitude on paper charts, and ϱ is the mass density. Here, we put $\varrho = 2.7$ gr/cm^3. Equating eqns. (3) and (4), we obtain

$$\frac{3}{8}\varrho\, A_V(t)^2 = \frac{W(M_S)\, 10^{-Bt}}{4\pi l R^2} K\left(\frac{t}{t_s}\right) \qquad \text{for } t>t_S. \tag{5}$$

After rearranging the terms of eqn. (5) and taking the logarithm, we obtain

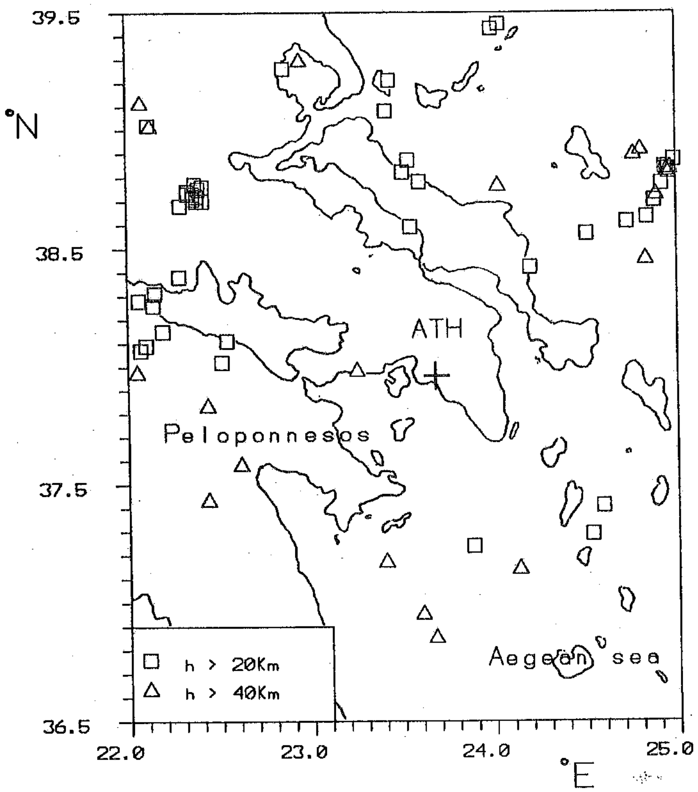

Fig. 1 — Epicenters of earthquakes used in the coda analysis. Cross shows station ATH.

$$\log_{10} I + Bt = F(t), \tag{6}$$

where

$$F(t) = \log_{10}\left[\frac{W(M_S)}{4\pi I R^2}\, K\left(\frac{t}{t_S}\right)\right] - \log_{10}\left[\frac{3}{8}\,\pi\, A_V(t)^2\right]. \tag{7}$$

We estimate I and B for each seismogram by the least squares method.

DATA ANALYSIS

The data used in this study were read from seismograms recorded at the ATH station during the interval 1982-1985. We selected 55 local earthquakes with magnitudes ranging from 3.8 to 5.2. Only earthquakes with focal depths equal to or larger than 20 km were considered to avoid contamination by low frequency surface waves. The source parameters of the events taken from the Monthly Bulletins of the National Observatory of Athens are listed in the Table. We used local magnitudes determined by the Greek network for Ms instead of surface wave magnitude. The earthquake epicenters are plotted in Fig. 1.

For each event recorded on the paper chart, the envelope of the displacement amplitude of the vertical component of motion, and the predominant frequency of the wavetrain were measured manually in successive five-second windows centered at lapse time t after the earthquake origin time, and spaced ten seconds apart. To estimate the predominant frequency, we counted the number of zero-crossings of the seismic trace within the window interval, and divided the result by twice the window length, following Herrmann (1980). In this way each seismogram

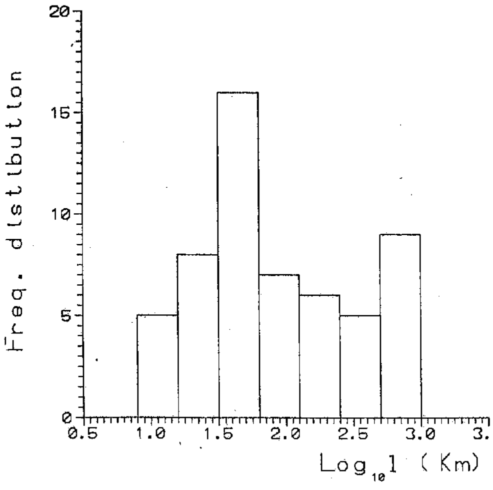

Fig. 2 — Plot of scattering coefficient (km^{-1}), which is the reciprocal of the mean free path. (Added to Fig. 2 of Kosuga, 1992).

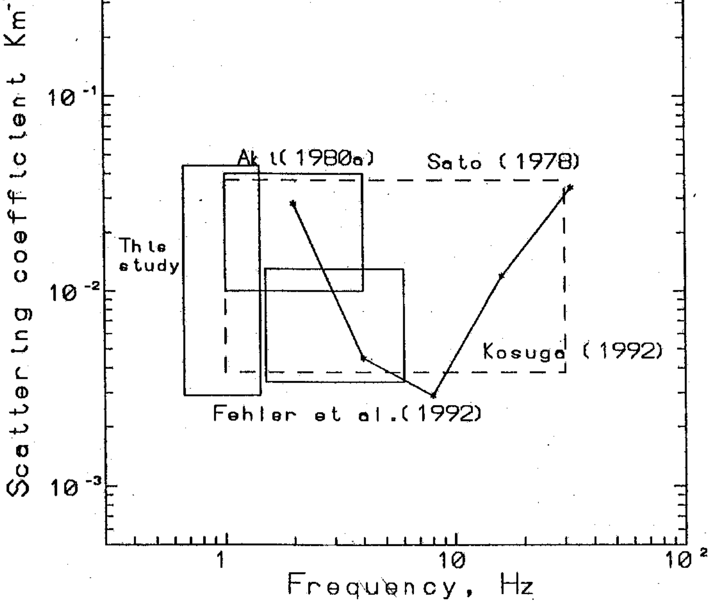

Fig. 3 — Histogram of mean free path (in km).

was replaced by a series of pairs of displacement amplitude A (t) and predominant frequency f_p (t). The halved displacement amplitudes A (t) were then transformed into velocity trace amplitudes A_V (t) by deconvolving the instrument response. The lapse time window was selected to start at $2t_S$, where t_S is the S-wave travel time. Displacement amplitudes were measured until the record amplitude fell below 5 mm, which still exceeded the noise level.

Predominant frequencies range from 0.66 to 1.42 Hz, and the lapse time ranges up to 180 sec. The parameters $\log_{10} l$ and B were estimated by the least squares method for all the selected events.

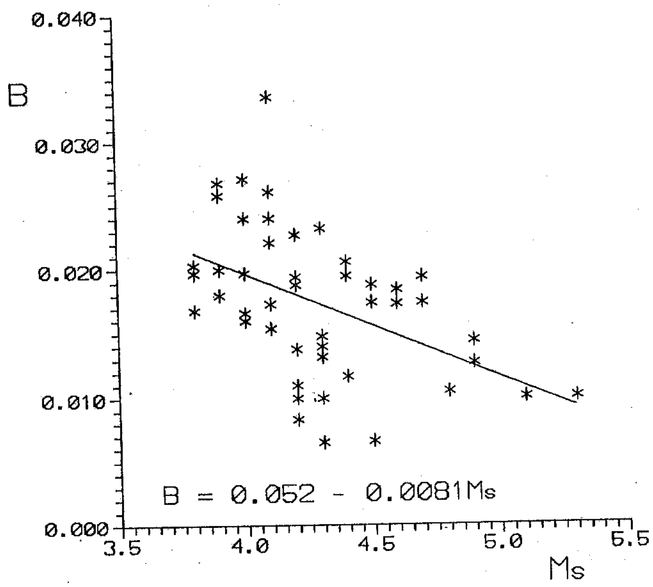

Fig. 4 — Plots of coda attenuation B [s^{-1}] against earthquake magnitude, where straight line is the linear regression.

RESULTS AND DISCUSSION

We estimate the mean free path under central Greece to be $10^{1.95 \pm 0.59}$ km, about 89.1 km. This represents an average value over the region encompassed by the coda waves from many different local earthquakes. The corresponding scattering coefficient is about 0.01 km^{-1}. A histogram of mean free path is shown in Fig. 2. The value of the scattering coefficient obtained here for Greece is consistent with values in Japan reported by Sato (1978), Aki (1980), Dainty et al. (1987), Fehler et al. (1992) and Kosuga (1992), shown in Fig. 3.

The values of $\log_{10} l$ (in cm), mean free path (in km), and B are tabulated in the Table for each seismogram. We plot B and mean free path against magnitude in Figs. 4 and 5, respectively. It is clear that the value of B decreases with increasing magnitude as shown in Fig. 4. The linear regression represented by a solid line in Fig. 4 is given by

$$B = 0.0520\ (\pm 0.0021) - 0.0081\ Ms. \qquad (8)$$

Two plausible explanations may be given for these observations. First, large earthquakes are relatively richer in low-frequency waves than small earthquakes and thus suffer comparatively smaller attenuation. Second, the scattering volume encompassed by the wavefield becomes larger with increasing magnitude, so that the seismic waves sample deeper portions of the earth which might be characterized by weaker attenuation. The large scatter observed in Fig. 5, especially for magnitudes in the range 3.8 to 4.5, may be due to the uncertainty of estimating such small magnitudes.

The plot of mean free path against focal depth in Fig. 6, shows large scatter and we cannot find a focal depth dependence except for four deep events. Mean free paths measured from the analyses of deeper earthquakes seem to show slightly larger values but this may be an artifact associated with the small size of the data set.

Table — **List of source parameters of earthquakes used in the present study and coda parameters estimated: h, depth in km; Ms, earthquake magnitude; *l*, mean free path in cm; $B=Q_C^{-1}\log_{10} e$, attenuation parameter in s^{-1}.**

DATE	GMT TIME (h m s)	LAT (°N)	LON (°E)	h (km)	Ms	$\log_{10} l$	B
1982 JAN 3	002110.0	38.8	24.9	20.0	4.6	7.107	.0173
1982 JAN 5	003036.7	38.8	25.0	25.0	4.9	6.761	.0144
1982 MAR 1	084301.6	38.5	24.8	31.0	4.1	7.270	.0154
1982 MAR 27	224534.0	38.1	22.5	20.0	4.3	6.118	.0232
1982 APR 25	090958.6	38.9	24.8	33.0	4.1	6.139	.0261
1982 AUG 27	014110.0	36.9	23.7	107.0	4.2	7.552	.0138
1982 OCT 8	170954.3	38.0	22.5	22.0	3.9	7.307	.0123
1982 NOV 27	002030.3	38.0	22.0	31.0	3.9	7.853	.0082
1982 DEC 14	192356.8	38.6	24.8	21.0	4.8	6.150	.0105
1982 DEC 29	025503.0	37.4	24.7	20.0	3.8	7.980	.0072
1983 JAN 12	035544.2	38.8	25.0	32.0	4.3	7.852	.0100
1983 FEB 4	055136.3	38.1	22.1	20.0	4.3	6.929	.0140
1983 FEB 24	004350.5	37.3	24.5	20.0	4.5	6.778	.0174
1983 FEB 28	160414.3	38.8	24.9	28.0	5.3	6.205	.0175
1983 MAR 11	083735.9	38.8	25.0	32.0	4.4	7.347	.0117
1983 APR 6	124914.7	38.7	22.3	20.0	3.9	7.425	.0180
1983 APR 7	015153.3	38.7	22.3	21.0	4.0	7.191	.0146
1983 APR 8	123516.2	38.8	22.4	159.0	4.0	7.841	.0081
1983 APR 8	132908.6	38.7	22.4	22.0	4.1	7.013	.0240
1983 APR 10	013846.2	38.7	22.4	21.0	4.0	6.664	.0271
1983 APR 29	042702.9	38.8	22.4	21.0	4.1	6.540	.0336
1983 JUN 19	141806.0	38.7	25.0	40.0	3.9	6.554	.0200
1983 AUG 7	120534.5	38.3	22.1	20.0	3.9	6.780	.0268
1983 AUG 20	081532.7	37.9	22.6	41.0	4.2	6.902	.0194
1983 SEP 15	000501.2	38.7	22.4	21.0	4.2	7.778	.0110
1983 SEP 19	011814.5	38.7	22.3	23.0	5.1	6.694	.0167
1983 SEP 29	035240.7	38.7	22.4	22.0	4.0	6.937	.0166
1983 SEP 22	041748.2	38.8	22.4	21.0	3.9	6.808	.0199
1983 OCT 7	041405.9	38.0	23.3	122.0	4.5	7.990	.0066
1983 OCT 15	204325.3	38.9	23.5	22.0	4.0	6.710	.0240
1983 OCT 31	140414.8	38.2	23.0	103.0	4.2	7.679	.0083
1983 OCT 31	211142.1	39.8	24.4	20.0	4.7	6.494	.0174
1983 NOV 9	095744.3	29.1	23.4	24.0	4.5	6.637	.0188
1983 DEC 18	125116.8	39.4	24.0	23.0	4.3	7.703	.0065
1983 DEC 26	030353.1	37.2	23.9	21.0	3.8	6.272	.0168
1984 JAN 16	225401.9	38.2	22.2	27.0	3.8	7.759	.0068
1984 FEB 16	224034.1	38.1	22.1	25.0	3.8	6.634	.0197
1984 FEB 17	211955.8	39.2	23.4	21.0	4.4	6.620	.0195
1984 MAR 3	014634.3	38.7	24.9	27.0	4.0	7.561	.0028
1984 APR 1	201700.9	38.9	25.0	26.0	4.0	7.457	.0058
1984 OCT 10	211121.5	37.0	23.6	81.0	4.7	6.649	.0194
1984 NOV 6	195226.6	38.7	23.0	41.0	4.3	6.500	.0202
1984 NOV 9	095123.7	38.3	22.1	22.0	3.9	6.257	.0258
1984 DEC 16	120809.2	37.2	24.1	125.0	4.2	7.938	.0100
1984 DEC 23	015956.5	37.8	22.4	36.0	4.1	7.399	.0173
1985 JAN 23	054838.8	38.8	24.9	21.0	3.8	6.362	.0203
1985 MAR 10	145208.6	38.6	24.7	24.0	4.0	6.668	.0197
1985 MAR 12	095109.5	39.4	24.0	25.0	5.1	6.118	.0188
1985 JUN 6	043729.7	38.8	23.5	27.0	4.2	6.792	.0227
1985 JUN 7	075052.2	38.4	24.2	29.0	4.2	6.568	.0188
1985 JUN 14	000045.4	37.4	22.4	72.0	4.0	7.024	.0160
1985 JUN 21	001522.6	38.6	23.5	28.0	4.4	6.319	.0206
1985 AUG 28	003341.3	37.2	23.4	75.0	4.6	6.900	.0184
1985 SEP 21	101309.6	39.0	22.0	41.0	4.9	6.546	.0127
1985 OCT 8	204900.0	38.3	23.7	20.0	4.3	6.409	.0148

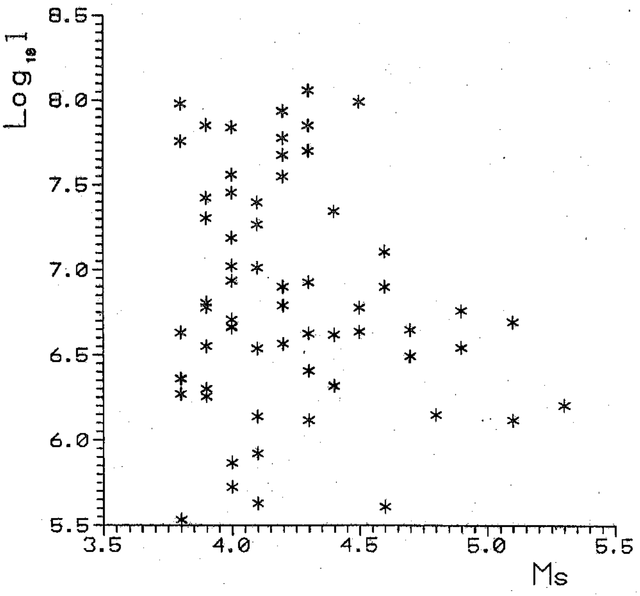

Fig. 5 — Plots of mean free path (in km) against earthquake magnitude.

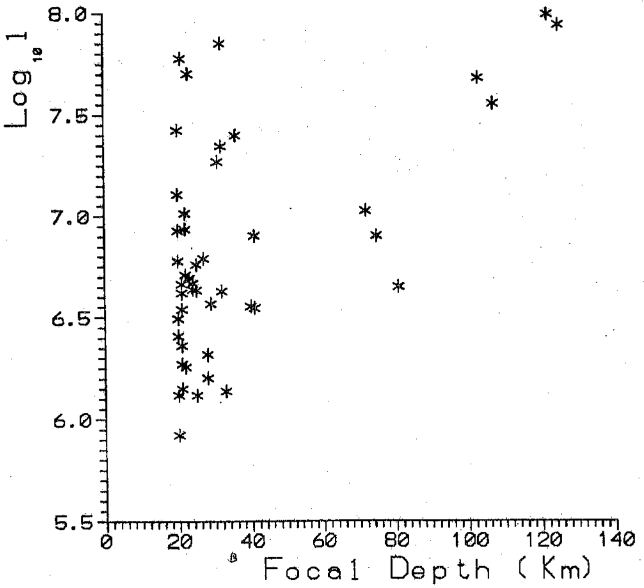

Fig. 6 — Plots of mean free path (in km) against focal depth (in km).

In this study we estimated the mean free path under central Greece based on the single isotropic scattering model. At increasing lapse times the single scattering assumption becomes inadequate and we need to consider multiple scattering for the excitation of coda. A better approach may be given by the multiple isotropic scattering model, (Hoshiba 1991; Zeng et al., 1991), for the simultaneous measurement of the mean free path and separation of scattering and intrinsic attenuation. Fehler et al. (1992) proposed the multiple lapse time window analysis. Such a development will be necessary in further studies of the crust inhomogeneity under central Greece.

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