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SOME ASPECTS OF SENSITIVITY ANALYSIS IN ONE DIMENSIONAL MAGNETOTELLURIC MODELLING

Abstract. Sensitivity analysis for a certain class of 1D MT modelling is done to highlight a few points of principle. The analysis reveals that (1) skin depth at crossover point frequency in 1D MT response (explained in the text) roughly estimates the top of the second layer for a certain class of three layer earth models; (2) the MT signal can feel the presence of an inhomogeneity when it is beyond the skin depth level; (3) a linear relation exists between the depth and thickness of the middle layer for some three layered H and K type earth models for the same percentage anomaly; (4) sensitivity drastically falls when the resistivity contrast exceeds 100; (5) the relation between thickness of the middle layer for a three layered H type model and the frequency at which the apparent resistivity becomes minimum is also linear; (6) the relation between thickness of the second layer for both K and H type models and the percentage anomaly at minimum or maximum frequency (explained in the text) is nonlinear; (7) the ratio of the wavelength at the perturbation centroid frequency (explained in the text) and the target bed thickness varies within a few orders of magnitude; (8) sensitivity of MT response for resolving a resistive bed in between two conductive bed becomes low, and vanishes for a certain class of models; (9) one or two kilometers of conductive overburden significantly reduces the sensitivity of the MT response from well inside the upper mantle upto 300 km to 500 km depth where the olivine structure of the mantle silicates changes to different phases of the spinel structures.

INTRODUCTION

The behavior of the magnetotelluric apparent resistivity response for several thousand one dimensional earth models is studied to highlight a few points of principle. Cagniard models (1953) are computed using the procedure given by Keller and Frischknecht (1966).

In the literature of electrical and electromagnetic methods, it is well documented that the ground element contribution to the total signal reaches a peak at a certain depth or distance, starting from a negligible contribution from the near surface zone; the contribution to the signal then gradually goes down with distance or depth from the source. (Dakhnov, 1962; Roy and Apparao, 1971; Oldenburg, 1978; Edward et al., 1984; Gomez-Trevino, 1987). The sensitivity factors of geoelectrical tools are given different names in different areas of geophysics. The terms lateral investigation characteristics of induction logging, depth of investigation characteristics in the d.c. resistivity method (Roy and Apparao, 1971), and Frèchet kernel in d.c. resistivity and magnetotellurics (Oldenburg, 1978, 1979) carry more or less the same sense. A certain electrode separation or a certain frequency in an EM or MT signal can see a particular depth to the maximum extent (Laird and Bostick, 1970). Information content from above and below a particular depth will drop down sharply. This sensitivity function, or Frèchet kernel or depth of investigation characteristics (DIC) will never be like the Dirac delta function (Edward et. al., 1984) nor can the averaging kernel in Backus Gilbert inversion (Backus-Gilbert, 1968, 1970) be made as good as a Dirac delta type function when reducing the Backus-Gilbert spread

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function to a small odd value. Therefore, in geoelectrical methods, the information is obtained by the receiver from a reasonably broad zone; and there is a scope to study the sensitivity of the MT signal. If a perturbation either in resistivity or in the thickness is inserted in the model, it must be reflected in the response curve. An assessment has been made of the sensitivity of MT response problems from different angles. In this paper we have restricted our studies to 1D models.

The Perturbation Centroid frequency (f_{pc}) is expressed as follows: if a certain perturbation in resistivity or conductivity is inserted in a model, an EM or MT signal of a certain frequency will see that perturbation to the maximum extent (Laird and Bostick, 1970). This frequency is termed the perturbation centroid frequency. If a certain perturbation in an otherwise homogeneous earth model is inserted to make it a three layered earth, at one particular frequency, the difference in apparent resistivities due to perturbed and unperturbed models becomes maximum. The depth of that perturbed zone is the depth of perturbation. It is different from the depth of investigation defined by Spies (1989). The terms depth of penetration, depth of investigation, depth of detection and depth of exploration exist in the EM and MT literature. The two definitions of depth of investigation by Spies (1984) and Gomez-Trevino (1987) are not identical. Entering into debate is beyond the scope of this paper. We are using depth of perturbation as synonymous to depth of investigation and depth of penetration is assumed to be equal to skin depth. The apparent resistivities are computed here from the frequency domain analysis of 1D models (Spies and Eggars, 1986; Keller and Frischknecht, 1966).

Depth of perturbation or depth of investigation is computed for the three layered earth model from

$$P=\frac{\rho_a(\rho_1\,,\ \rho_2+\Delta\rho_2\,,\ \rho_3\,,\ h_1\,,\ h_2)-\rho_a(\rho_1\,,\ \rho_2\,,\ \rho_3\,,\ h_1\,,\ h_2)}{\rho_a(\rho_1\,,\ \rho_2\,,\ \rho_3\,,\ h_1\,,\ h_2)}\,,$$

where $\Delta\rho_2$ is the increment given to the perturbed model; h_1 , h_2 are, respectively, the thickness of the first and second layer; f_{pc} is obtained for the maximum value of P. Depth to the target d is the depth to the centre of the target bed whose resistivity or thickness was changed to determine f_{pc} ; it is taken as the depth of perturbation.

Oscillations in the magnetotelluric apparent resistivity curves are shown in sensitivity functions by Gomez-Trevino (1987) and have been mentioned by Spies and Eggars (1986). Oscillations in MT and EM apparent resistivity and phase curves are shown by Cagniard (1953), Wait (1962). Point 'A', p. 623, fig. 7 in Cagniard (1953) is the point where oscillation ends and the MT curves give information about the earth model. The frequency at which the MT response (apparent resistivity or phase) reaches the point 'A' is termed as the crossover point frequency f_{cr} . f_{max} and f_{min} are, respectively, the frequencies at which the apparent resistivity becomes maximum for a K type ($\rho_1<\rho_2>\rho_3$), and minimum for an H type ($\rho_1>\rho_2<\rho_3$) model, where ρ_1 , ρ_2 and ρ_3 are, respectively, the resistivities of the first, second and third layer in a three layered earth model.

Percentage anomaly (PA) is computed from

$$PA=\frac{|\,\rho_1-(\rho_a)_{min.}\ \text{or}\ (\rho_a)_{max}\,|}{\rho_1}\times 100\%,$$

where $(\rho_a)_{min.}$ and $(\rho_a)_{max.}$ are the minimum and maximum apparent resistivities for a three layer H type and K type model. Several thousand models are tested for accurate estimation of P_{max} , PA_{max} , f_{min} , and f_{max} .

Some interesting properties of the 1D MT response are noted and outlined in this paper.

RESULTS AND DISCUSSION

Model-1

(a) For a certain class of H type models, the estimated skin depth at crossover point frequency

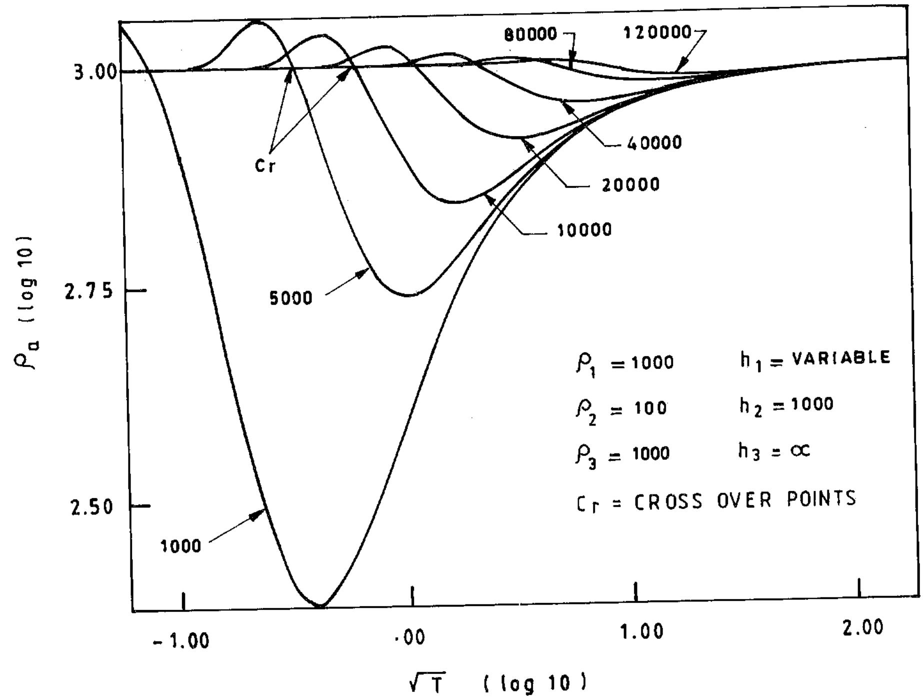

Fig. 1 — Magnetotelluric sounding curve with gradual increase in depth of the target bed for an H-type model, showing cross-over point frequencies.

f_{cr} becomes nearly equal to the depth to the top of the second layer (Table 1 and Fig. 1).

(b) Magnetotelluric signals can start feeling the presence of a bed at a certain depth which is beyond the skin depth of the signal (Spies, 1989).

Model-2

In this section a series of three layered models are assumed, with a middle layer at different depths from the surface. The percentage anomaly (PA) is computed. It is observed that there is a linear relation between the depth to the top of the second layer (h_1) and its thickness (h_2) for the same PA in layered earth magnetotellurics. Models are examined for arbitrarily chosen 3%, 5% and 8% anomalies. Thickness of the second layer is increased in small steps

Table 1 — Relation between skin depth at crossover point frequency and depth to the top of the target for an H type model ($\varrho_1 = 1000.0\ \Omega m$, $\varrho_2 = 100.0\ \Omega m$, $\varrho_3 = 1000.0\ \Omega m$, h_1 = variable $h_2 = 1000.0$ m).

FIRST LAYER THICKNESS	SKIN DEPTH (δ)	CROSSOVER POINT FREQUENCY f_{cr}
1000.00	1027.28	156.250000
2000.00	2054.53	39.068879
3000.00	3082.21	17.327027
4000.00	4110.57	9.709791
5000.00	5136.39	6.250000
6000.00	6158.66	4.397788
7000.00	7178.05	3.283144
8000.00	8195.25	2.555423
9000.00	9210.72	2.051718
10000.00	10224.75	1.687449
20000.00	20312.14	0.473472
30000.00	30337.41	0.227250
40000.00	40323.89	0.135386
50000.00	50283.69	0.090667

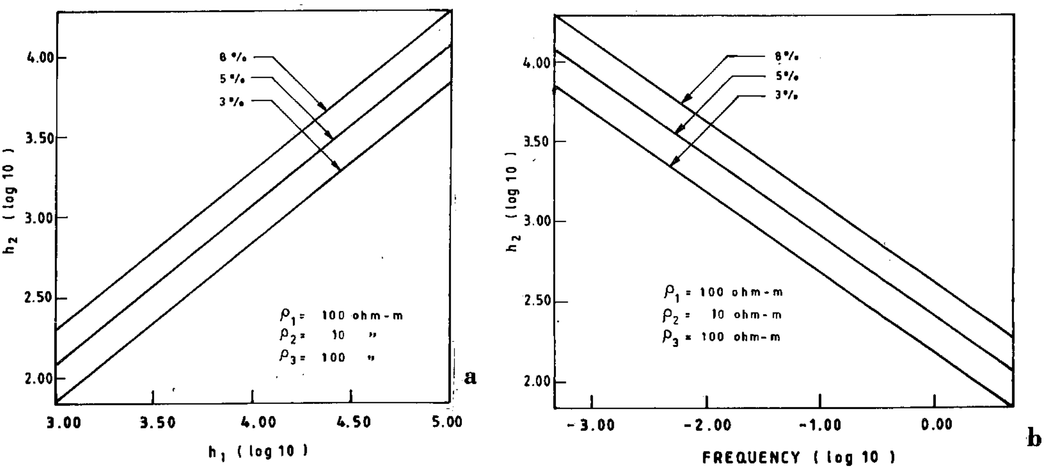

Fig. 2 — Relations for an H type three layered earth model for the same percentage of MT apparent resistivity anomaly (PA); linear relation for 3%, 5% and 8% anomalies: a) between h_2 and h_1 ; b) between h_2 and f_{min} .

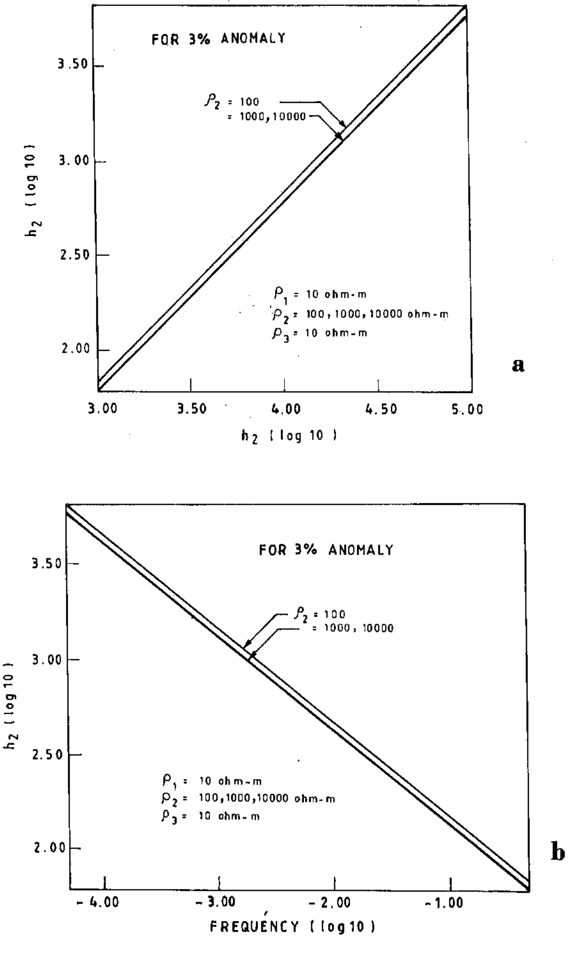

Fig. 3 — Relations for a K type three layered earth model for 3% Anomaly (PA) for varying resistivity contrast: a) between h_2 and h_1 ; b) of h_2 versus f_{max} .

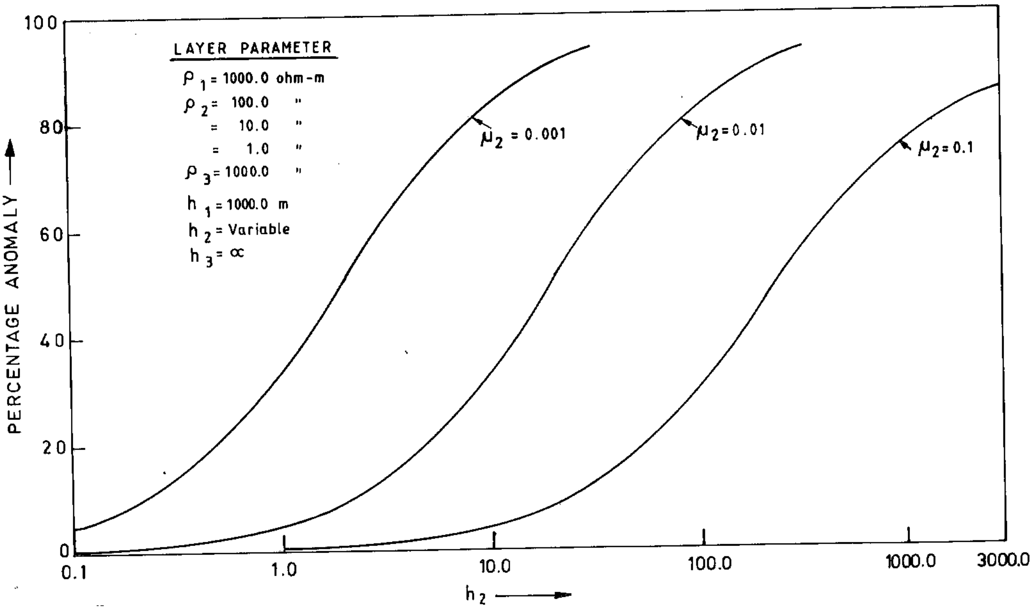

Fig. 4 — Variation of Percentage anomaly (PA) with h_2 for an H type model for varying resistivity contrast.

for finer adjustment of percentage anomaly.

Fig. 2a shows the nature of the interdependence of h_1 and h_2 for models with $\varrho_1 = 100.0$ ohm-m, $\varrho_2 = 10.0$ ohm-m and $\varrho_3 = 100.0$ ohm-m. One model dependent estimate is presented to give the reader a rough semi-quantitative idea. For every 1000 m increase in depth to the target bed (second layer), its thickness should be increased by 6.8 m, 11.5 m and 19.0 m for 3%, 5% and 8% anomalies, respectively. The relation between h_1 and h_2 for the same PA exists for H type three layer models. Fig. 2b shows that the same linear relation exists between the thickness of the second layer and f_{min} . f_{min} shifts with ϱ_1 for the same resistivity contrast ($\varrho_1 : \varrho_2 : \varrho_3$) and same thickness ratios ($h_1 : h_2$). For a 10 times increase in ϱ_1 , f_{min} increases 10 times. Figs. 3a and 3b show that for the resistive target, the sensitivity vanishes when the resistivity contrast ($\varrho_1 : \varrho_2$) exceeds 100. The relations h_2 vs h_1 and h_2 vs f_{max} are linear for K type models also. Figs. 4 and 5 show the nonlinear variation of PA with variation of the second layer thickness h_2 for H type and K types, respectively. The curves show quantitatively the thickness required for H type and K type models to generate the same percentage anomaly. For 20% anomaly, the bed thickness should be 5 m in the case of a conductive bed, and 500 m in the case of resistive beds, for μ_2 (ϱ_2 / ϱ_1) $= 0.01$ and 100, respectively. It is observed that beyond the resistivity contrast of 100 for a K type model, the MT curve fails to resolve any further contrasts of higher order.

Resolving a resistive bed in between two conductive beds is a difficult proposition. Figures 6a and 6b show that the sensitivity goes down to the minimum level for a certain clas of KH type models, where both apparent resistivity and phase curves totally failed to see the changes in the ϱ_2 for nearly three orders of magnitude.

Model-3

The relation between thickness of a target bed and the wavelength of an MT signal for the detectability of the bed is studied. It is well known that the resolving power of an electromagnetic wave, in delineating thin beds, is frequency or wavelength dependent. Wavelength is the distance over which the phase rolls through an angle 2π. Therefore $\lambda = \sqrt{2\pi}/\beta$, where $\beta = \sqrt{\omega\mu\sigma/2}$, ω, μ and σ are well known parameters in electromagnetism. It reduces to $\lambda = \sqrt{10^7 \varrho_t / f}$, where $f = f_{min}$ or f_{max} , where ϱ_t is the longitudinal resistivity of the target bed

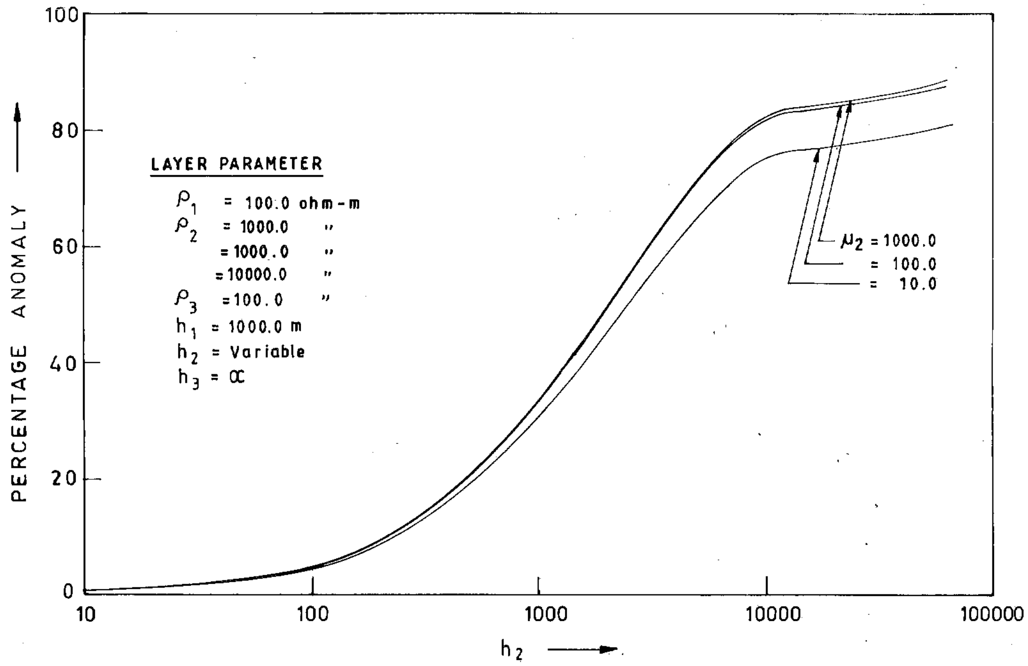

Fig. 5 — Variation of Percentage anomaly (PA) with h_2 for a K type model for varying resistivity contrast.

(Vanayan, 1967; Paul and Nasar 1987). H and K type three layered models are examined. The frequency ranges for minimum apparent resistivity in the case of an H type, and maximum in the case of K type earth models, are determined, and the minimum and maximum are searched for at close intervals to sharpen the estimate of f_{min} or f_{max}, respectively. The values of h_2/λ are given in Table 2. The points to be noted are (i) the wavelength at f_{min} or f_{max} is several orders (10^3—10^5) of magnitude more than the thickness of the target for certain classes of models; and (ii) h_2/λ is same for the conductive and resistive target as long as ϱ_2/ϱ_1 or ϱ_1/ϱ_2 remains the same. The ratio ϱ_2/ϱ_1 or ϱ_1/ϱ_2 has inverse relation with h_2/λ (see Table 2).

Model-4

In this section, the effect of the conductive overburden on the sensitivity of the magnetotelluric field response for layered earth models down to 1000 km from the surface is examined. Seven and eight layer earth models are assumed for models with and without sediments. The assigned model parameter are as follows: (i) Sediments: resistivity $\varrho_0=10.0$ Ωm; thickness $h_0=2.0$ km. (ii) Granitic/granodioritic crust: $\varrho_1=10000.0$ Ωm; $h_1=10.0$ km. (iii) Basaltic/gabbroic/amphibolite facies/granulite facies lower crust: $\varrho_2=2000.0$ Ωm; $h_2=15.0$ km. (iv) Low resistive zone at the base of the lower crust: $\varrho_3=300.0$ Ωm; $h_3=5.0$ km. (v) Upper mantle pyrolite: $\varrho_4=1000.0$ Ωm; $h_4=80.0$ km. (vi) Asthenosphere: garnet or spinel lherzolite; $\varrho_5=400.0$ Ωm; $h_5=280.0$ km down to olivine-spinel phase transition zone. (vii) Mantle below phase transition- spinel lherzolite and β-phase of garnet lherzolite: $\varrho_6=10.0$ Ωm; $h_6=600.0$ km down to upper mantle lower mantle boundary. (viii) Lower mantle: perovskite structure: $\varrho_7=0.05$ Ωm; h_7=infinity. The values of the model parameters are assumed on the basis of the information obtained from Vanzijl (1979), Kurtz and Garland (1976), Kurtz (1982), Jones (1982), Haak and Hutton (1986), Roy et al. (1989). Sensitivity of the MT sounding response is qualitatively tested by varying ϱ_1 to ϱ_6, successively, and keeping all other parameters fixed. ϱ_1 to ϱ_6 are varied within the following ranges: (i) $\varrho_1=1000.0$ to 20000.0 Ωm. (ii) $\varrho_2=500.0$ to 4000.0 Ωm. (iii) $\varrho_3=100.0$ to 500.0 Ωm. (iv) $\varrho_4=100.0$ to 1000.0 Ωm. (v) $\varrho_5=100.0$ to 500.0 Ωm. (vi) $\varrho_6=5.0$ to 25.0 Ωm.

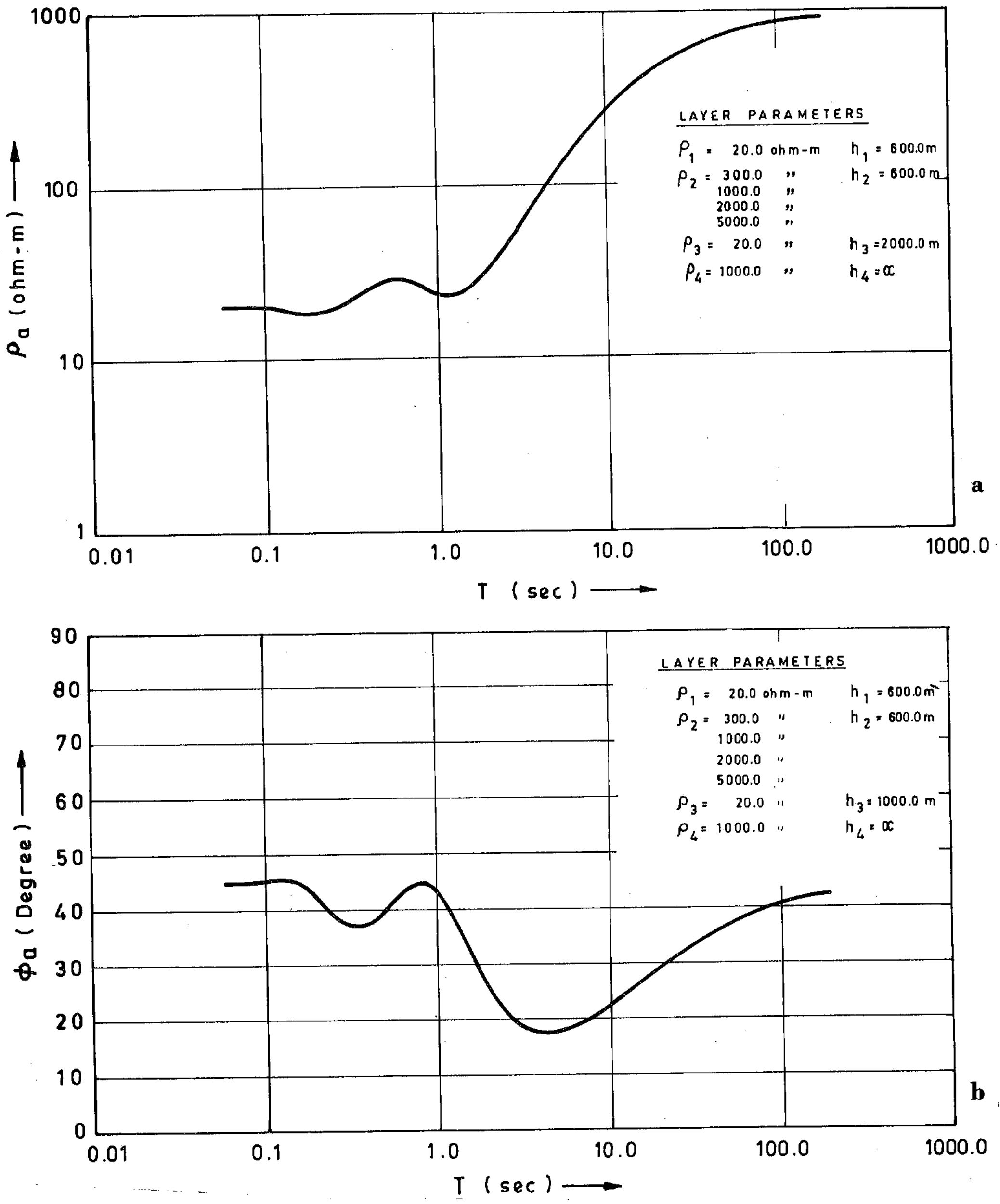

Fig. 6 — Magnetotelluric curves for four layer model showing total lack of resolution: a) apparent resistivity curves; b) phase curves.

Figs. 7a, 7b, 7c, 8a, 8b and 8c are MT apparent resistivity responses, with the granitic body as the top layer, and Figs. 7a', 7b', 7c', 8a', 8b' and 8c' are the MT responses for the same earth model with the additional 2.0 km of conducting sediments of resistivity 10.0 Ωm. The thickness of the sediments in a basin can be as high as 6.0 to 7.0 kms. These diagrams qualitatively show the sensitivity of the MT response when perturbation is introduced into only one parameter, keeping all other parameters fixed. The layer thicknesses assumed in this model are generally realistic layer thicknesses beneath the continental crust. It is observed that even 2.0 km of sediments of average resistivity 10.0 Ωm can mask the sensitivity of the MT response almost totally down to 40.0 km and partially to 100.0 km. Figs. 8c and 8c' show that sensitivity

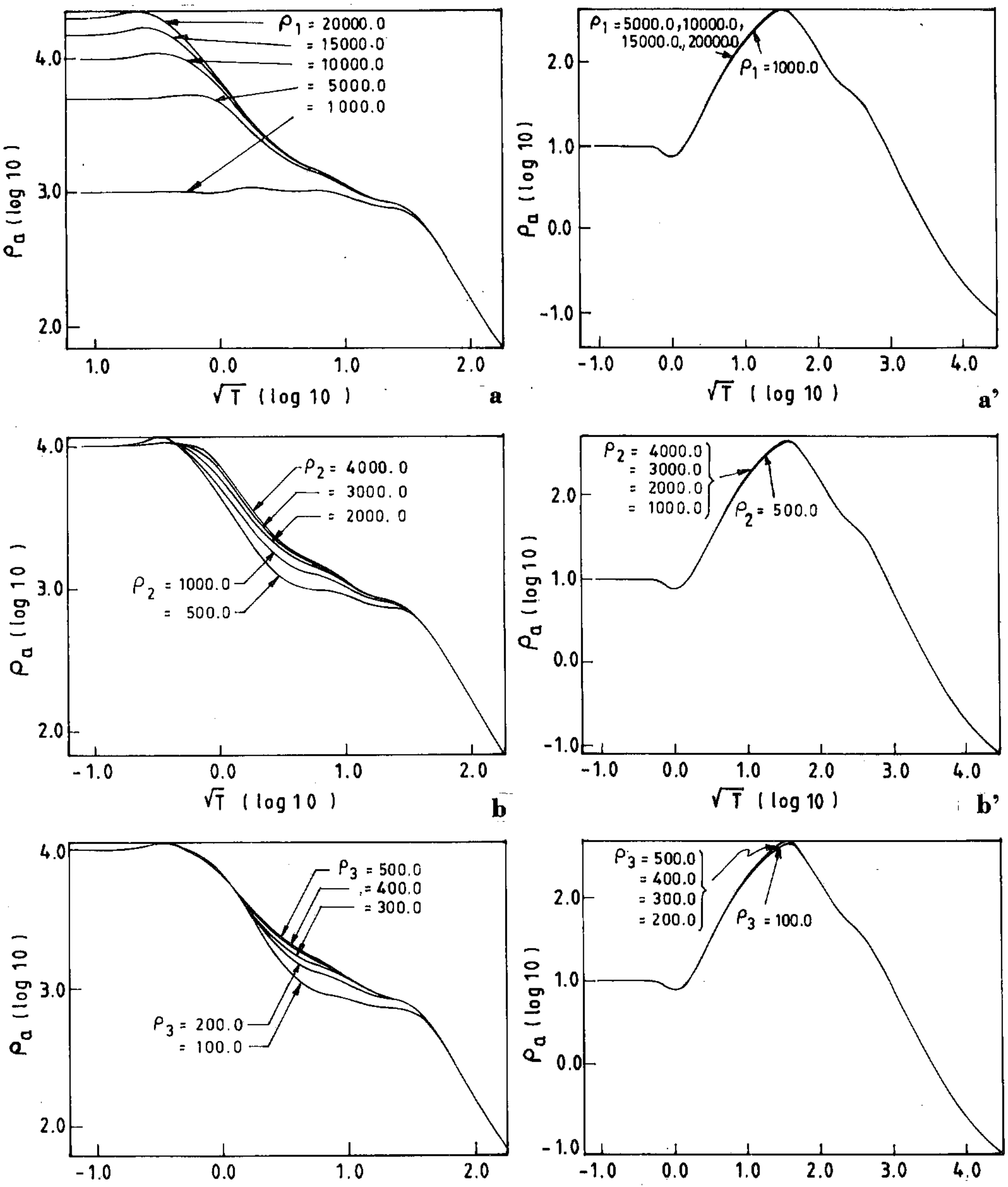

Fig. 7 — MT sounding curves for seven/eight layered earth models, with and without overburden sediments: a, b, c are for models without overburden; a', b', c' are the models with overburden. (a and a' for varying ϱ_1 ; b and b' for varying ϱ_2 ; c and c' for varying ϱ_3).

remained unaffected beyond 400 km depth in the model with top sediment layer. This semi-quantitative exercise is model dependent, and innumerable models will be needed for any quantitative analysis. This experiment demonstrates that MT observation sites should be fixed right over the granitic/granodioritic/amphibolite or granulite exposed windows (Beamish, 1991). Sensitivity enhances if high frequency components can be preserved for deeper entry into the earth. This model was taken specifically to show the nature of the variation in MT response by varying only one parameter, keeping all other parameters fixed. The situation will be worse for a more conductive and thicker overburden, and if the real resistivities at different depths below an Archaean craton are less than those assumed in the model.

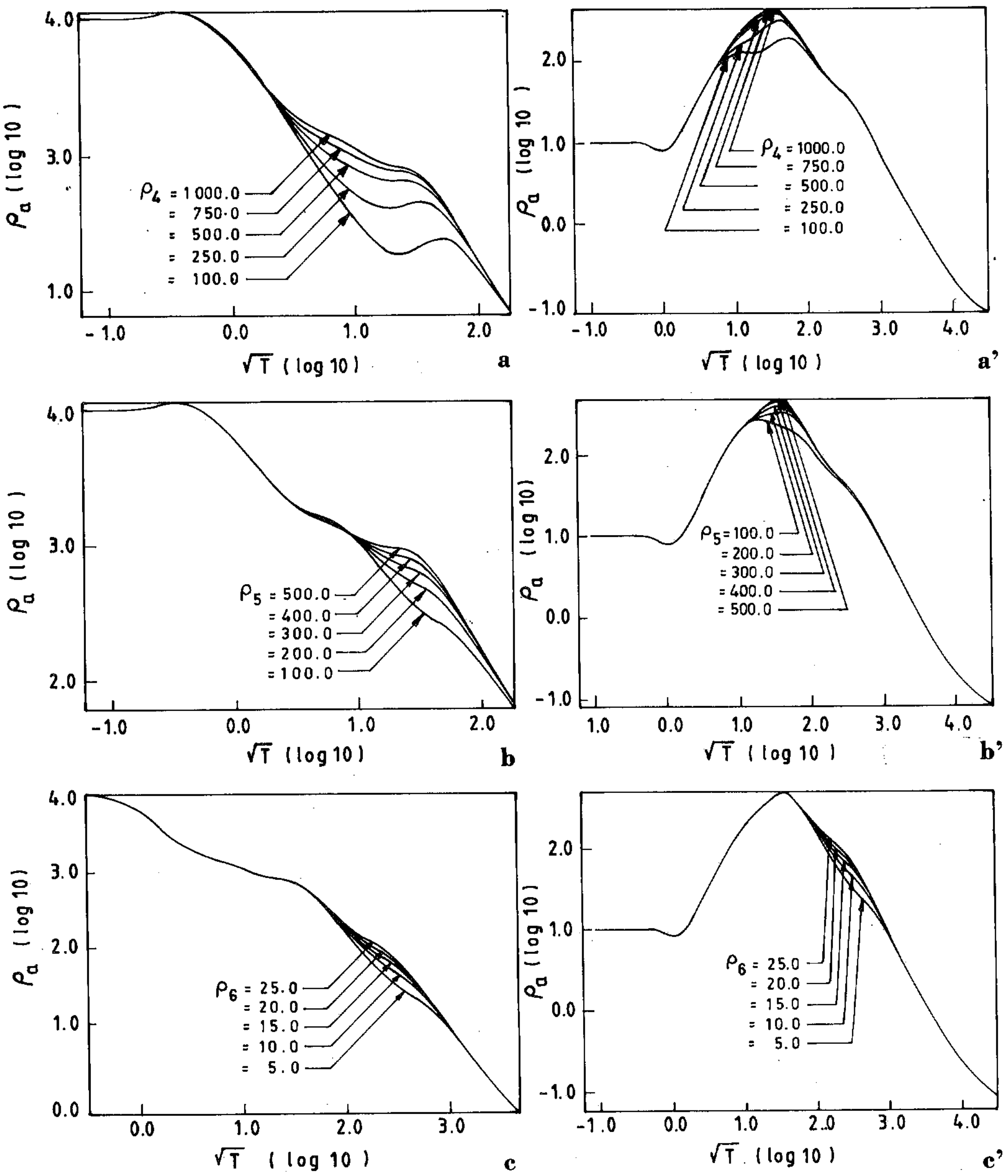

Fig. 8 — MT sounding curves for seven/eight layered earth models, with and without overburden sediments: a, b, c are for models without overburden; a', b', c' are the models with overburden. (a and a' for varying ϱ_4 ; b and b' for varying ϱ_5 ; c and c' for varying ϱ_6).

Table 2 — Relation between the ratio of second layer thickness and wavelength at f_{min} or f_{max} and μ_2 (ϱ_2 / ϱ_1).

μ_2		h_2/λ
CONDUCTIVE TARGET	RESISTIVE TARGET	
0.100	10.0	0.001490
0.010	100.0	0.000428
0.001	1000.0	0.000134

SUMMARY

Some aspects of the magnetotelluric response for layered earth models were studied for sensitivity analysis. Four classes of 1D earth models were examined and the following points recorded:

(i) Depth of perturbation at perturbation centroid frequency and depth of penetration (skin depth) are not the same.

(ii) For a certain class of model, the skin depth at f_{cr} can roughly tell the depth to the top of the second layer (target bed) in a three layered earth model. It is also observed that magnetotelluric signals can feel the presence of a target bed even when it lies at a depth greater than the skin depth.

(iii) Depth to the top of the second layer, in a three layer model, and the thickness of the second layer have a linear relation for a wide class of models. The relation between f_{min} and h_2 is also linear.

(iv) For K type models, i.e., for a resistive target, the sensitivity of the MT response goes down when $\mu_2 \geq 100.0$, but the relations h_2 vs h_1 and h_2 vs f_{max} are linear.

(v) The relation between the percentage anomaly and the layer thickness is nonlinear for both K and H type models. For the same percentage anomaly, the bed thickness of the conductive bed in a H type model ($\mu_2 = 1/100$) will be 100 times less than that for a K type model ($\mu_2 = 100$).

(vi) For a certain class of K type models, the MT response lacks resolution totally.

(vii) Two kilometers of conductive sediments in a continental crust and upper mantle earth model can reduce the sensitivity of the MT response to 400.0 km. The depth limit for the sensitivity will depend upon all the layer resistivities and thicknesses.

(viii) Wavelengths at f_{max} or f_{min} for K and H type models are 10^3 to 10^5 times larger than the thickness of the second layer, which has a cause and effect relation with f_{max} and f_{min}. The ratio h_2/λ has an inverse relation with the resistivity contrast, and is independent of whether the target is more resistive or more conductive in comparison to the host rock, so long as ϱ_2/ϱ_1 in one case is equal to ϱ_1/ϱ_2 in the other.

These observations are model dependent and cannot be generalised.

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