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MULTIFREQUENCY ELECTROMAGNETIC RESPONSE OF A MAGNETICALLY PERMEABLE COVER-TARGET SYSTEM

Abstract. Multifrequency EM response of a cover-target system represented by two concentric spherical shells has been analysed, theoretically, for uniform field excitations. Some numerical results have been presented to show the effects of changing (i) the product of conductivity and thickness and (ii) the magnetic permeability values of the inner or outer shell. It is found in multifrequency EM measurements that (i) the imaginary component of the response factor shows the maximum sensitivity to the changes in physical and geometrical properties of the cover and/or target bodies; (ii) a magnetic shell is reflected earlier than the corresponding nonmagnetic ones.

The study may find relevance in delineating (i) pyrhotite possessing ferromagnetic properties and (ii) thin conducting deposits separated by resistive formations.

INTRODUCTION

Some recent studies on applied geoelectromagnetics have shown that multifrequency measurements afford the possibility of resolving conducting overburden and ore formations. Negi and Saraf (1981, 1984, 1989), Saraf and Negi (1983, 1986), Nagubai and Saraf (1991) have examined the feasibility of the separation of the responses due to overburden and underlying structures in multifrequency measurements. Negi and Saraf (1984) assumed a model of two concentric spherical thin shells. However, the effect of the magnetic contrast of a magnetic body with the overlying or surrounding bodies has relevance when the ferromagnetic (magnetized ores) materials are associated with the geological formations. Though almost all sulphide ores are nonmagnetic, some varieties of pyrhotite possess ferromagnetic properties due to their origin and thermal and mechanical history. Tarkhov (1965) emphasised another case of practical importance in EM Induction prospecting and that is the situation when the thin cover is insulated from the conducting target. Hence, in this paper an attempt is made to model the geological reality of multiple conductors in nongalvanic contact by assuming composite overburden-ore formations. Changes in the product of conductivity and thickness of the thin shell and magnetic permeability values have been demonstrated through numerical examples.

FORMULATION

The physical system under consideration consists of two concentric thin spherical shells of radii B (outer) and A (inner); thickness D_1 (outer), D_2 (inner); conductivities σ_1 (outer), σ_2 (inner) and magnetic permeabilities μ_1 (outer), μ_2 (inner) as shown in Fig. 1.

Following Negi and Saraf (1984), the reflection factor for uniform field considerations is given by

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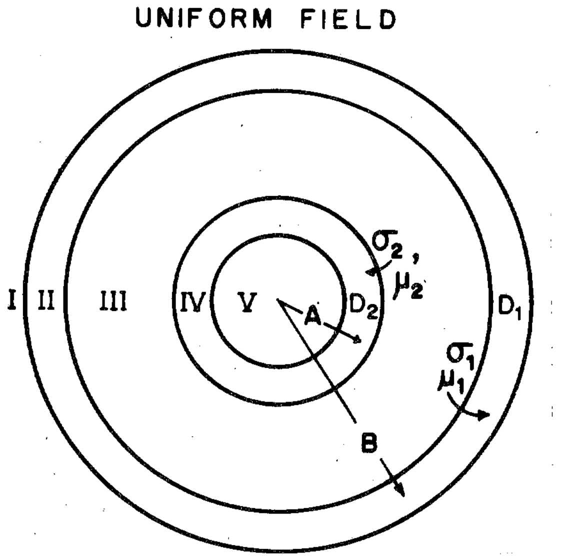

Fig. 1 — Concentric two shell (in non-galvanic contact) model placed in uniform E.M. field.

$$R_T = 1 - \left[\frac{3\ (1+p)}{\{3+\nu_2\ D_1 \cdot B\ (1+p)\}} \right], \tag{1}$$

where

$$p = \frac{\nu_4 \cdot D_2 \cdot A}{3+\nu_4\ D_2 \cdot A} \left(\frac{B}{A} \right)^3,$$

$\nu_4 = i\ w\ \mu_2\ \sigma_2$ and

$\nu_2 = i\ w\ \mu_1\ \sigma_1$.

DISCUSSIONS OF THE RESULTS

Some numerical results have been obtained to demonstrate the variations in the reflection factor (expression given by eqn. (1)) against the frequency of the EM waves. For plotting purposes, the dimensionless terms K_i $(=\sigma_i\ \mu_i\ D_i)$ have been used to facilitate the application in various situations representing the actual field problems. In the EM literature, normalisation is usually used for perspicuity and brevity.

Change in K_1 parameter

The following two sets of problems have been identified for demonstrating the changes in the K_1 parameter:

Change in K_1 when $(\mu_1 = \mu_2 = \mu_0)$

Here it is assumed that the inner and outer shells are magnetically impermeable i.e. the effects of changing the magnetic properties have been ignored.

In this particular case, $K_1 = \sigma_1\ D_1\ \mu_1$ (i.e. $K_1 = \sigma_1\ D_1$). Hence, any change in the value of

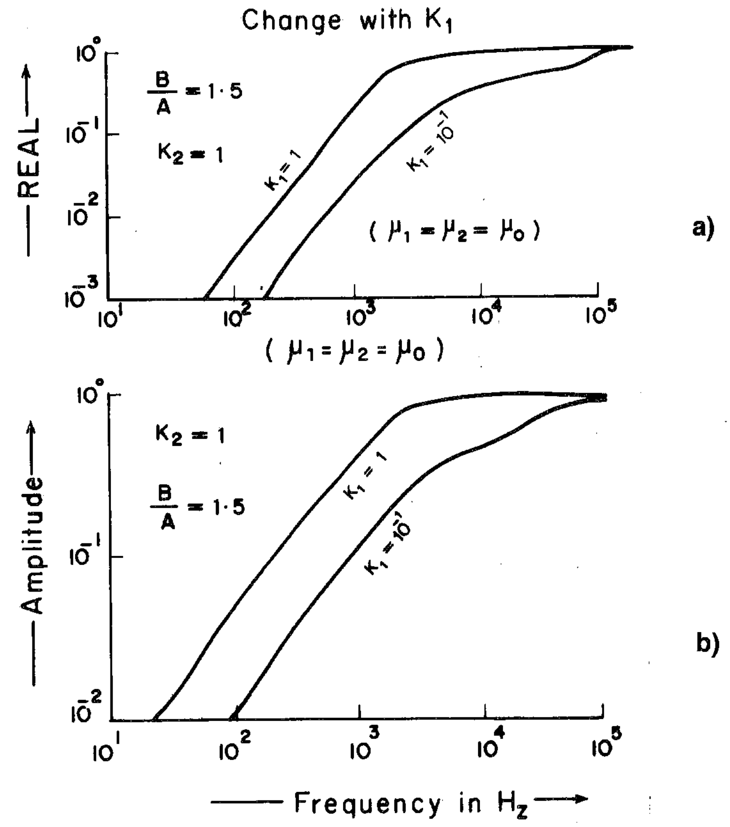

Fig. 2 — Variation of the real (a) and amplitude (b) components of the reflection factor against frequency of the E.M. waves for different values of K_1 $(\sigma_1 D_1 = 1, 10^{-1})$, $\mu_1 = \mu_2 = \mu_0$, $K_2 = 1$, $B/A = 1 \cdot 5$.

K_1 represents a change in σ_1 D_1 (i.e., product of conductivity and thickness parameter) only.

Behaviour of real and amplitude components

In Fig. 2, real (2a) and amplitude (2b) components of the reflection factor have been plotted against the frequency of the EM waves for different values of K_1 $(=\sigma_1 D_1 , \mu_1 = \mu_0)$.

It is observed that when $K1 = K2 = 1$, the variation patterns of both real and amplitude components behave like a single shell model. However, when $K_1 < K_2$, both the shells contribute to the response and one finds a slight deviation from the single shell response.

Behaviour of imaginary component

In Figs. 3a and 3b, the imaginary components have been plotted against frequency for different values of K_1 $(K_1 = \sigma_1 D_1 , \mu_1 = \mu_0)$.

It is found that:

(i) When $K_1 < K_2$, the imaginary components of the response factor exhibit the capability to separate out the responses due to cover (outershell) and target (innershell). This can be seen in the form of two peaks (Fig. 3a, $K_1 = 10^{-3}$ and $K_2 = 1)$ in different frequency zones. The first peak in the low frequency range (i.e. $F < 10^4$ Hz) is mainly due to induction in the inner shell, as the outer shell behaves like a transparent body at these low frequencies. The second peak at higher frequencies $F > 10^4$ Hz) corresponds to the contribution from the outer shell only. At these frequencies, almost all the EM energy will be dissipated in the outer shell only.

(ii) It is seen that the peak value due to the outer shell is almost twice the peak value due

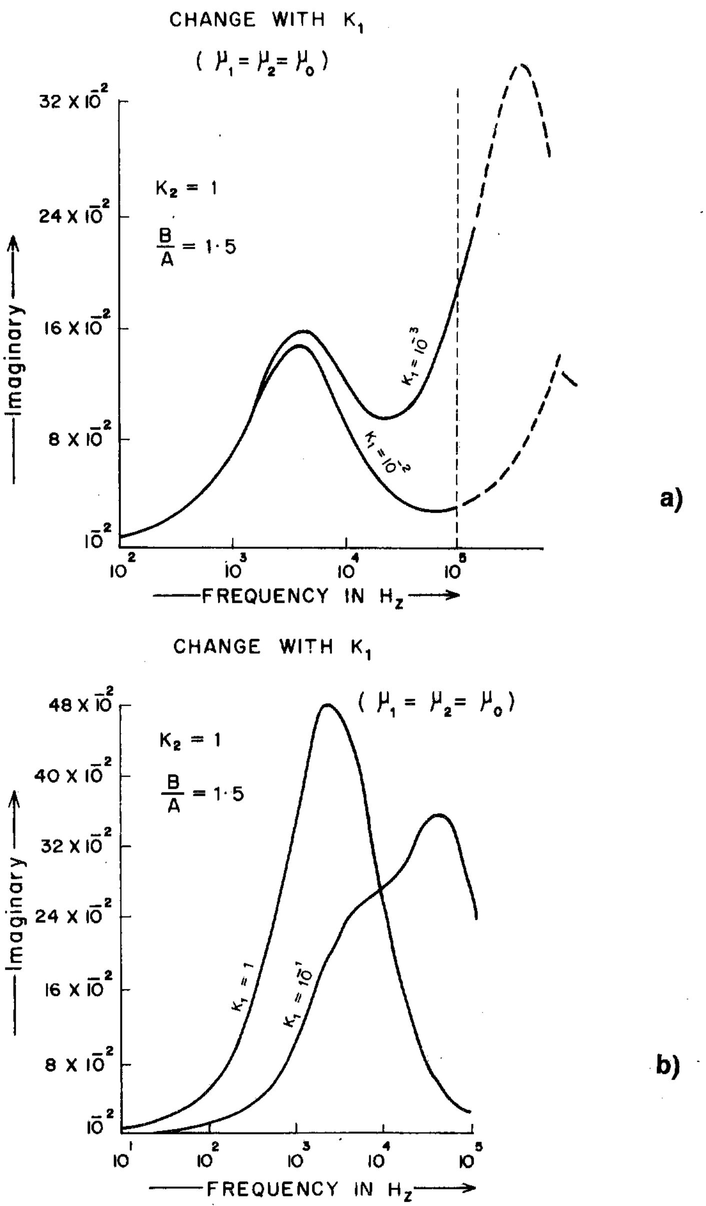

Fig. 3 — Variation of the imaginary component of the reflection factor against frequency of the E.M. waves for different values of K_1 : a) σ_1 $D_1=10^{-2}$, 10^{-3}; b) σ_1 $D_1=10^{-1}$, 1. $\mu_1=\mu_2=\mu_0$, $K_2=1$, $B/A=1\cdot5$.

to the inner shell (when $K_1 << K_2$). However, the change in the value of K_1 from 10^{-3} to 10^{-1} and 1 (Fig. 3b), indicates that the effect of the outer shell is not always the same when the value of the *K1* parameter $(K_1=\sigma_1\ D_1)$ is also changed.

(iii) When it is assumed that $K_1=K_2=1$ (Fig. 3b), the imaginary - frequency variation pattern, similarly to the corresponding real (Fig. 2a) and amplitude (Fig. 2b) components, behaves like a single shell model.

Behaviour of the phase component

In Figs. 4a and 4b, the variation of the phase component against frequency has been plotted

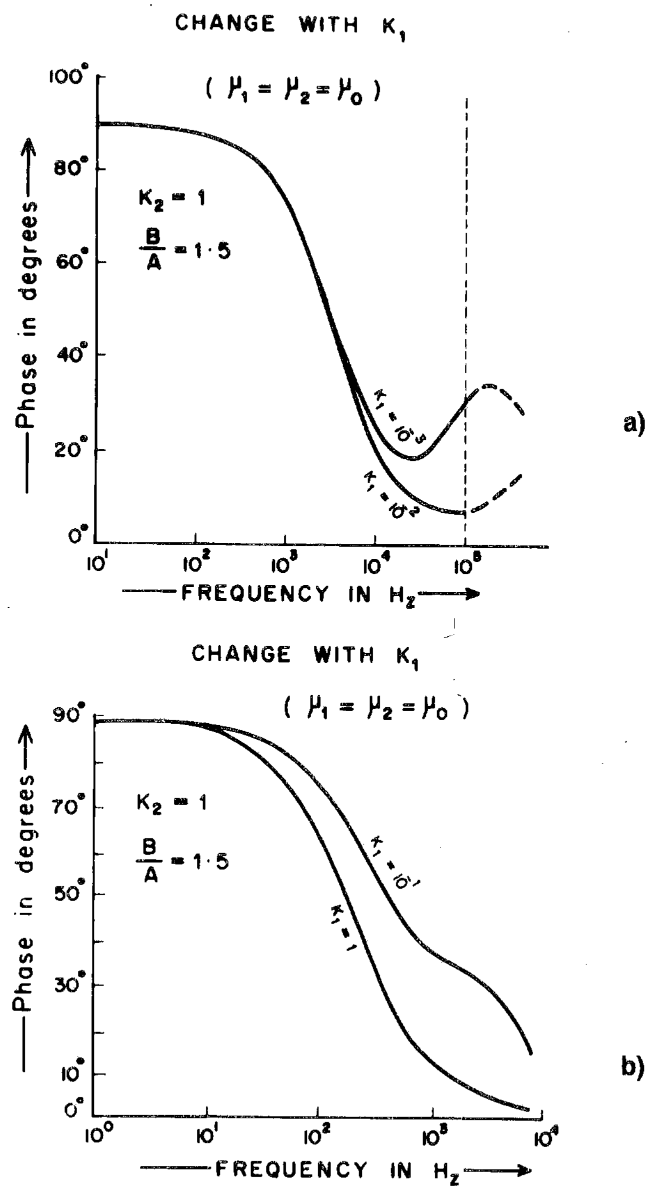

Fig. 4 — Variation of the phase component of the reflection factor against frequency of E.M., waves for different values of K_1 : a) σ_1 $D_1=10^{-2}$, 10^{-3} ; b) σ_1 $D_1=10^{-1}$, 1.
$K_2=1$, $\mu_1=\mu_2=\mu_0$, $B/A=1\cdot 5$.

for different values of K_1 $(=\sigma_1$ D_1 , $\mu_1=\mu_0$).

Similarly to the imaginary component, here also contributions from both the shells are seen when $K_1<K_2$. When $K_1=K_2=1$ the composite system behaves like a single shell model.

An interesting result is seen in Figs. 4a and 4b. On increasing the value of K_1 $(=\sigma_1$ D_1), the values of the phase component decreases, while in a similar situation, the other components (Figs. 2 and 3) of the reflection factor increase with an increase in the value of K_1 . Such a change may be utilized in checking the results obtained by multifrequency EM methods.

It is evident from Figs. 2, 3 and 4 that the imaginary component of the reflection factor is more sensitive to changes in the geometrical/physical properties of the system. Hence, here after, the discussions will be confined to the variations in the imaginary component only.

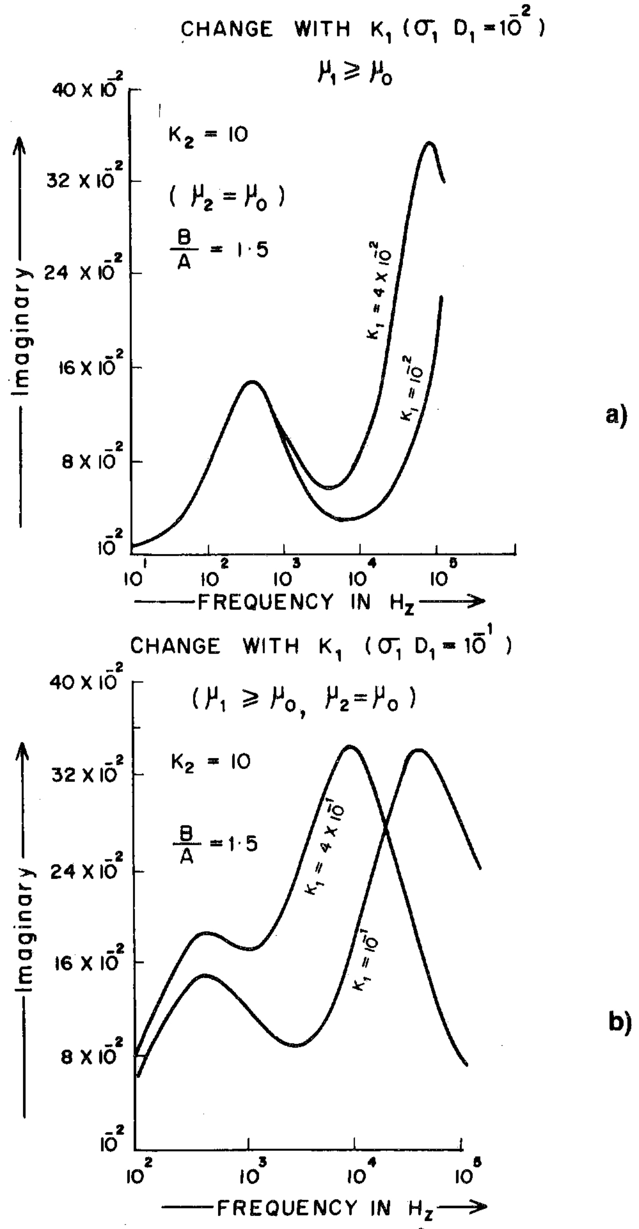

Fig. 5 — Variation of imaginary component of the reflection factor against frequency of the E.M. waves for different values of K_1 : a) $\mu_1=\mu_0$, 4 μ_0 ; σ_1 $D_1=10^{-2}$; b) $\mu_1=\mu_0$, 4 μ_0 ; σ_1 $D_1=10^{-1}$. $K_2=10$, $\mu_2=\mu_0$, $B/A=1 \cdot 5$.

Change in K_1 parameter when $\mu_1 \geqslant \mu_0$

The simplifying assumption that the values of magnetic permeability of the ore body and the surrounding medium are always the same becomes too ideal in the interpretation of geoelectromagnetic data. A maximum change in the magnetic permeability of the order of 5 to 10 would be anticipated in massive magnetites (Ward, 1961). Hence, any attempt to ignore the contrast in magnetic permeabilities of the cover and target bodies may lead to erroneous conclusions.

In this section, it is assumed that the outer shell with the K_1 parameter (with different μ_1 values but σ_1 D_1 values constant) having values 10^{-2} to 4×10^{-2}, covers a nonmagnetic but

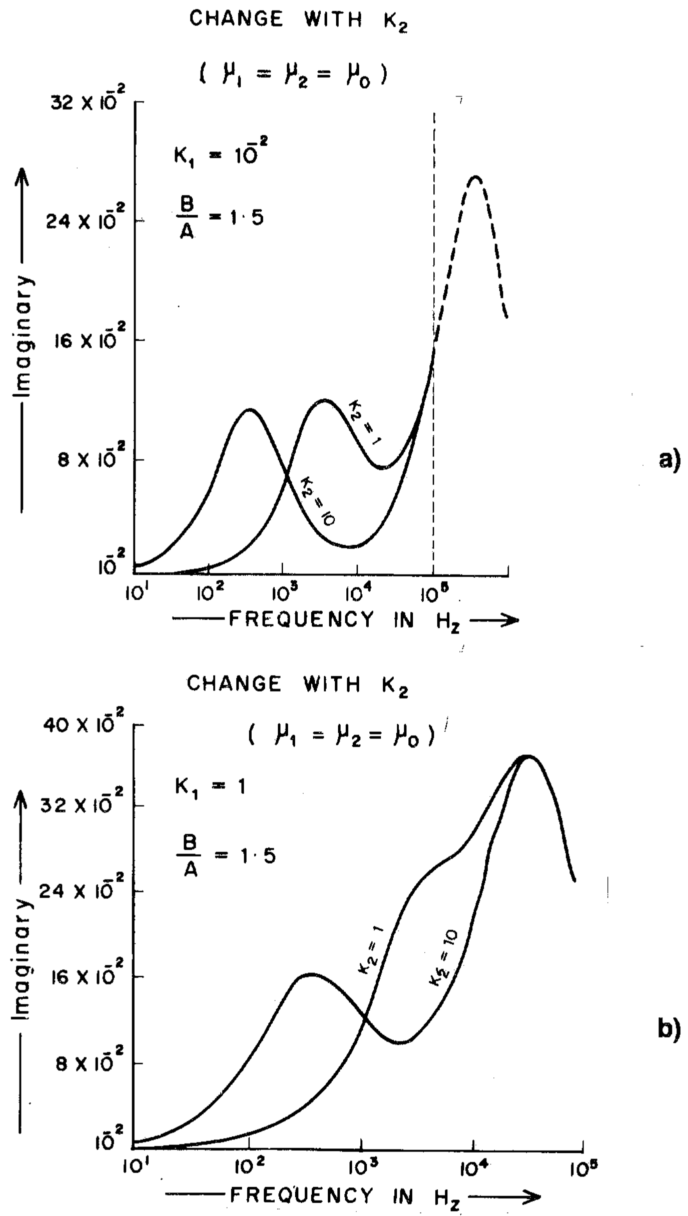

Fig. 6 — Variation of the imaginary component of the reflection factor against frequency of the E.M. waves for different values of K_2 : a) $K_1 = 10^{-2}$; b) $K_1 = 1$.
$\sigma_2 D_2 = 1, 10, \mu_1 = \mu_2 = \mu_0$; $B/A = 1 \cdot 5$.

highly conducting target ($K_2 = \sigma_2 D_2 = 10$ and $\mu_2 = \mu_0$).

(i) In Fig. 5a, $\sigma_1 D_1 = 10^{-2}$ (and hence $K_1 = \sigma_1 D_1 \mu_1 = 10^{-2}$) represents the case of nonpermeable cover while $K_1 = \sigma_1 D_1 \mu_1 = 4 \times 10^{-2}$ represents the case $\mu_1 = 4 \mu_0$ (i.e., a magnetic cover). In both the above mentioned cases one finds that the peak values due to changes in parameter K_1 shift towards the lower frequency band with a change from nonmagnetic to magnetic cover. In other words, the magnetic shell covering a conducting target reflects earlier in the frequency variation pattern in comparison to the corresponding nonmagnetic outer shell.

(ii) A noticeable change in the variation patterns of the imaginary component is observed in changing the value of parameter K_1 from 10^{-2} (Fig. 5a) to 10^{-1} (Fig. 5b) (in both cases,

$K_2=10$), showing that a change in the conductivity* thickness parameter of the outer shell (i.e., K_1) affects the resolution of the outer shell, significantly, in multifrequency measurements. And this change is further enchanced in the case of magnetically permeable cover (i.e., 4×10^{-2} in Fig. 5a to 4×10^{-1} in Fig. 5b).

(iii) It is evident from Fig. 5b that a nonmagnetic ($\mu_1=\mu_0$) outer shell with low value of K_1 ($\sigma_1 D_1=10^{-1}$) covering a target (with $K_2=\sigma_2 D_2=10$ and $\mu_2=\mu_0$) resolves better than the corresponding case of a magnetically permeable ($\mu_1=4\mu_0$) cover.

(iv) In Figs. 3a and 5a (the change in K_1 is due to a change in $\sigma_1 D_1$ in Fig. 3a, while in Fig. 5a it is due to a change in magnetic permeability μ_1) one finds several similarities in the variation patterns of the imaginary component viz:

In both cases (whether one changes $\sigma_1 D_1$, keeping $\mu_1=\mu_0$, or μ_1, keeping $\sigma_1 D_1$ constant):

(a) cover and target are well resolved provided $K_1<K_2$;

(b) the effects of changing K_1 are seen in the high frequency bands, and

(c) the magnitude of the peak value due to the outer shell is more than the peak value due to the inner shell (provided $K_1<K_2$);

(v) on increasing the value of K_1 in Figs. 3a (from $K_1=10^{-3}$ to $K_1=10^{-2}$), the trend of the imaginary component shifts towards the higher frequency side. Similarly to Fig. 5a, on changing the value of K_1 from 10^{-2} to 4×10^{-2}, the trend shifts towards the lower frequency side.

Change in K_2 parameter

Change in K_2 (when $\mu_2=\mu_0$)

Here it is assumed that both cover and target bodies are magnetically impermeable. Hence, change in the value of K_2 ($=\sigma_2 \mu_2 D_2$) represents a change in $\sigma_2 D_2$ (product of conductivity and thickness of inner shell).

In Figs. 6a and 6b, the imaginary component of the reflection factor is plotted against frequency of the EM waves. The outer shell is also assumed to be nonmagnetic (i.e. $K_1=\sigma_1 D_1 \mu_1$, the term K_1 represents change in $\sigma_1 D_1$ only as $\mu_1=\mu_0$).

In Fig. 6a, $\frac{K_2}{K_1} \simeq 10^2$ (i.e. $\sigma_2 D_2/\sigma_1 D_1 \simeq 10^2$).

Such situations are usually obtained in areas where low resistive sulphide formations ($\sigma_2 \simeq 1 \rightarrow 10^{-1}$ *mhos/m*) are covered by high resistive younger volcanics ($\sigma_1 \simeq 10^{-2}$ *mhos/m*) (Strangway 1973).

The following points emerge:

(i) in Fig. 6a, similarly to 3a, both the shells are well resolved (mainly because $K_1<K_2$) in different frequency zones.

(ii) Change in the value of K_2 is reflected only in the lower frequency band ($F<10^4$ *Hz*).

(iii) On comparing the similar results shown in Fig. 3a (where a change in K_1 ($=\sigma_1 D_1$) is demonstrated), one finds that unlike Fig. 3a, the increase in the value of K_2 shifts the peak value towards the lower frequency side. When $K_2>>K_1$, the major part of the EM energy transmits through the outer shell and the amount of induced current generated increases significantly. Thus, the contribution from the inner shell or target body reflects distinctly. When the ratio of K_2/K_1 decreases, the resolution of the inner shell also decreases significantly.

Change with K_2 (when $\mu_2 \geqslant \mu_0$)

In this case, the target body is assumed to be magnetically permeable ($\sigma_2 D_2$ term is kept constant). Hence, any change in K_2 represents a change in the μ_2 value only (in Fig. 7a:

(a) $K_2=30$, $=\sigma_2 \mu_2 D_2$; $\sigma_2 D_2=10$; Hence $\mu_2=3\mu_0$

and

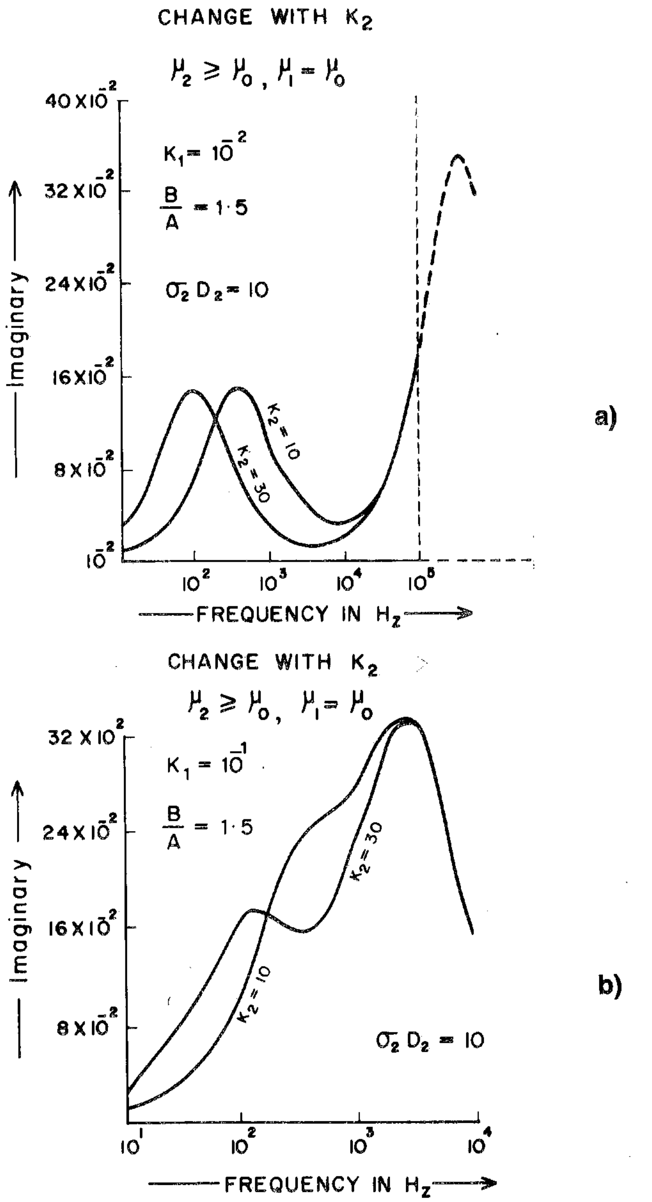

Fig. 7 — Variation of imaginary component of the reflection factor against frequency of the E.M. waves for different values of K_2 : a) $K_1=10^{-2}$; b) $K_1=10^{-1}$.
$\mu_2=\mu_0$, $3\ \mu_0$, $\sigma_2\ D_2=10$; $\mu_1=\mu_0$, $B/A=1\cdot5$.

(b) $K_2=10$, $\sigma_2\ D_2=10$, $\mu_2=\mu_0$).

The following points may be noted:

(i) Similarly to Fig. 6a (change due to $\sigma_2\ D_2$ only), in Fig. 7a also, both the shells are well resolved in multifrequency measurements (In Fig. 7a, a change in K_2 means a change in μ_2 only).

(ii) The effect of changing K_2 is seen only in the lower frequency range.

(iii) Similarly to Fig. 5a, the magnetic shell ($K_2=30$, $\mu_2=3\ \mu_0$) reflects earlier in the frequency scale than the corresponding nonmagnetic shell $(K_2=10,\ \mu_2=\mu_0)$.

(iv) On changing the value of K_1 from 10^{-2} (Fig. 7a) to 10^{-1} (in Fig. 7b), the variation pattern changes drastically i.e., variation or $\sigma_1\ D_1$ parameter in the outer shell affects the variation pattern due to the inner shell $(F<10^4\ Hz)$ also.

v) In Fig. 7b, two cases are dealt with:

(a) $K_1=10^{-1},\ K_2=10=\sigma_2\ \mu_2\ D_2\ ,\ \sigma_2\ \mu_2=10\ ;\ \sigma_1\ D_1=10^{-1}$, (i.e. $\mu_1=\mu_0=\mu_2$),

(b) $K_1=10^{-1},\ K_2=30=\sigma_2\ \mu_2\ D_2\ ;\ \mu_1=\mu_0\ ,\ \sigma_2\ D_2=10,\ \mu_2=3\ \mu_0$

Comparison of the trends in the above two cases reveals that a magnetic target (case b) is resolved better than the nonmagnetic one (case i) (Here, other physical and geometrical parameters are kept constant).

CONCLUSIONS

In a number of situations in applied geophysics, one is interested in the EM response of a target body covered by overlying formations. Such a cover-target composite system is relevant under the following circumstances:

(i) Searching for underlying mineral deposits covered by less conducting weathered formations,

(ii) Delineating sulphur deposits buried in oceans.

It is also well recognised that some massive sulphide deposits are surrounded by a halo of less conducting material. Sometimes the thickness of the surrounding medium is sufficient to mask the response due to the target. Hence, in this paper an attempt is made to model such situations where the cover-target composite system can be resolved in multifrequency measurements. Change in the variation pattern due to another relevant parameter, usually ignored in geoelectromagnetic measurements - the magnetic permeability - has also been accounted for in this paper. It is shown through numerical results that ignoring magnetic permeability in delineating ferromagnetic bodies may cause erroneous results.

Our study may be helpful in describing discrimination windows (i.e. frequency zones) for favourable or unfavourable conditions of transmission of EM energy from the overlying formations, and in separating out the response due to the target body in multifrequency EM measurements.

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