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GRAVITY INVERSION OF CLOSED TWO-DIMENSIONAL BODIES

Abstract. An inversion scheme to trace the boundary of a closed two-dimensional body from its gravity anomalies is suggested. The body is replaced by a series of juxtaposing prisms. The initial values of the depths to top and bottom of these prisms are calculated and improved iteratively, using the method of ridge regression, until the anomalies of the model best fit the gravity anomalies. The input to the computer consists of the gravity anomalies, the density contrast, and approximate mean depth of the body and the horizontal boundaries of the body. The method differs from shemes using the polygonal cross-sections, in that the initial values of the parameters defining the cross-section of the body need not be supplied to the computer as the input.

INTRODUCTION

Inversion of gravity and magnetic anomalies employing optimization techniques is now a routine affair. In the current geophysical literature, inversion is taken to imply replacement of cross-sections of two-dimensional bodies by N-sided polygons, assigning the initial values of the coordinates of their vertices and modification of these coordinates through an appropriate optimization procedure. Several inversion schemes built on these lines differ only by the optimization technique used.

Radhakrishna Murthy and Bhaskara Rao (1982) have suggested that the anomalous bodies can be represented by a series of juxtaposing vertical prisms and their gravity anomalies can be inverted for the depths to the top and bottom of the prisms. It appears that the model of juxtaposing prims has a unique advantage over the polygonal model, in that the initial values of the parameters defining the outline of the body need not be provided as an input to the computer. Inversion schemes can be designed with a built-in mechanism to determine these parameters by the computer itself. Such inversion schemes were reported earlier (Radhakrishna Murthy et. al., 1988; Radhakrishna Murthy and Rao, 1989) for extreme cases of two-dimensional bodies viz., bodies with a flat bottom, bodies with a flat top and density interfaces. In this paper, a generalised case of a closed two-dimensional body with irregular top and bottom is considered and a scheme is suggested for the inversion of its gravity anomaly. The model of juxtaposing prisms is used and the depths to the top and bottom of prisms are determined. The input to the computer consists of the gravity anomalies, sampled at equal intervals along a profile, and the body's density contrast, probable mean depth and horizontal boundaries.

NOTATION

N_{obs} = Number of observation points on the profile

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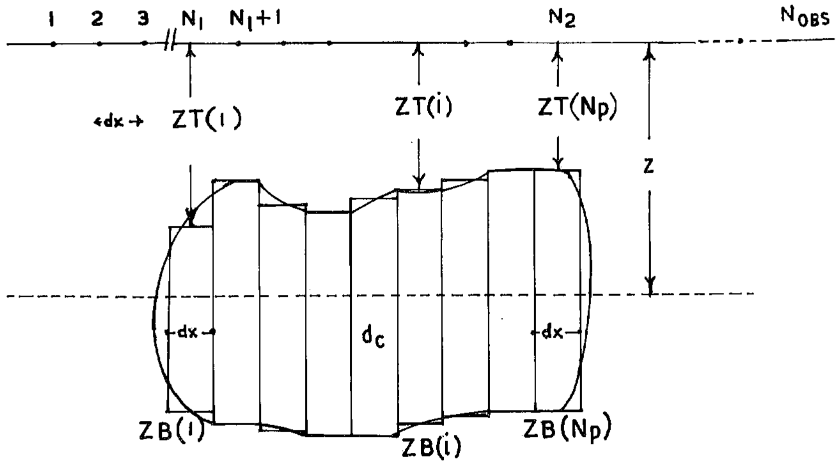

Fig. 1 — Series of juxtaposing finite prisms replacing the two-dimensional body of irregular cross-section.

N_p = Number of prisms = $N_{obs}/2$

$X(i) = X_i$ = Distance of the ith observation/anomaly point measured from an arbitrary reference on the profile - usually the one at the extreme left.

dx = Station spacing.

Z = Mean depth of the anomalous body.

$ZT(i) = ZT_i$ = Depth to the top of the body at the ith station.

$ZB(i) = ZB_i$ = Depth to the bottom of the body at the ith station.

d_c = Density contrast, gm/c^3

$dZT(i) = dZT_i$ = Increments to depths to the bottom of the body at the ith station.

$g_{obs}(i) = g(X_i)$ = Observed gravity anomaly at the ith station, in milliGal.

$g_{cal}(i)$ = Calculated gravity anomaly at the ith station of the interpreted model.

THE METHOD

A closed two-dimensional body and its approximation by vertical prisms are shown in Fig. 1. Each prism has a width equal to the station spacing and is placed symmetrically to one of the anomalies on the profile. Since there are N_{obs} anomaly points, the maximum number of parameters that can be determined from the profile is only N_{obs} and hence the number of prisms to be considered is resctricted to $N_p = N_{obs}/2$.

The density contrast of the body and its approximate mean depth are assumed. The position of the first prism is also assumed, by specifying N_1, the station number below which it lies. The program calculates the initial values of the depths to top and bottom of each of the N_p prisms and improves them iteratively using Marquardt's optimization procedure.

For buried structures, whose density information is unknown, it appears reasonable to develop methods which can estimate the density contrast also. But, the number of parameters to be determined is controlled by the number of observations. In the chosen model, the number of parameters works out to be equal to the number of observations on the profile. Thus, there is no way of determining the density contrast. If the density contrast is not known, the profiles can be inverted for different density contrasts and the best possible solution can be selected as in any inversion and modelling scheme.

The method also requires knowledge of the horizontal limits of the body's extension. For

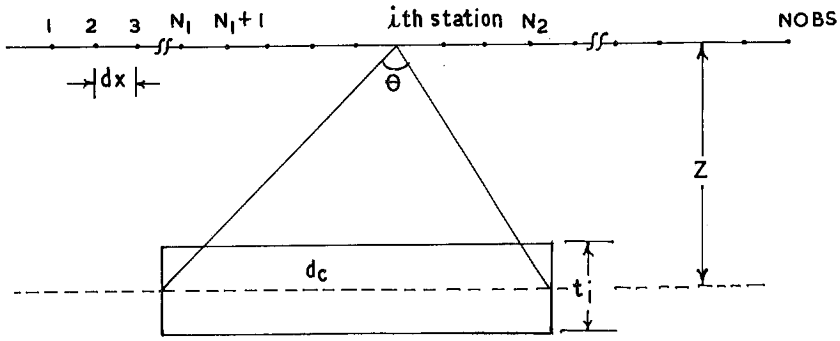

Fig. 2 — The finite horizontal sheet model. Z is the mean depth and d_c is the density contrast.

this, N_1, the station number corresponding to the position of the first prism is specified. N_2, the station number corresponding to the position of the last prism is then $N_2=N_1+N_p$. Thus, the body is assumed to be contained completely between the distances $X_{N1}-dx/2$ and $X_{N2}+dx/2$. X_{N1} and X_{N2} can be approximately equated to the positions on the profile at which the profile tends to show a steep rise or a gradual fall. The horizontal extension of the body can be assumed much larger than it could actually be; in which case the inversion scheme locates prisms of zero thickness, where there is no mass below any station.

The approximate mean depth Z can be assigned based on the characteristics of the profile and employing rule-of-thumb interpretations of the gravity anomalies of cylinders and horizontal plates. It should be mentioned that Z is never used in inversion, except in the calculation of initial depths to the top and bottom of the prisms. Its choice does not generally affect the results, although a spurious or unrealistic choice can lead the solution to a local minimum or a saddel point.

ESTIMATION OF INITIAL PARAMETERS

Initial values of the depths to top ZT and bottom ZB of the various prisms are made in two stages with two separate assumptions. First, the anomalies are considered to be caused by a set of plate-like masses (the masses being concentrated along a plane), one below each station from N_1 to N_2 (Fig. 2). The initial values of depths Z_i ($i=N_1$ to N_2) of these masses are equated to that of Z, while the initial values of their masses, M_i are calculated (Fig. 2) using the equation (Rao and Murthy, 1978)

$$M_i=d_c\cdot t_i\cdot dx=\frac{[g(i)]\ dx}{2G\theta}\ (i=N_1 \text{ to } N_2), \tag{1}$$

where

$$\theta=\arctan\ [(X_i-X_{N1}+dx/2)/Z_i]\ +\arctan\ [(X_{N2}-X_i+dx/2)/Z_i],$$

and $t_i=ZB_i-ZT_i$ is the thickness of the body below the ith station.

From the distribution of M_i, the gravity anomaly at any station k can be calculated by the formula (Rao and Murthy, 1978)

$$g_{cal}(k)=2G\sum_{i=1}^{Np} M_i\ (\theta_2-\theta_1)/dx, \tag{2}$$

where

$$\theta_2-\theta_1=[\arctan\ (P_1/Z_i)-\arctan\ (P_2/Z_i)],$$

$$P_1=X_k-X_{N1+i-1}+dx/2$$

and

$$P_2 = X_k - X_{N1+i-1} - dx/2.$$

The difference $dg(k) = g(k) - g_{cal}(k)$ at any station k is due to the error in the calculated masses and their depths. The errors in the anomaly dg(k) can be written as

$$dg\ (k) = \sum_{i=1}^{N_p} \frac{\partial\ g\ (k)}{\partial\ M_i} dM_i + \sum_{i=1}^{N_p} \frac{\partial\ g\ (k)}{\partial\ Z_i}\ dZ_i\ , \tag{3}$$

where

$$\begin{aligned} \frac{\partial\ g\ (k)}{\partial\ M_i} &= 2\ G\ [\arctan\ (P_1\ /\ Z_i\ ) - \arctan\ (P_2\ /\ Z_i\ )] \\ &\qquad \text{if } Z_i > 0, \\ &= 2\ \pi\ G \text{ if } Z_i = 0 \text{ and } |X_k - X_{N1+i-1}| < dx/2, \\ &= \pi\ G \text{ if } Z_i = 0 \text{ and } |X_k - X_{N1+i-1}| = dx/2, \\ &= 0 \quad \text{if } Z_i = 0 \text{ and } |X_k - X_{N1+i-1}| > dx/2, \end{aligned} \tag{4}$$

and

$$\frac{\partial\ g\ (k)}{\partial\ Z_i} = 2\ G \left(\frac{M_i}{dx} \left[\frac{P_1}{P_1^2 + Z_i^2} - \frac{P_2}{P_2^2 + Z_i^2} \right] \right) . \tag{5}$$

Eqn. (3) is a linear equation with N_{obs} unknowns, N_p ($= N_{obs}/2$) parameters relating to the errors in depths and the other N_p relating to errors in the masses. The increments dM_i and dZ_i can be calculated by framing the necessary normal equations and solving them using Marquardt's algorithm. The necessary system of simultaneous equations is

$$\begin{aligned} \sum_{i=1}^{N_{obs}} \sum_{k=1}^{N_{obs}} \frac{\partial\ g\ (k)}{\partial\ a_j} \cdot \frac{\partial\ g\ (k)}{\partial\ a_i}\ (1 + \delta\lambda)\ da_i \\ = \sum_{k=1}^{N_{obs}} dg\ (k) \cdot \frac{\partial\ g\ (k)}{\partial\ a_j} \\ \text{for } j = 1,\ 2,\ \ldots,\ N_{obs}\ , \end{aligned} \tag{6}$$

where

$$da_i = dM_i$$

$$da_{i+Np} = dz_i\ , \qquad \text{for } i = 1,\ 2,\ \ldots,\ N_p\ ,$$

and λ = Marquardt's damping factor.

The increments dM_i and dZ_i- are added to M_i and Z_i , respectively. The procedure is repeated several times. The thicknesses of the various prisms are then given by

$$t_i = ZB_i - ZT_i = M_i/(d_c \cdot dx),$$
$$(i = 1,\ 2,\ \ldots,\ N_p). \tag{7}$$

In the second step, the gravity anomaly component at any station (say ith) due to the estimated mass below it is explained by a vertical prism. If the width dx of the prism is further assumed to be small compared to its thickness t_i, the component dg of the gravity anomaly produced by it can be written as (Rao and Murthy, 1978)

$$dg = 2\ G\ d_c\ dx\ ln\ (ZB_i\ /\ ZT_i\ ). \tag{8}$$

This component of gravity anomaly is actually produced by the mass M_i concentrated over a horizontal length dx and is given by

$$dg = 4\ G\ d_c\ t_i\ arctan\ (dx/2Z_i\ ). \tag{9}$$

From eqns. (8) and (9),

$$ZT_i = [t_i\ /\ (e^A - 1)]\ , \tag{10}$$

where

$$A = \frac{2\ t_i\ arctan\ (dx/2Z_i)}{dx}\quad .$$

Eqn. (10) helps to determine the initial value of ZT_i below the ith station. ZB_i is then calculated as

$$ZB_i = ZT_i + t_i\ . \tag{11}$$

METHOD OF REFINIMENT

The algorithm described above finds the depths to the top and bottom of the juxtaposing prisms, that the body is viewed to be made of. These depths are only approximate. The composite gravity anomaly g_{cal} (k) of all these prisms, at any point k, using these approximate depths differs from the observed anomaly g_{obs} (k). These initial estimates of the depths are improved by the Marquardt's algorithm so that the objective function Σdg^2 (k) becomes a minimum, where dg(k) is the difference between observed and calculated anomalies.

The gravity anomaly g_{cal} (k) = g_{cal} (X_k) at any point P (X_k) due to all the prisms can be written as (Rao and Murthy, 1978)

$$g_{cal}\ (k) = 2\ G\ d_c \sum_{i=1}^{N_p} \Big[ZB_i\ (\theta_4 - \theta_3) - ZT_i\ (\theta_1 - \theta_2)$$
$$+\ [P_1\ ln\ (r_4\ /\ r_1\ ) - P_2\ ln\ (r_3\ /\ r_2\ )] \Big]\ , \tag{12}$$

with

$$r_1^2 = ZT_i^2 + P_1^2\ ,$$
$$r_2^2 = ZT_i^2 + P_2^2\ ,$$

$$r_3^2 = ZB_i^2 + P_2^2 ,$$

$$r_4^2 = ZB_i^2 + P_1^2 ,$$

$$\theta_4 - \theta_3 = \arctan (P_1 / ZB_i) - \arctan (P_2 / ZB_i)$$

and

$$\theta_1 - \theta_2 = \arctan (P_1 / ZT_i) - \arctan (P_2 / ZT_i).$$

The difference between observed and calculated anomalies dg(k) at any station k, can be written as

$$dg(k) = \sum_{i=1}^{N_p} \frac{\partial g(k)}{\partial ZT_i} dZT_i + \sum_{i=1}^{N_p} \frac{\partial g(k)}{\partial ZB_i} dZB_i , \tag{13}$$

where

$$\frac{\partial g(k)}{\partial ZT_i} = -2 G d_c [\arctan (P_1 / ZT_i) - \arctan (P_2 / ZT_i)] , \tag{14}$$

and

$$\frac{\partial g(k)}{\partial ZB_i} = 2 G d_c [\arctan (P_1 / ZB_i) - \arctan (P_2 / ZB_i)] . \tag{15}$$

From the eqn. (13), the increments dZT_i and dZB_i can be calculated by framing the necessary normal equations and solving them using the Marquardt's algorithm. The normal equations are

$$\sum_{i=1}^{N_{obs}} \sum_{k=1}^{N_{obs}} \frac{\partial g(k)}{\partial a_j} \cdot \frac{\partial g(k)}{\partial a_i} (1+\delta\lambda) (da_i) \tag{16}$$

$$= \sum_{k=1}^{N_{obs}} dg(k) \cdot \frac{\partial g(k)}{\partial a_j} ,$$

$$\text{for } j=1, 2, \ldots, N_{obs} ,$$

where

$$da_i = dZT_i$$

and

$$da_{i+Np} = dZB_i \quad \text{for } i=1, 2, \ldots, N_p ,$$

so that

$$\frac{\partial g(k)}{\partial a_i} = \frac{\partial g(k)}{\partial ZT_i}$$

and

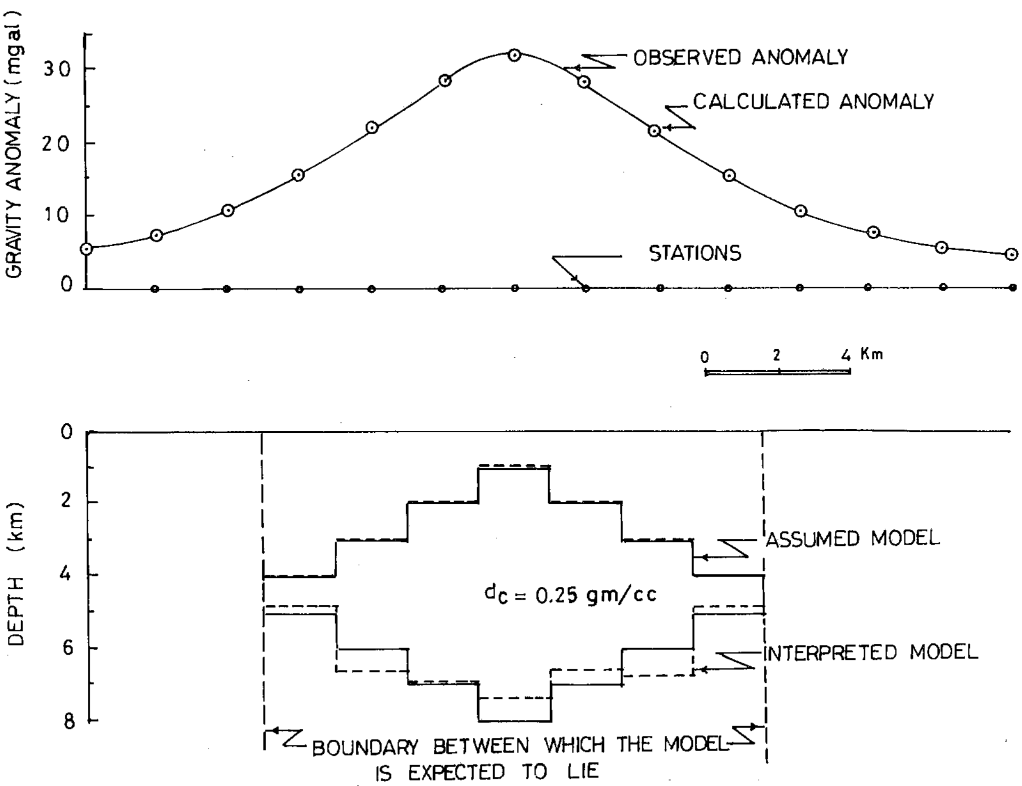

Fig. 3 — Interpretation of the gravity anomaly of seven juxtaposing prisms of varying thicknesses.

$$\frac{\partial g(k)}{\partial a_{i+Np}} = \frac{\partial g(k)}{\partial ZB_i} \cdot$$

$$for\ i = 1,\ 2,\ ...,\ N_p$$

The increments dZT_i and dZB_i are added to ZT_i and ZB_i respectively. The procedure is repeated until the difference between observed and calculated anomalies shows a minimum.

Computer programs can be developed using the method outlined in the foregoing pages imposing constraints, if necessary, where the depth of the body is known at any point on the profile.

EXAMPLES OF INTERPRETATION

Three examples are presented here to study the efficiency of the inversion scheme.

Fig. 3 shows a synthetic gravity profile over an assumed model of seven juxtaposing prisms of varying thickness with a density contrast of 0.25 gm/c^3 and the model interpreted by the proposed scheme. A mean depth of 3 km was assumed for calculating the initial values of the depths to top and bottom of each prism. The interpreted model is in good agreement with the assumed model and no differences are noticed between the observed and calculated anomalies. The objective function which is initially 13.4 units has become insignificant at the end of 10th iteration, when the program is terminated. The depths to the top are fairly in good agreement whereas some errors in the depths to the bottom are however noticed. This example shows that the gravity anomalies are less susceptible to variations in the depths to bottom, in comparison

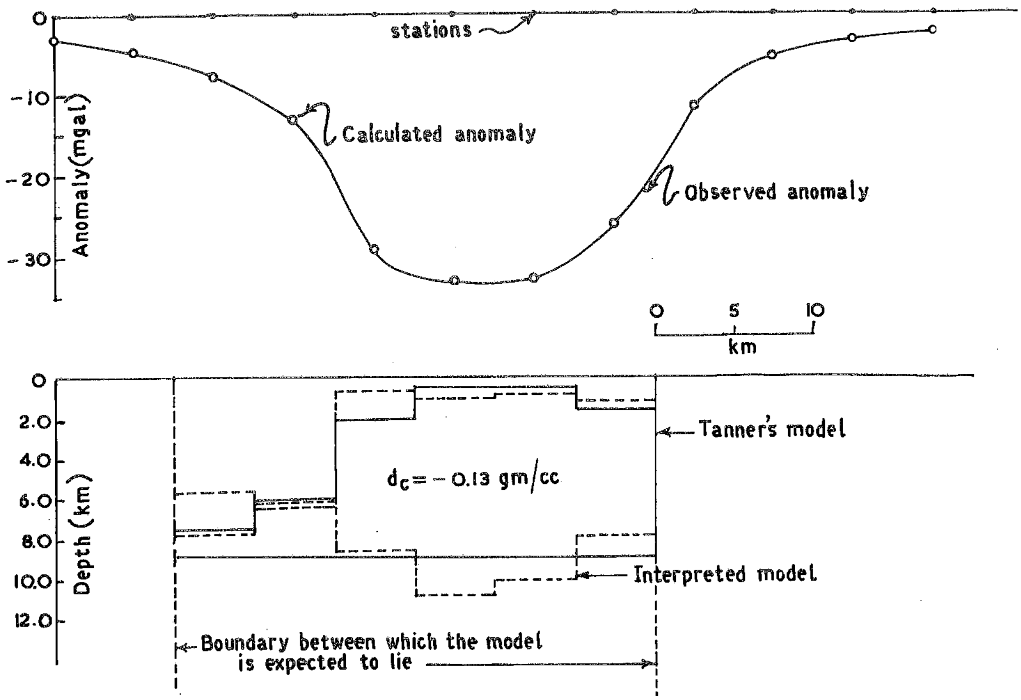

Fig. 4 — Interpretation of residual gravity profile over the Weardale granite body (Tanner, 1967).

to the variations in the depths to the top of the prisms.

Fig. 4 shows the inversion of the residual gravity profile of twelve stations over Weardale granite body (Tanner, 1967) by the proposed scheme. The station spacing is 5.0 km an the assumed density contrast is -0.13 gm/c^3. A total of six prisms is assumed, with the first placed below the third station. The interpretation is also compared with that of Tanner. The two interpretations agree well to a large extent and the minor discrepancy is due to the difference in the nature of the assumptions made in either the methods. In the Tanner's method, for example, the bottom is assumed to be a flat one and the depth to its top at one point is also maintained to be a constant. In the present method no constraints were imposed to the bottom of the model, showing that the proposed method works well even in extreme cases.

Fig. 5 shows the gravity profile in South East Minnesota reported by Craddock et. al. (1963). Very little geological information existed below this profile and anomaly was attributed to possible basic igneous rocks of density 2.9 gm/c^3 with the precambrian crystalline basement of density 2.7 gm/c^3. The basement and the igneous intrusions were belived to be under a cover of sediments having a thickness of 3.2 km and density of 2.3 gm/c^3. The interpretation obtained by the present method is shown in Fig. 5. It is clearly seen from the figure that the fit between the observed and calculated anomalies is not satisfactory. This figure also shows the model derived by Al-Chalabi (1972), who also reported a bad fit between the observed and calculated anomalies by this inversion scheme. The structure obtained in the present interpretation is close to the one by Al-Chalabi, although they are derived by different techniques. The minor difference may be attributed to the regional gradient assumed by Al-Chalabi (1972). It may be emphasised that no initial estimates of the body parameters are supplied in the present method, unlike in the exercise of Al-Chalabi (1972). It can also be seen that the present scheme works well even if the data is associated with some errors/regional component.

DISCUSSION

It is clear from the various examples studied that the model of juxtaposing finite prisms

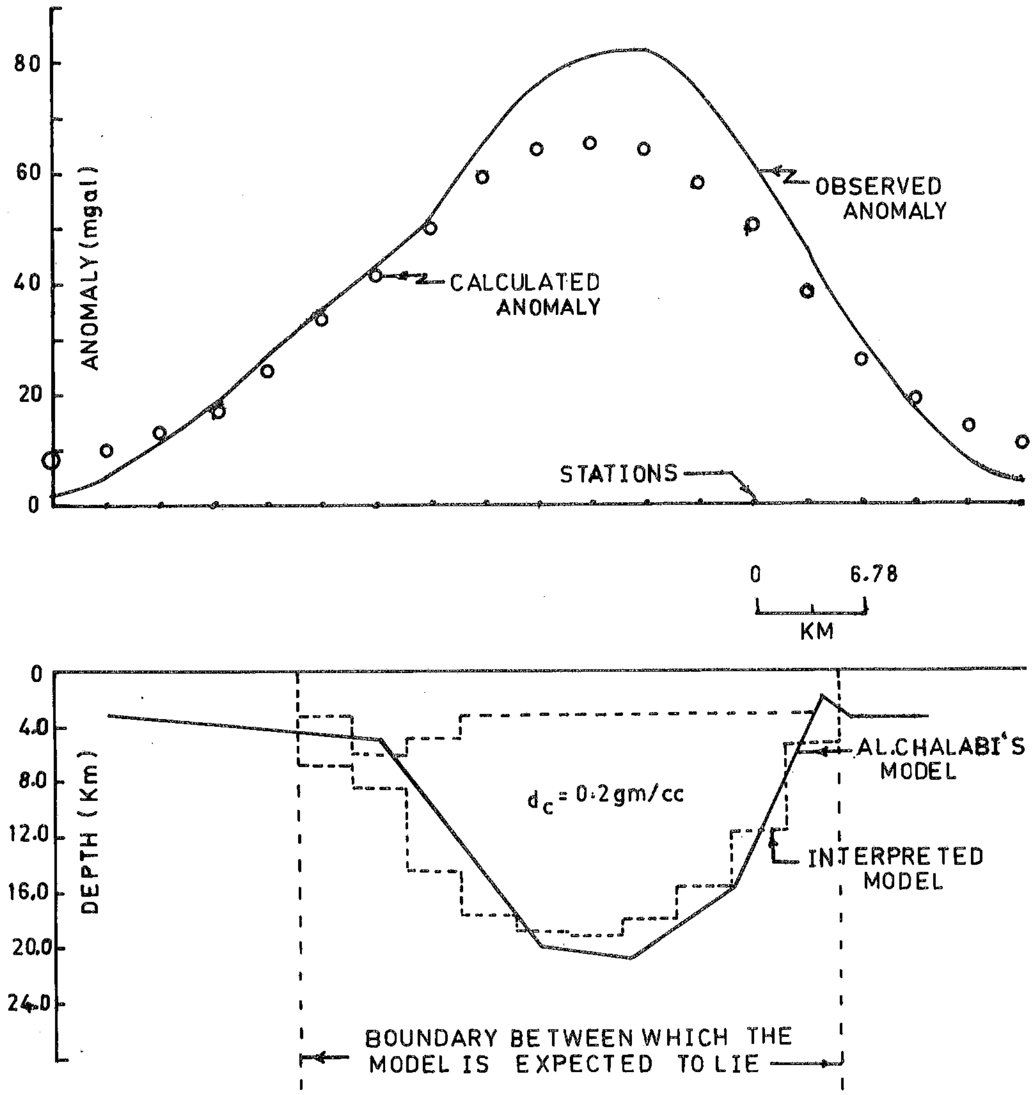

Fig. 5 — Interpretation of gravity profile over basic igneous rocks in South East Minnesota.

is capable of generating the initial values of the depths to the bottom and top and improving them iteratively. It can be easily understood that such a facility does not exist with the polygonal model, popularly used in inversion schemes. The use of the model of juxtaposing prisms is thus more advantageous than that of the polygonal model. It must be clear that the present inversion scheme is flexible and can accomodate a wide variety of geological structures by providing suitable constraints. An irregular body can be viewed as a series of prisms and its gravity anomalies inverted without imposing any restrictions on their depths to top and bottom. In case of an outcropping sedimentary basins, the prisms can be constrained to have a zero depth only for their tops. Again a flat-bottom granite body can be interpreted with its bottom having a fixed value.

The correct choice of the initial estimates is needed for convergence to an acceptable solution. Too much departure of the initial estimates from the actual values makes the problem to diverge or converge to a local minimum or a saddle point, where the corresponding interpretation may not be acceptable geologically. From the acceptability of interpretation of the examples presented, it can be inferred that the method used for estimating the initial values of the depths to top and bottom of the prisms is working efficiently. However it is a common feature of the various synthetic examples studied that depths to top of the structure are always accurately recovered, whereas a little error can persist in the depth to bottom of the structure.

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