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A NEW METHOD FOR MEASURING THERMAL CONDUCTIVITY OF ROCKS: THE "SANDWICH METHOD", FIRST RESULTS

Abstract. A new method for rock thermal conductivity measurement is proposed. A constant power flat source heats a sandwiched pair of rock disks and the temperature differences across the specimens are measured in the steady state. Replication of the experiment with a set of disk pairs of different thicknesses allows the effect of contact thermal resistance to be eliminated. The method strongly reduces lateral heat losses.

INTRODUCTION

The simplest way of measuring thermal conductivity λ of rocks is by using a device that will satisfy the Fourier law. If one face of an unlimited flat plate of thickness H (Fig. 1) is heated by a source emitting a heat flow q (Wm^{-2}) and the other one is in contact with a heat-absorbing surface, the plate's thermal conductivity ($Wm^{-1}\ K^{-1}$) is given by (Fig. 1)

$$\lambda = \frac{q}{\Delta T/H} , \tag{1}$$

where ΔT (K) is the difference in temperature which is established between the two surfaces in a steady state.

In actual practice, this simple set up is subject to two important sources of error: a) the contact resistance between the plates and the heating and cooling surfaces, b) lateral heat losses, given the fact that, under real conditions, the plate's extent is limited.

Contact resistance depends on the thermal conductivity of the interposed fluid - generally air - and on the smoothness of the surfaces facing each other; according to Beck (1965) they should be smoothed to 0.02 mm.

Lateral losses depend on the overall thickness of the interposed set of materials between the heat source and the cooling surface, on the intensity of the heat source, on the specimen's diameter, and the duration of the experiment.

Extensive literature on the subject and, especially, a recent review by Beck (1989) indicate that all methods proposed thus far, whether absolute or relative, differ in the way they deal with or try to minimize such sources of error.

The present work proposes a new method by which the effect of contact thermal resistance can be eliminated and lateral losses considerably reduced, by reducing total thickness of the set of materials to a minimum.

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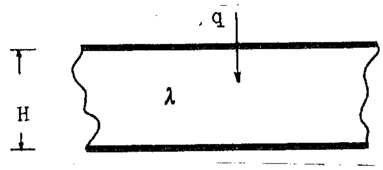

Fig. 1 — Fourier law.

THEORY

Consider a rock disk of thickness H and conductivity λ_r which is in contact with a heating surface C and a cooling surface F through two films of fluid of thickness x/2 and conductivity λ_c (Fig. 2).

Regardless of lateral heat losses, a study of steady state heat propagation through such a set can be approached in two different ways.

1st method

We know that, in the steady state, the total thermal resistance or resistivity R of a set of materials in series is equal to the sum total of those of each separate component r_i :

$$R=\sum_i r_i \, . \tag{2}$$

With reference to Fig. 2, this equation can be written as

$$\frac{H+x}{\lambda}=\frac{H}{\lambda_r}+\frac{x}{\lambda_c} \, , \tag{3}$$

where λ is the apparent conductivity of the set, i.e.,

$$\lambda=\frac{q}{\Delta T/(H+x)} \, . \tag{4}$$

Now, since the thermal conductivity of air is always less than that of any rock, we can write

$$\lambda_c=\frac{\lambda_r}{n} \, , \; n>1. \tag{5}$$

Therefore, from eqn. (3)

$$\lambda=\lambda_r\frac{H+x}{H+nx} \, . \tag{6}$$

From eqn. (6) we know that apparent conductivity λ is, among other things, a function of H, i.e.,

$$\lambda=\lambda\;(H). \tag{7}$$

Eqn. (6) and therefore eqn. (7) represents a hyperbole having the following properties:

$$x>0; \quad \frac{d\lambda}{dH}>0; \quad \lim_{H\to\infty} \lambda=\lambda_r$$

$$x=0 \quad \lambda=\lambda_c$$

and is represented in Fig. 3.

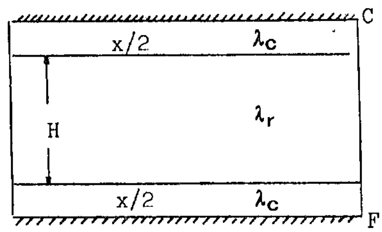

Fig. 2 — Schematic representation of contact resistance in heat propagation from heat source C to the cold face F.

The graph shows that as the specimen thickness H increases, the value of λ increasingly approaches that of λ_r. That means that if H is large enough one will obtain λ_r just by using eqn. (1).

The problem is then to determine how large H must be. To do so, let us write eqn. (6) in the form

$$\lambda = \alpha\, \lambda_r\,, \tag{8}$$

where

$$\alpha = \frac{H+x}{H+nx}\,, \tag{9}$$

from which we obtain

$$\frac{H}{x} = \frac{\alpha\, n-1}{1-\alpha}\,. \tag{10}$$

For example, for a 5% error in λ_r, i.e. if $\alpha=0.95$ and for $n=100$ (air conductivity is lower than rock conductivity by two orders of magnitude), we get $H/x=1880$; hence, if $x=0.02$ mm, $H=3.7$ cm.

This thickness is to be regarded as acceptable in the steady state only if lateral losses can be eliminated or compensated for.

One possible alternative is to make a certain number of determinations of λ for low values of H (<1 cm) and to determine λ_r by means of eqn. (6).

Apparent conductivity λ is given by eqn. (4). Hence, in principle, it is not computable,

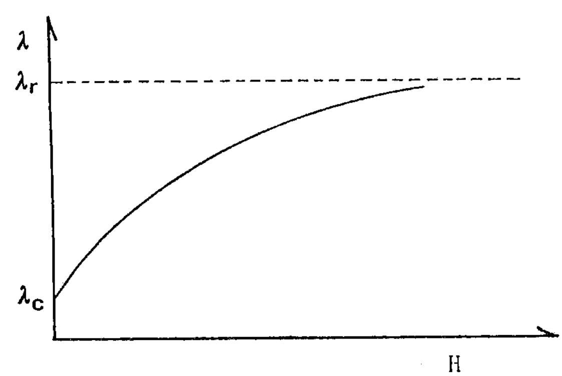

Fig. 3 — Graph representing eqn. (6).

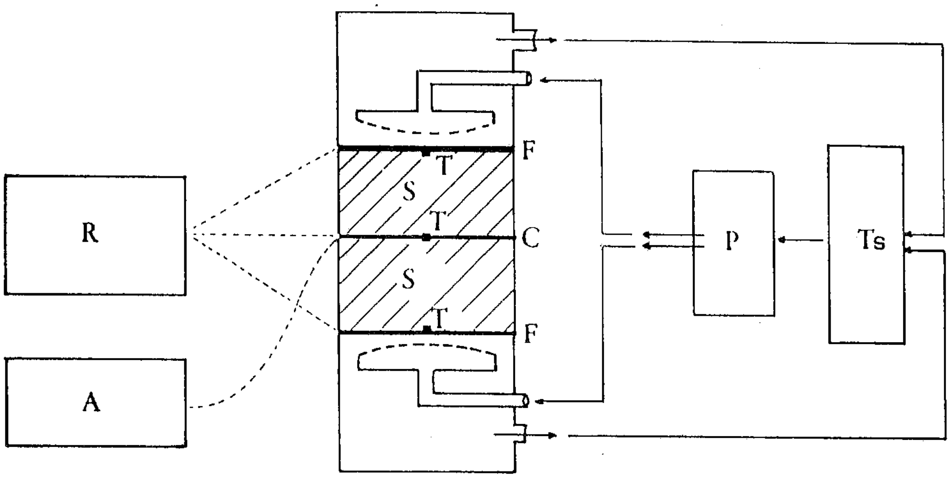

Fig. 4 — Device measuring thermal conductivity by the "sandwich" method. R - temperature recorder; A - stabilized power source; C - heat source; F - cooling heads; S - rock specimens; T - thermoelements; P - pump; Ts - thermostat.

since x is unknown; but if x « H, then x can be neglected in eqn. (4).

Now, assume we only have the results of three measurements (λ_1, H_1), (λ_2, H_2), (λ_3, H_3); these can be substituted in each instance in eqn. (6) and, by simple calculation, we get

$$\lambda_r = \frac{\begin{vmatrix} H_1\lambda_1 & \lambda_1 & 1 \\ H_2\lambda_2 & \lambda_2 & 1 \\ H_3\lambda_3 & \lambda_3 & 1 \end{vmatrix}}{\begin{vmatrix} H_1 & \lambda_1 & 1 \\ H_2 & \lambda_2 & 1 \\ H_3 & \lambda_3 & 1 \end{vmatrix}} . \tag{11}$$

If we have a larger number of measurements, we can obtain a more probable value of λ_r .

2nd method

The set in Fig. 2 can be regarded as a triple layer through which the heat released by the hot source C passes with flow density q (W m^{-2}). Let us denote by ΔT_i the temperature differences established in the steady state through each separate layer i. Then we can write

$$\Delta T_1 = \frac{q\cdot x}{\lambda_c\cdot 2}$$
$$\Delta T_2 = \frac{q\cdot H}{\lambda_r} \; , \tag{12}$$
$$\Delta T_3 = \frac{q\cdot x}{\lambda_c\cdot 2}$$

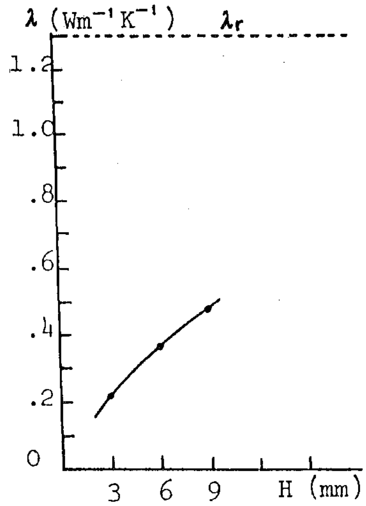

Fig. 5 — Fitting apparent thermal conductivity measurements by means of hyperbole.

and, by summing

$$\Delta T = q\left(\frac{x}{\lambda_c} + \frac{H}{\lambda_r}\right), \tag{13}$$

or else

$$\Delta T = \frac{qH}{\lambda_r} + \frac{qx}{\lambda_c} \ . \tag{14}$$

Therefore, after performing a series of at least 2 determinations of ΔT as H changes, the straight line which best approximates the representative points has, as its intercept,

$$q \cdot x/\lambda_c \ ,$$

from which the contact resistance x/λ_c can be derived; and, as its slope,

$$tg\,\beta = \frac{q}{\lambda_r} \ , \tag{15}$$

from which λ_r is obtained.

FIRST EXPERIMENT

Fig. 4 presents the block diagram of the device used to carry out the first experiments. A source of heat C, consisting of a copper wire formed into flat concentric coils, is energized by direct current from a stabilized power source A. It is sandwiched between two rock disks S which, in turn, are contained between two cooling heads F. Three thermistors measure temperatures with a sensitivity of 0.001 °C at the centre of the source and of the two contact surfaces between the disks and the heads. A pump forces a fluid from a thermostat to the heads; the fluid sprinkles the contact surfaces of the heads and flows back to the thermostat.

No action was taken to control lateral losses.

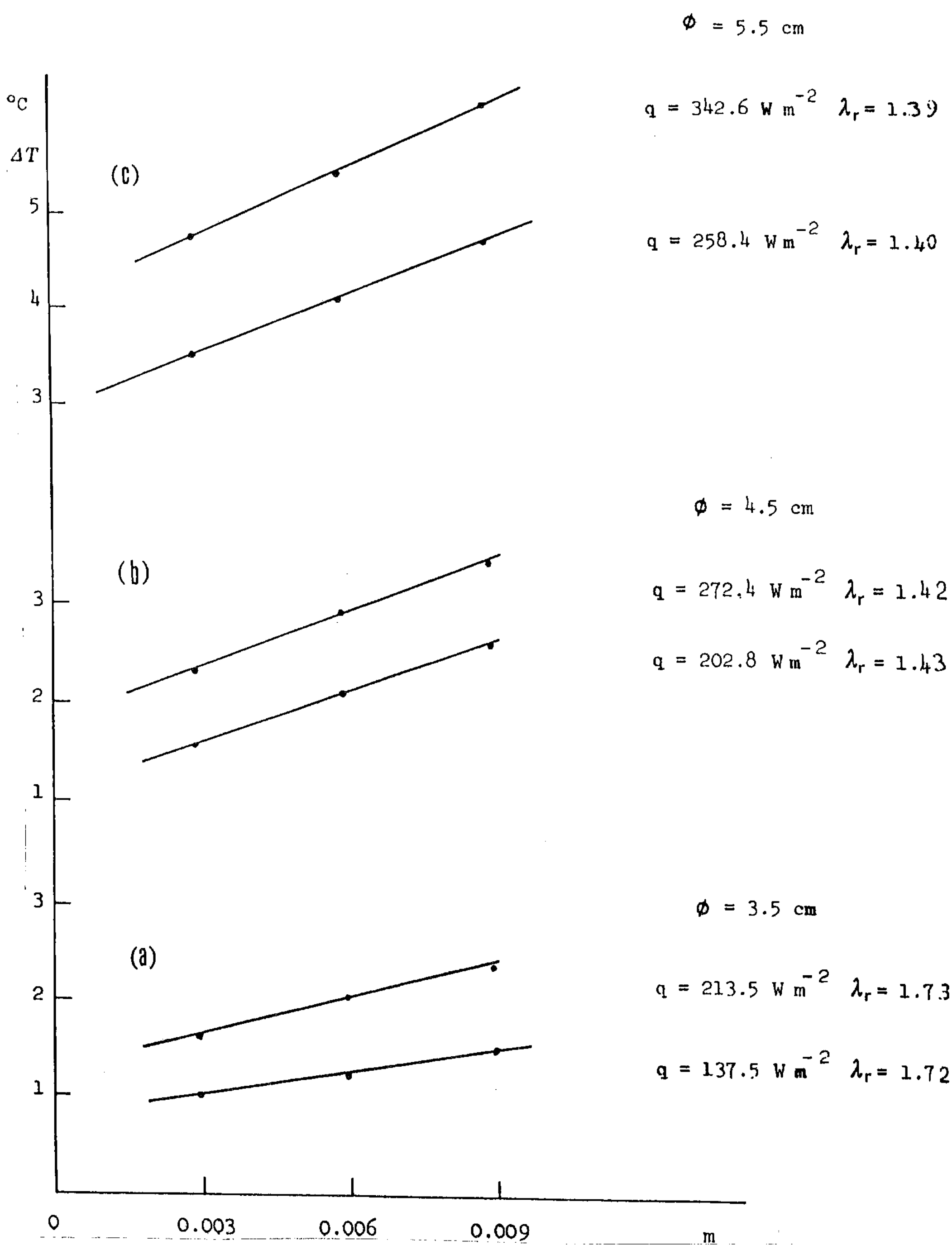

Fig. 6 — Fitting ΔT (H) by means of straigth lines. λ_r obtained from the slope using eqn. (15).

Fused quartz disks (international standard) were used to take measurements for calibration of the methods.

Fused quartz conductivity in mcal/cm s °C is given by (Ratcliffe, 1959)

$$\lambda_{FQ} \times 10^3 = 3160 + 4.6\ T - 0.016\ T^2, \tag{16}$$

where T is in °C.

For a 26.7 °C temperature, eqn. (16) gives $\lambda_{FQ} = 3.29$ mcal/cm s °C which holds 1.37 $Wm^{-1}\ K^{-1}$.

1ST METHOD

By way of example, Fig. 5 shows the results obtained by using three pairs of disks 4.5 cm in diameter and 3, 6, and 9 mm thick. With these data and using eqn. (11), we get $\lambda_r = 1.30\ Wm^{-1}K^{-1}$. The difference between this value and the reference is 0.07 $Wm^{-1}K^{-1}$, which is about 5 per cent of the latter.

2ND METHOD

Pairs of disks, 3.5, 4.5, and 5.5 cm in diameter and 3, 6, and 9 mm thick, were utilized. Fig. 6 shows the results of these experiments which were obtained by also letting power output change. In Fig. 6a, the λ_r values obtained for the 3.5 cm in diameter disks are too high; it would therefore appear that the measurements are affected by lateral losses.

Figures 6b and c show the results obtained with the 4.5 and 5.5 cm diameter disks; the conductivities obtained are respectively $\lambda_r = 1.42$, 1.43 and 1.39, 1.40 $Wm^{-1}K^{-1}$, which differ from the reference value $\lambda_{FQ} = 1.37\ Wm^{-1}K^{-1}$ by less than 3 per cent. This small difference indicates that lateral losses are reduced to acceptable levels but not completely eliminated.

CONSIDERATIONS

Due to the long duration of the measurements and the length of the set of materials, all current methods for measuring thermal conductivity in the steady state require a guard ring in order to prevent lateral losses.

With the "sandwich method", having reduced the set length to just 1-2 cm, and using specimens that are at least 4-5 cm in diameter, lateral losses have been practically eliminated.

This result seems to demonstrate that lateral losses are mainly due to the length of the measurement set (other than the diameter).

Further experiments will be useful to define the best diameter in the case of high conductivity rocks.

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