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ALTERNATING CURRENT AT DIFFERENT FREQUENCIES: A RESISTIVITY SURVEY ON THE EASTERN HILL OF SELINUNTE (SICILY)

Abstract. Electrical measurements with alternating current taken in the same area at different frequencies may provide indications about the areal extension of the anomaly source as well as its depth (skin effect). We present examples of measurements obtained in the Selinunte archaeological district, using alternating currents up to 10 kHz and a parallel disposition of the fixed transmitting dipole and the mobile receiver (method of rectangle).

THEORETICAL CONSIDERATIONS

The study consists in the analysis of the distribution of the electrical field produced in the ground by the injection of an alternating current with frequency up to 10 kHz.

The electric field generated by two current probes A and B can be calculated according to the distant action theory without considering displacement and the induced currents characteristic of Maxwell's theory, since we utilise frequencies up to 10 kHz and limit the linear dimensions of the survey area to 20 metres.

At a frequency of 1/T = 10 kHz for a resistivity $\varrho = 50$ ohm · m the skin depth δ in metres is

$$\delta = 500\,(T\,\varrho)^{\frac{1}{2}} = 500\,(10^{-4}\cdot 50)^{\frac{1}{2}} \simeq 35\ m,$$

and $\lambda = 2\pi\delta \simeq 220$ m $>>$ 20 m is the wavelength of the electromagnetic field (Kunetz, 1958).

Moreover, if t is the time taken by the electromagnetic waves to cover the distance of 20 metres between the current probes A and B, and T is the period of the current used, T/t = 220/20, and therefore T = 11 t.

In these conditions, currents at up to 10 kHz can be thought of as almost stationary and the displacement current negligible.

The solution of stationary current distribution problems is mathematically identical to the solution of electrostatic problems that have the same geometry, and thus all the methods developed in electrostatics are applicable to electrodynamics (Panofsky and Phillips, 1950).

In electrostatics: In electrodynamics:

$$\mathbf{D} = K\,\epsilon_0\,\mathbf{E} \qquad\qquad \mathbf{j} = \sigma\,(\mathbf{E} + \mathbf{E'}),$$

Manuscript received January 15, 1992; accepted March 17, 1992.

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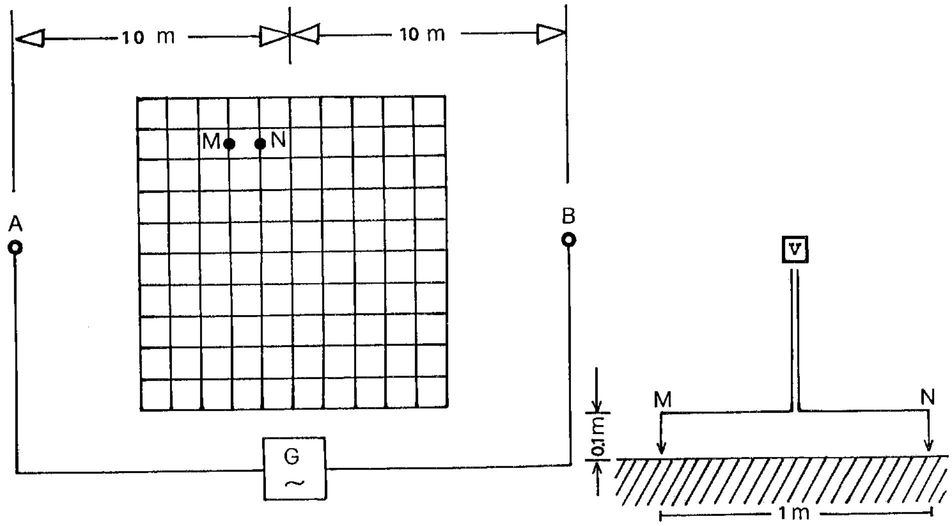

Fig. 1 — Measuring Devices. The MN axis is kept parallel to the AB axis.

where **E'** is the non conservative field applied.

In the almost stationary hypothesis div $\mathbf{j}=0$, the divergence of such fields gives:

$$div\ (K\ \epsilon_0\ \mathbf{E})=\varrho, \qquad div\ (\sigma\ \mathbf{E})=div\ (-\sigma\ \mathbf{E}'),$$

where ϱ is the density of free charges of the field **D**,
σ is the conductivity and
div $(-\sigma\ \mathbf{E}')$ is the source of the field σ **E**.

MEASUREMENT SYSTEM

Measurements were taken inside the field area outlined by the grid in Fig. 1.

The method consists of working out the ratio between the potential difference, measured between two points M and N, and the potential difference calculated at the same points for a medium with resistivity equal to 1 produced by the same current.

For a homogeneous medium, the potential difference measured between two points M and N is

$$V\ (N)-V\ (M)=(\varrho \cdot i/(2\ \pi))\ (1/AN-1/BN-1/AM+1/BM), \tag{1}$$

where A and B are the positions of the current electrodes.

The potential difference calculated for $\varrho=1$ between the same points M and N is

$$Vc\ (N)-Vc\ (M)=(1 \cdot i/(2\ \pi))\ (1/AN-1/BN-1/AM+1/BM). \tag{2}$$

The ratio between the corresponding members of the eqns. (1) and (2) gives the resistivity of the examined medium.

The ratio between the potential difference measured in a.c. and that calculated in eqn. (2) for i=i-effective gives the apparent resistivity between two points M and N at a chosen frequency.

When the ground is formed of two horizontal layers, the potential will be calculated using

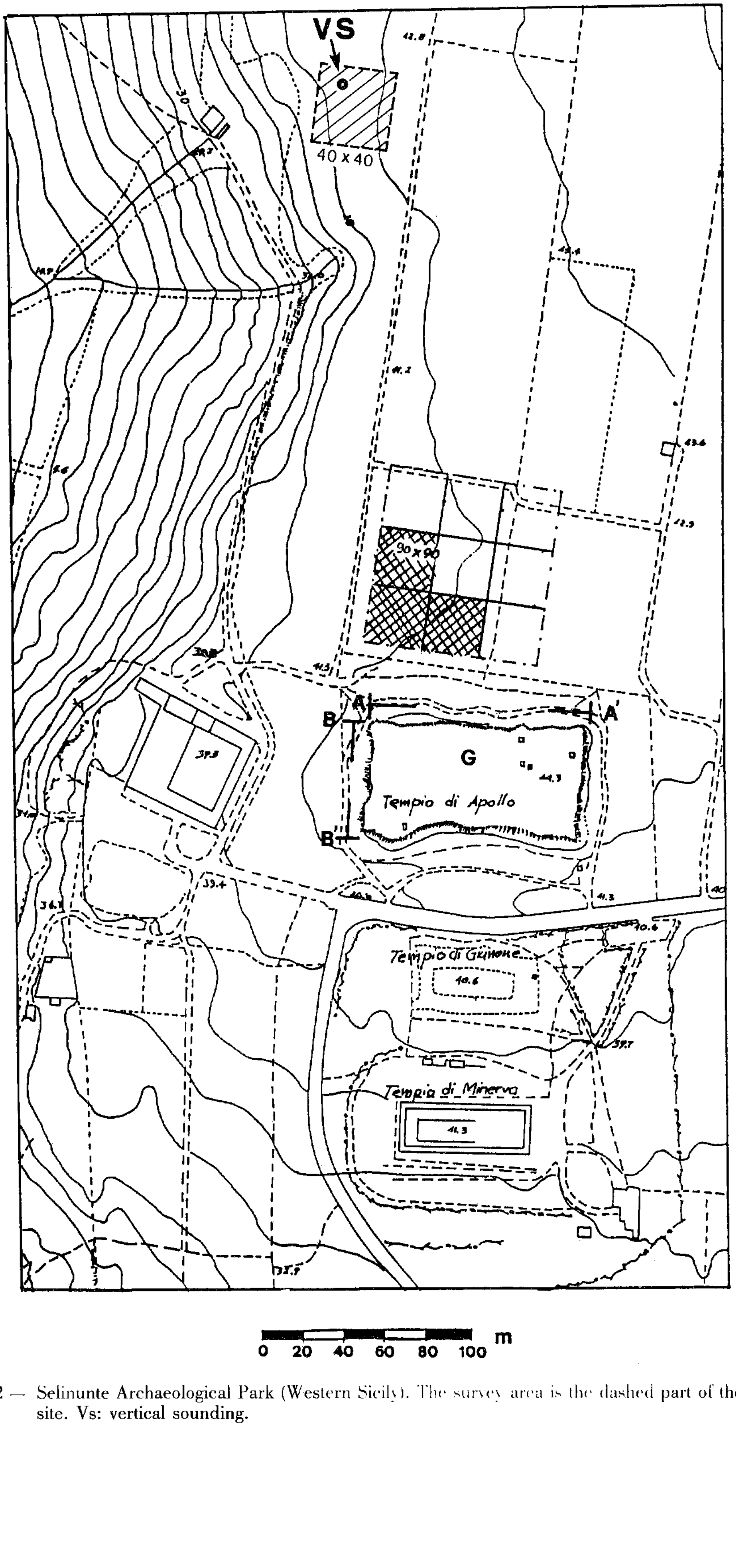

Fig. 2 — Selinunte Archaeological Park (Western Sicily). The survey area is the dashed part of the 40×40 test-site. Vs: vertical sounding.

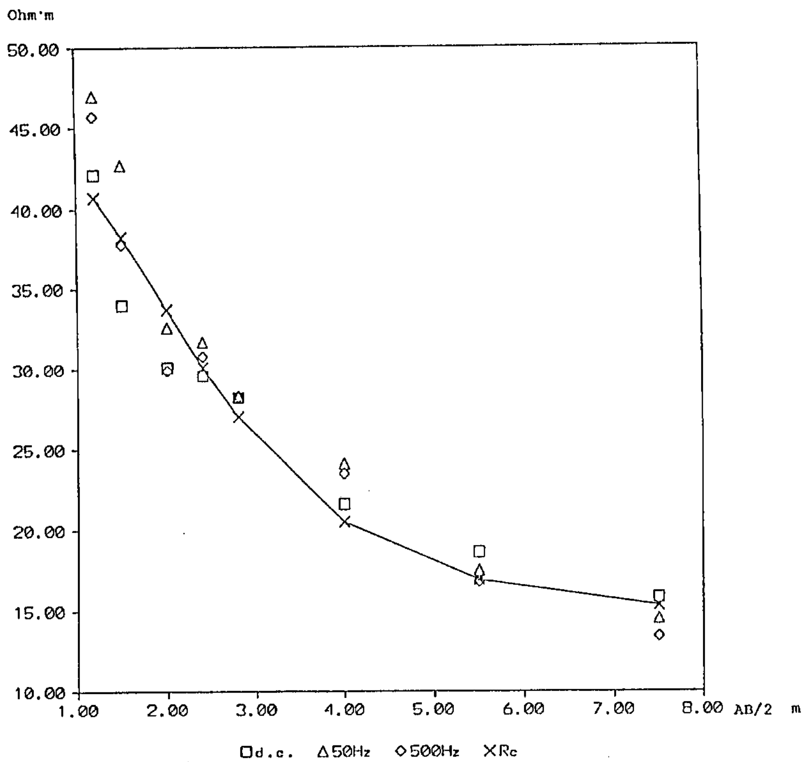

Fig. 3 — Graphic of the calculated apparent resistivity and experimental values of vertical sounding taken in three different ways.

the method of the image sources:

$$Vc\ (N) - Vc\ (M) = V\ (x_N\ ,\ y_N\ ) - V\ (x_M\ ,\ y_M\ ), \tag{3}$$

where

$$V\ (x,\ y) = K\left[\frac{1}{|r_1|} - \frac{1}{|r_2|} + \sum_{m=1}^{\infty} 2R^m\left[\left(r_1^2+\left(2\ mz\right)^2\right)^{-\frac{1}{2}} - \left(r_2^2+\left(2\ mz\right)^2\right)^{-\frac{1}{2}}\right]\right],$$

$R = (\varrho_2 - \varrho_1)\ /\ (\varrho_1 + \varrho_2)$ reflection coeficient,
ϱ_1 = resistivity of the first layer,
ϱ_2 = resistivity of the second layer,
z = height of the first layer,

$PB = r_1 = ((a-x)^2 + y^2)^{\frac{1}{2}}$ $\qquad A \equiv (-a,\ 0),$

$P \equiv (x,\ y)$

$PA = r_2 = ((a+x)^2 + y^2)^{\frac{1}{2}}$ $\qquad B \equiv (\ a,\ 0),$

$K = 1 \cdot i/(2\pi)$ for $i = i$—effective.

Instruments employed

The measurement system is comprised as follows:

Table — Vertical sounding: experimental and calculated values of the apparent resistivity.

AB/2	MN/2	d.c.	50 Hz	500 Hz	R_c
1.2	.40	42.1	47.0	45.7	40.7
1.5	.40	34.0	42.7	37.8	38.3
2.0	.40	30.1	32.6	29.9	33.7
2.4	.40	29.6	31.7	30.8	30.1
2.8	.40	28.2	28.3	28.3	27.0
4.0	.40	21.6	24.1	23.5	20.5
5.5	.40	18.6	17.5	16.8	16.9
7.5	.40	15.8	14.5	13.4	15.3

Column R_c shows apparent resistivity values calculated with
$\varrho_1=44$ Ohm·m resistivity of the first layer,
$\varrho_2=14$ Ohm·m resistivity of the second layer,
z = 1.2 m height of the first layer

- a power converter with a 50 Watt output fed by a 12 Volt battery that generates sinusoidal currents with 50 Hz, 500 Hz, 5 kHz, 10 kHz, 20 kHz, and 50 kHz frequencies connected to the probes located at A and B;
- a digital voltmeter with sensitivity of 10^{-4} Volt and terminals connected to the electrodes M and N forming a perpendicular coil with the ground with dimensions less than 0.1 m^2 (Fig. 1).

The influence of the induced potential is negligible even at 50 kHz provided that the cables from the current generator to the probes A and B are at least 2 metres from the surveyed area. A series of measurements were taken inside the field outlined by the grid of Fig. 1. A coil - insulated from the soil - formed by the electrodes M and N and a connecting cable, was placed normal to the ground surface and oriented in all possible directions. With frequencies up to 50 kHz, although a voltmeter with a sensitivity of 10^{-4} Volt was used, no voltage was measurable even keeping the coil at few centimetres from the soil.

EXPERIMENTAL MEASUREMENTS

The survey was conducted in the dashed part of the 40 m×40 m test-site, located in Selinunte

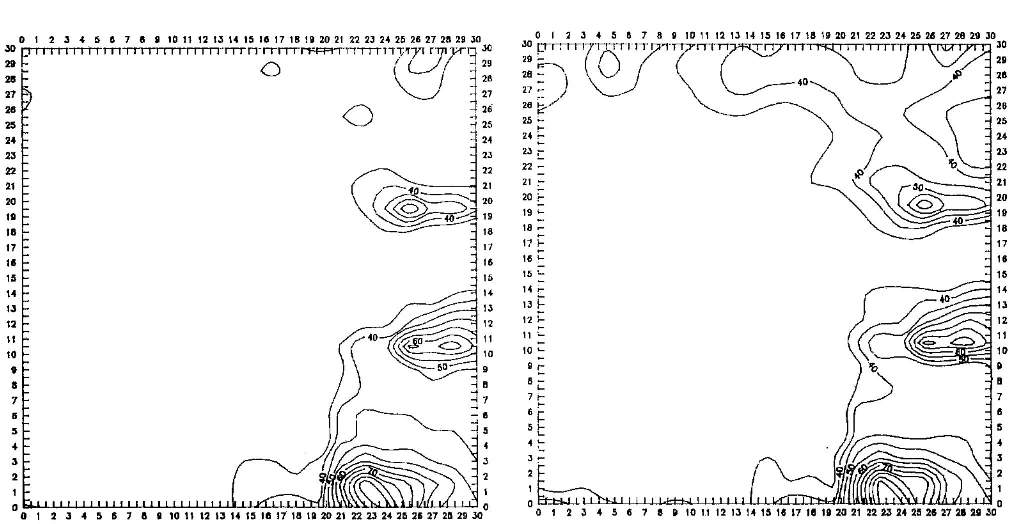

Fig. 4 — Apparent resistivity contours (ohm·m) obtained for the dashed area inside the 40×40 test-site.

Archaeological Park (Western Sicily, see Fig. 2).

Measurements were taken at nine contiguous 10×10 m^2 areas covering 30×30 m^2. Current probes A and B were set at points lying 20 metres apart to allow the survey of an area of 100 m^2. An alternating sinusoidal current was applied to these probes and the reading of the voltage produced was made at frequencies of 500 Hz and 10 kHz.

The current between A and B was unchanged during the measurement time of the voltage between M and N - at a distance MN=1 m - taken every metre along ten lines parallel to AB spaced one metre apart. In this way, 100 voltage values between M and N were measured for each 100 sqm area in about one hour.

A vertical sounding was done in the same 40 m×40 m area in three different ways: direct current, and alternating current with 50 Hz and 500 Hz frequencies. The apparent resistivity values obtained are shown in the Table and in Fig. 3.

Resistivity maps in Fig. 4 were calculated using in eqn. (3) the same values of ϱ_1, ϱ_2 and z employed to obtain Column R_c of the Table.

On the right side of both maps, higher values of resistivity are shown; to find out if these higher values are due to archaeological buried structures, a test excavation will be necessary.

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