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TEMPERATURE DEPENDENCE OF THE THERMAL PARAMETERS OF SOME ROCKS

Abstract. The dependence of thermal parameters (conductivity λ, diffusivity k and specific heat c_p) on temperature for eight rock samples coming from different deep geological formations of the Travale (Tuscany) geothermal area is investigated. The measurements of these thermal parameters were done by Mongelli et al. (1982). The results of the analysis by least square fitting techniques in the temperature range $20° C \leqslant T \leqslant 240° C$ have shown that: i) the dependence of λ on T has the same hyperbolic mathematical form as that worked out by other authors for most crystalline rocks; ii) the dependence of *k* on T disclosed by our analysis gives the same hyperbolic canonical form of λ on T; iii) a new temperature dependence of c_p can be derived by the functional relations λ (T) and *k* (T) through the well known expression $c_p = \lambda / \varrho k$.

INTRODUCTION

Many geological, geophysical and petrological problems require detailed information on the thermal properties of rocks (conductivity λ, diffusivity k and specific heat c_p) in their subsurface environments. In many cases their variation with increasing depth into the crust, or with increasing pressure and temperature, is needed. Generally, measurements of the thermal parameters are carried out on rock samples in the laboratory, i.e., under standard conditions: room temperature and atmospheric pressure. As a consequence, a great deal of effort has gone into developing techniques and models that can predict the pressure and/or temperature related effects on the thermal parameters of the rocks (see Zoth and Haenel, 1988 and references therein).

It must be pointed out that the separate effects of pressure and temperature on thermal parameters are quite different. Generally the pressure effect is relatively small. Somerton (1992) maintains that the pressure dependence of λ, k and c_p can be neglected for the whole crust.

At present, the effect of temperature on each of the three thermal parameters is known to different degrees of reliability. While a unique empirical relation between λ and T holds good for a lot of materials (Powell et al., 1966; Horai, 1971; Tkach and Yurchak, 1972; Cermak and Ryback, 1982 and others) the temperature dependence of both k and c_p is poorly known, and different models have been proposed for different rocks (Hanley et al., 1978 ; Somerton, 1992).

Mongelli et al. (1982) carried out laboratory measurements of the thermal parameters of eight sample rocks coming from a deep hole drilled in Tuscany (Italy) in the temperature range 20° C - 240° C.

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In the present work we have revisited their data to find the empirical functional dependence of λ, k and c_p on temperature for all the sample rocks.

THERMAL PARAMETERS

The parameters under room conditions of the eight sample rocks examined by Mongelli et al. (1982) are presented in Tab. 1. The thermal conductivity λ and diffusivity k were measured by the transient method of the cut-core (Mongelli, 1968), whereas the specific heat c_p was calculated from the ratio

$$c_p = \frac{\lambda}{\varrho k}, \tag{1}$$

where ϱ is the rock density.

Thermal conductivity

The dependence of thermal conductivity on temperature is controlled by two mechanisms: lattice and radiative conductivity. According to many authors (see e.g. Somerton, 1992 and references therein), below 500° C the thermal conductivity is due almost entirely to lattice vibrations, and decreases with increasing temperature (approximately with 1/T). Above this temperature, a significant amount of heat is transferred by radiation. The effect of increasing pressure is to cause a slight increase in lattice conductivity.

For most rocks the empirical relationship between thermal conductivity λ and temperature T is of the type (see e.g. Cermak and Ryback, 1982)

$$\frac{\lambda_o}{\lambda\ (T)} = 1 + aT, \tag{2}$$

where λ_0 is the thermal conductivity at T=0° C and atmospheric pressure.

Zoth and Haenel (1988) reported similar hyperbolic experimental expressions for many metamorphic, acid, basic and ultrabasic rocks valid up to 1300° C.

To relate by the best fit technique the λ-values given by Mongelli et al. (1982) to the temperature T at which the laboratory measurements were carried out, we have taken eqn. (2) as the reference formula. The thermal conductivities λ^* of the sample rocks measured at room temperature (20°C) are listed in the forth column on Table 1.

Table 1 - Sample rocks and the respective thermal parameters (λ^* = thermal conductivity; k* = thermal diffusivity; c_p^* = specific heat) measured under standard conditions (room temperature, about 20°C, and atmospheric pressure).

No	sample	$\varrho^* \times 10^{-3}$ (kg m^{-3})	λ^* (W m^{-1} K^{-1})	k $^* \times 10^7$ (m^2 s^{-1})	c_p $^* \times 10^{-3}$ (J kg^{-1} K^{-1})
1	grey limestone of carbonatic complex	2.71	1.63	8.89	0.67
2	grey calcareous dolomite of carbonatic complex	2.71	2.28	11.30	0.75
3	white-rose quartzite with quartzitic elements (anagenite) of metamorphic complex	2.60	3.79	12.50	1.16
4	grey micaceous quartzite of metamorphic complex	2.73	1.89	7.78	0.90
5	grey-green phyllite of the metamorphic basement	2.67	1.69	6.40	0.99
6	micaceous schistose quartzite of metamorphic complex	2.73	1.35	6.90	0.72
7	micaschists plagioclasic of metamorphic basement	2.73	2.29	5.67	1.48
8	gneiss of the deep regional basement	2.69	1.37	7.63	0.67

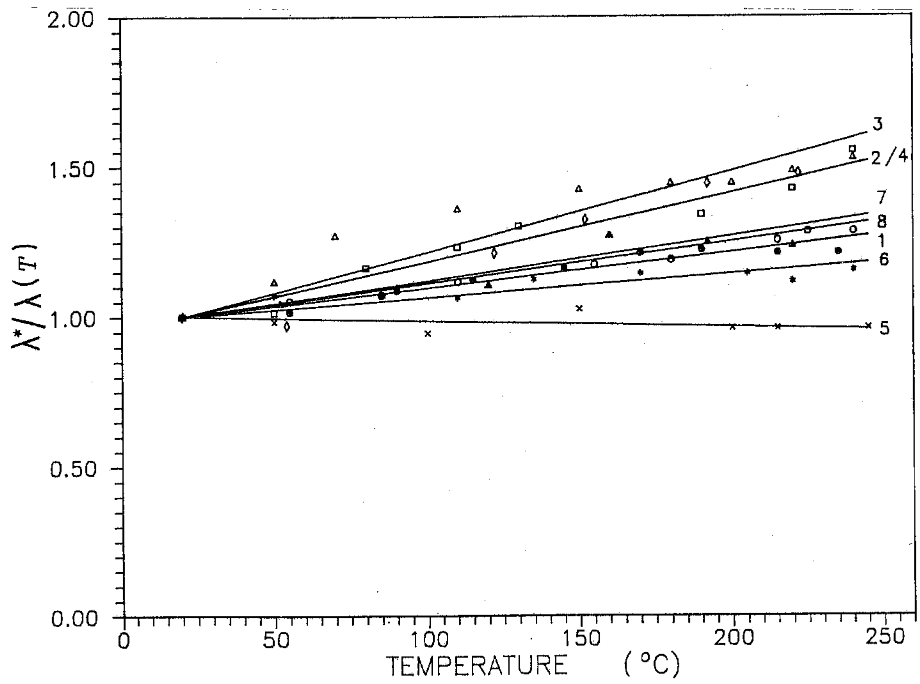

Fig. 1 — The straight line best fit between the ratio λ*/λ (T) and the temperature T for each of the eight sample rocks.

According to eqn. (2), we estimated the best fit lines between the ratio λ*/λ and (T-20). The straight lines of the regression are shown in Fig. 1; the slopes and the r correlation coefficients of each sample are listed in Table 2.

Table 2 - The parameters of the correlation (2) between λ*/λ and (T-20)°C.

sample No.	slope a ($°C^{-1}$)	correlation coefficient r (percent)
1	0.0013	99.87
2	0.0023	99.18
3	0.0027	98.14
4	0.0023	98.29
5	−0.0002	−71.99
6	0.0008	96.86
7	0.0014	97.91
8	0.0012	98.97

As it can be seen, eqn. (2) holds good for each sample rock. Save for the phyllite, all slopes and all correlation coefficients are positive. This points out that, in general, the thermal conductivity decreases with increasing temperature. Owing to the variation in composition of the examined sample rocks, their slopes are quite different, ranging from 0.0008 for the micaceous quartzite to 0.0027 for the white-rose quartzite. So, the eight straight lines diverge slowly from each other for increasing T.

The behaviour of phyllite is not unusual: constant values with changing temperature or negative slopes were found for rocks with high structural disorder, like non-crystalline materials in the glass-state (Ross et al., 1981).

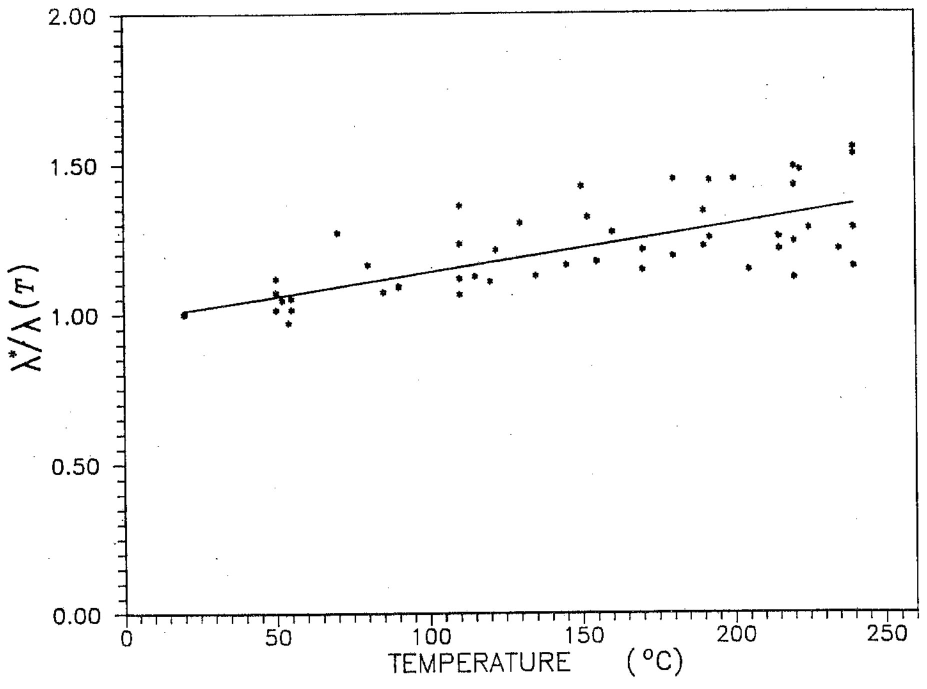

Fig. 2 — The best fit line of the observed points (λ*/λ (T), T) for all the examined sample rocks, apart the phyllite (sample No. 5).

In Fig. 2 the plot of all paired values (λ*/λ, T) for the examined samples is shown. The straight line represents the best fit of all data points; its equation is given by

$$\frac{\lambda^*}{\lambda\ (T)} = 1 + 0.0015\ (T-20) \qquad for\ 20°C \leqslant T \leqslant 240°C, \tag{3}$$

whose correlation coefficient is equal to 70.85 percent. As it can be seen, it is less than any other correlation coefficient of the individual samples owing to the scattering of the points around the regression line with increasing T (Fig. 2).

Thermal diffusivity

To show the effect of temperature T on the thermal diffusivity k, each data set of the paired values (k*/k, T) was tested by a mathematical model of type (2); that is, the observed thermal data were approximated by the following equation

$$\frac{k^*}{k\ (T)} = 1 + a'\ (T-20). \tag{4}$$

The k*-values are listed in the fifth column of Table 1.

The estimated parameters a' of the correlation analysis are given in Table 3. The regression straight lines are shown in Fig. 3.

Table 3 - The parameters of the linear correlation (4) between the ratio k*/k and T (in °C).

sample No	slope a' (°C^{-1})	correlation coefficient r (percent)
1	0.0024	99.80
2	0.0030	99.70
3	0.0037	99.76
4	0.0036	99.75
5	0.0020	99.26
6	0.0020	99.82
7	0.0017	98.72
8	0.0019	99.85

The linear dependence of the thermal diffusivity on temperature is characterized by very high correlation coefficients (more than 98 percent). The straight lines (4) have common trends: their slopes are all positive and range over the interval 0.17-0.37 °C^{-1}.

The set of the paired values (k*/k, T) of all samples is approximated by the following regression equation

$$\frac{k^*}{k\ (T)} = 1 + 0.0025\ (T-20) \qquad for\ 20°C \leqslant T \leqslant 240°C. \tag{5}$$

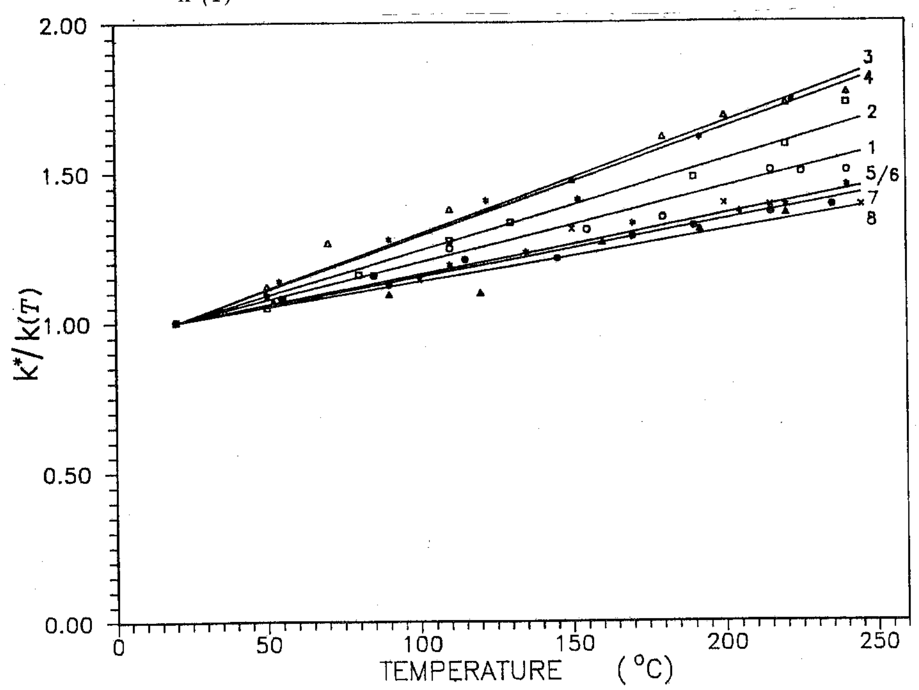

Fig. 3 — The straight line best fit between the ratio k*/k (T) and the temperature T for each of the eight sample rocks.

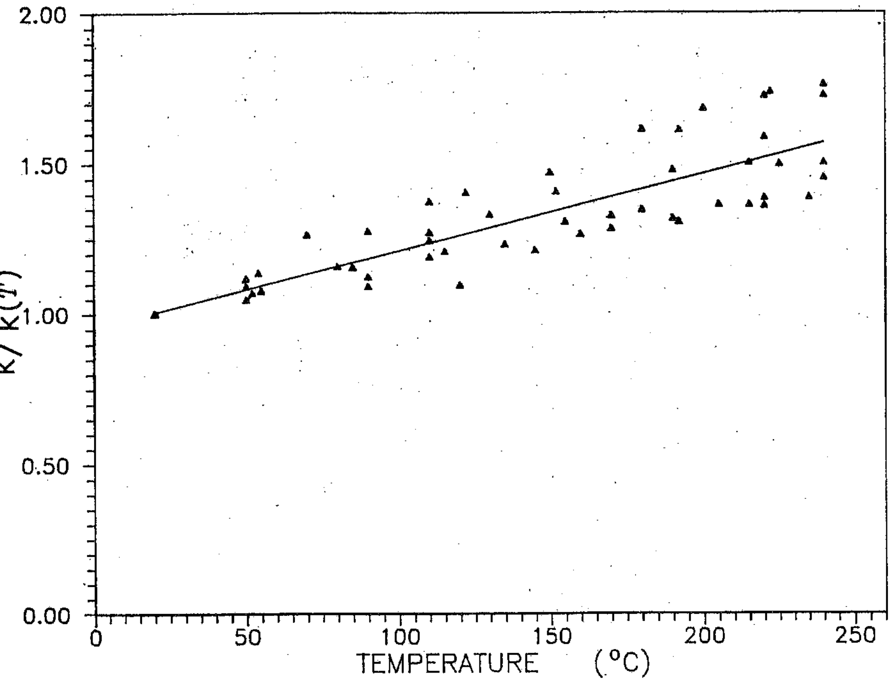

Fig. 4 — The best fit line of the observed points (k*/k (T), T) for all the examined sample rocks.

The estimated correlation coefficient r of the regression is 95.83 percent. Even though r is less than any other correlation coefficient shown in Table 3 (Fig. 4), nevertheless it is significantly high.

Specific heat

To express the dependence of the specific heat on temperature we followed two different paths.

First we tested the c_p values against T by the England (1982) expression

$$c_p = q\left(1 + q' \, T + \frac{q''}{T^2}\right), \tag{6}$$

where q, q', q'' are constants to be estimated by the least squares method.

The results of the regression analysis show that eqn. (6) does not fit very well all the examined data sets: in particular, the estimated r-coefficients of samples 4, 7 and 8 are less than 0.80.

On the other hand, the empirical relationships (2) and (4) via eqn. (1) place strong costraints on the mathematical expression of c_p (T). Indeed, assuming ϱ is a constant (the bulk density changes very slowly with depth), we have

$$c_p\,(T) = \frac{\lambda\,(T)}{\varrho k\,(T)} = \frac{\lambda^*}{\varrho k^*} \cdot \frac{1 + a'\,(T-20)}{1 + a\,(T-20)},$$

which can be rewritten in the form

$$c_p\,(T) = c_p{}^* \, \frac{1 + a'\,(T-20)}{1 + a\,(T-20)}, \tag{6}$$

where a and a' are the slopes of the linear regressions (2) and (4). Thus, we are concerned

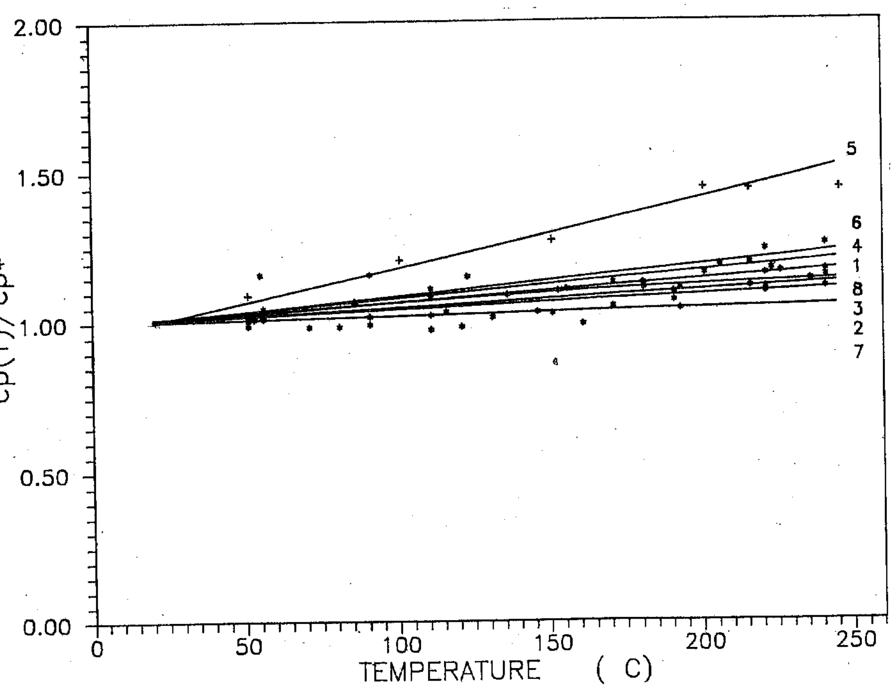

Fig. 5 — The straight line best fit between the ratio c_p^*/c_a (T) and the temperature T for each of the eight sample rocks.

with the values of the deviation χ (T) between the observed ratio c_p (T) $/c_p^*$ and the model fitting

$$\frac{1+a'\,(T-20)}{1+a\,(T-20)}\,, \tag{6}$$

for each individual sample.

In Table 4, the residual standar errors

$$RE=\sqrt{\frac{\sum \chi^2\,(T)}{n}}$$

are given.

Table 4 - The residual standard error of the observed ratio c_p (T)/c_p* and the model (6).

sample No	RE
1	0.0154
2	0.0251
3	0.0429
4	0.0649
5	0.1918
6	0.0242
7	0.0317
8	0.0253

The RE-values are significantly low, apart from that of the phyllite (sample No. 5). This

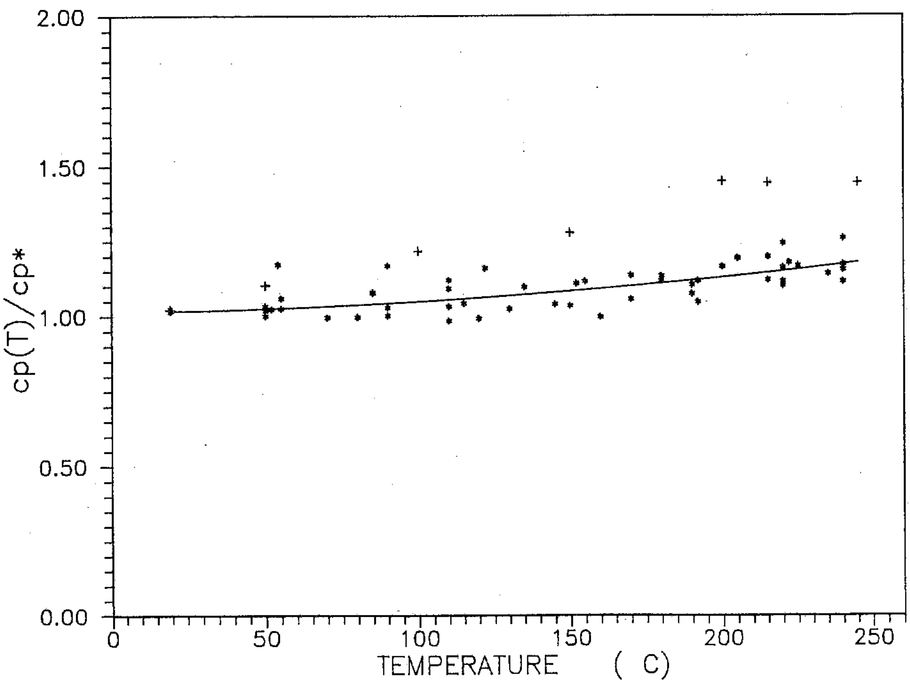

Fig. 6 — Comparison between the observed points (k*/k (T), T) for all the examined sample rocks and the curve given by eqn. (7).

means that the agreement between the observations and model (6) is good enough. In Fig. 5 the plot of the points (c_p/c_p^*,T) and the corresponding fitted curve of each individual sample are shown.

In Fig. 6 the curve of the general model

$$\frac{c_p\,(T)}{c_p^*} = \frac{1+0.0025\ (T-20)}{1+0.0015\ (T-20)} \quad for\ 20\ °C \leq T \leq 240\ °C, \qquad (7)$$

and the paired values (c_p / c_p^*, T) of all examined samples, are shown. The + symbol has been assigned to the points corresponding to the phyllite. The deviation of the latter points from the curve underlines once again the different behaviour of the phyllite.

The correlation coefficients between the first and second member of eqn. (7) are close to unity, as one would expect.

CONCLUSIONS

The results confirm that the thermal conductivity λ is linearly related to 1/T by an empirical correlation equation formally equal to that suggested by other authors (see e.g. Chermak and Ryback, 1982; Somerton, 1992 and references therein).

The dependence of k on T, and c_p on T have been tested by two new empirical models.

The former, namely the functional relationship k=k (T), has the same form as λ=λ (T), but generally is characterized by a greater decrease with increasing T.

The general mathematical form of the latter was deduced from the empirical models of both λ and k related to each other by the physical definition of k, and then tested by the observed pairs of data (c_p / c_p^*, T).

The least square fitting analysis has shown that for most data sets the mathematical model of the empirical relations between the thermal parameters λ, k, and c_p and the temperature T in the range (20 °C-240 °C) is good enough. The r correlation coefficients are significantly high, generally more than 90%.

If the hyperbolic relationships for the diffusivity and the specific heat can, like the conductivity, be extrapolated to temperature 1300 °C, then a better evaluation of i) the thermal data for the entire lithosphere in transient regime, and ii) the heat extractable from deep hot dry rocks would possible.

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